Philosophia Britannica:

OR,

A NEW and COMPREHENSIVE

SYSTEM

OF THE

NEWTONIAN

PHILOSOPHY, ASTRONOMY,

AND

GEOGRAPHY,

In a Course of Twelve Lectures,

With NOTES;

Containing the PHYSICAL, MECHANICAL, GEOMETRICAL, and EXPERIMENTAL PROOFS and ILLUSTRATIONS of all the Principal Propositions in every Branch of NATURAL SCIENCE:

ALSO

A particular Account of the INVENTION, STRUCTURE, IMPROVEMENT and USES of all the considerable INSTRUMENTS, ENGINES, and MACHINES;

With NEW CALCULATIONS relating to their NATURE, POWER, AND OPERATION.

The Whole collected and methodized from all the principal Authors, and public Memoirs;

And embellished with EIGHTY-ONE COPPER-PLATES.

By B. MARTIN.

THE THIRD EDITION.

IN THREE VOLUMES.

VOLUME I.

LONDON:

Printed for W. STRAHAN; J. & F. RIVINGTON; W. JOHNSTON; HAWES and Co. T. CARNAN and F. NEWBERY, jun. B. COLLINS, in Salisbury; W. FREDERICK, in Bath; And sold by the AUTHOR, at his House, in Fleet-street.

MDCCLXXI.

K

Philosophia Britannica:

OR,

A NEW and COMPREHENSIVE

SYSTEM

OF THE

NEWTONIAN

PHILOSOPHY, ASTRONOMY,

AND

GEOGRAPHY,

In a Course of Twelve Lectures,

With NOTES:

Containing the PHYSICAL, MECHANICAL, GEOMETRICAL, and EXPERIMENTAL PROOFS and ILLUSTRATIONS of all the Principal Propositions in every Branch of

NATURAL SCIENCE:

ALSO

A particular Account of the INVENTION, STRUCTURE, IMPROVEMENT and USES of all the considerable INSTRUMENTS, ENGINES, and MACHINES;

With NEW CALCULATIONS relating to their

NATURE, POWER, AND OPERATION.

The Whole collected and methodized from all the principal Authors, and public Memoirs;

And embellished with EIGHTY-ONE COPPER-PLATES.

By B. MARTIN.

THE THIRD EDITION.

IN THREE VOLUMES.

VOLUME [illegible]

LONDON:

Printed for W. STRAHAN; J. & F. RIVINGTON; W. JOHNSTON; HAWES and Co. T. CARNAN and F. NEWBERY, jun. B. COLLINS, in Salisbury; W. FREDERICK, in Bath; And sold by the AUTHOR, at his House, in Fleet-street.

MDCCLXXI.

PREFACE.

HAVING read and confidered the Defign of the feveral Books hitherto publifhed for the Explanation of the NEWTONIAN PHILOSOPHY, under the Titles of *Commentaries, Courfes, Effays, Elements, Syftems*, &c. I obferved not one of them all could be juftly efteemed a TRUE SYSTEM, or COMPLEAT BODY of this Science.

I WOULD not here be underftood to have any Defign of depreciating the Labours of thofe Gentlemen who have gone before me on this Subject; I only would intimate that their Views and Defigns were very different from mine, which is to prefent the World with a PHYSICO-MATHEMATICAL TREASURY of NATURAL KNOWLEDGE, upon the Principles of the *Newtonian Philofophy*; which I have been chiefly enabled to effect from the Abundance of Materials fupplied by the Authors of the Treatifes in the following Catalogue, each of which is excellently good in his Way.

So that if the Cafe be truly ftated, my Book may be looked upon as a *General Compendium* or *Abridgment* of all theirs. For as

 they

they have all lain before me, I have constantly collated them upon every particular Subject, as I proceeded through the Work, and selected the best of every Thing I could find for my own Composition.

Hence it is, that where some of them have been *very prolix*, my Labour has been to abridge or reduce them to a Compendium of Principles and Essential Propositions only. Thus *Keill* on the Subject of *Motion* and *Astronomy*, *Muschenbroek* on *Cohesion*, Sir *Isaac Newton* on *Optics*, &c. have all been treated.

As Redundancy on the one Hand has been avoided, so their Deficiencies on the other have been supplied. Thus *Keill*'s Introduction to *Philosophy* relates to the Mechanical Part only. Professor *Muschenbroek* has wholly omitted the grand Article of *Astronomy*, and the *Physical Causes* of the *Celestial Motions* and *Phænomena*. Dr. *Desaguliers* has thought fit to omit the most essential Science of *Optics*, and is almost wholly taken up on Mechanical and Hydrostatical Subjects. Yea, the best and most general Writer *S'Gravesande*, in his last large Edition of his *Elements*, falls far short of a general System or Body of *Philosophy*, in various important Subjects, as will easily appear, by comparing his Table of Contents with mine.

I TAKE no notice of leſſer Tracts, ſince few of them pretend to any thing Syſtematically; and they who do, propoſe it with reſtricted Views, either to the Theory only, or elſe to ſome principal Parts or Branches of the Science. Thus *Worſter*'s *Principles of Philoſophy* amount to little more than a Rehearſal of *Theſes*, with ſcarce any Demonſtrations throughout. *Helſham*'s *Lectures* are admirably good as far as they go. And the Lectures, lately publiſhed by the *Abbé Nollet*, are but a Part of his Work, and are indeed excellent Philoſophical Sermons or Declamations, and ſtrictly in the *French Style and Taſte**.

BUT what others have done is not ſo much the Deſign of this Preface to relate, as what I have done Myſelf in this Treatiſe, which I ſhall ſpecify under the following Heads.

FIRST, I have endeavoured to compile a *Compleat Syſtem of Philoſophy*, from the great Fund of Materials which the Writings and Inventions of the Moderns have ſupplied me with. Under this Head I have had regard only to the Choice and Propriety of the Subjects; and their Order and Diſpoſition, or

* The Lord *Roſcommon* obſerves,
The weighty *Bullion* of *one Sterling Line*,
Drawn to *French Wire*, would thro' *whole Pages ſhine*.

due Arrangement in the Body of the Work. Hence it is that I have admitted nothing that is not of a Physical Nature, and omitted nothing that is so, as far as the Limits of my prescribed Form would admit me to go. Nor are these so strait as not to be comprehensive of every material Branch of natural Science, or considerable Phænomenon, as will easily appear, from a View of the *Contents* and *Index*.

SECONDLY, What has been wrote upon each Subject in every Part of the Science has been carefully examined, and such an Extract made as was judged to give a sufficient Idea or Notion thereof to the Reader. And where I thought a more extensive View could be of Service, I have constantly referred to the best Authors on the Subject.

THIRDLY, If from the whole Account, given by others, of any particular Branch of Science, I could not collect what sufficed for an entire Sketch, I have endeavoured to supply those Deficiencies, in order to render every Part compleat, and save the Reader the Trouble of turning to several Authors for that Purpose. Remarkable Instances of this may be found under the Articles of *Wheel-Carriages*, *Hydraulics*, *Optics*, &c.

FOURTHLY, If the Method of representing or demonstrating any Subject or Proposition by others has not been judged so clear,

concise or natural as was necessary, I have proposed it in my own way, to answer this important End: For the greatest Excellency of a Demonstration is Conciseness and Perspicuity. The Want of this is a great Discouragement to Physical Enquiries, as I am but too well convinced of by my own Experience, and that of many others.

FIFTHLY, According to the different Nature of the Subject, a various Process has been made use of to explain or illustrate the same. Thus some Things require a *Physical*, others a *Mechanical*, some a *Geometrical*, and some an *Experimental Proof*, which have been accordingly applied, sometimes singly, sometimes two or more, and sometimes altogether. Thus from *Phænomena* we institute a Ratiocination concerning the Materiality, the Velocity, different Magnitudes, Colours, and other Affections of the Particles of Light. Then the Laws of Reflection and Refraction are explained by *Mechanical Principles* relating to the Motion and Action of Bodies one upon another. Again, The *Principles of Geometry* are call'd in to explain and account for the Effects of Mirrours, Lenses, Optic Instruments, the Rainbow, &c. And, Lastly, those various Phænomena are represented, and the Manner how they happen explained by *Experiments*.

Sixthly, I have all along applied the *Newtonian Geometry*, that is, the *Fluxionary Calculus*, for the Demonſtrations, which I think is with the greateſt Propriety and Juſtneſs in Phyſical Matters, becauſe the Principles of this Geometry are in themſelves ſtrictly Philoſophical, as I have ſhewn in its proper Place *. Beſides, the Proceſs is not only rendered by this means much more congruous and natural, but more evident and conciſe than by the common Methods of Demonſtration it could be. Of this we have a ſignal Inſtance in the Calculation of the Angle, which the Incident Ray makes with the laſt refracted Ray in a Drop of Water, as will appear by comparing what is here contained in *Scholium* to *Annot.* CXXIV. with what you find on this Head in other Authors.

Seventhly, As the Bounds of this Science have been of late Years vaſtly extended, ſo I have been very careful to treaſure up every uſeful Invention, and given a ſhort hiſtorical Account of the ſame, and the Names of the Authors. If in any of them I found an Improvement could be made, I have done it, as in the *Air-Pump*,

* See Vol. I. of *Mathematical* Institutions, juſt now publiſhed in the General Magazine of Arts and Sciences.

the *Microſcope*, &c. or hinted it to the Reader, as in Caſe of the *Air* or *Steam-Gun*.

EIGHTHLY, I find by common Experience nothing tends more to the Conception and enlivening an Idea, and fixing it in the Memory, than a good Delineation of the Figure or Form thereof. Hence it is that the Reader will here find a greater Number of Copper-Plate Figures than in any other Treatiſe of this Kind, of its Size. And if I tell him they were all done by the ingenious Mr. BOWEN (GEOGRAPHER to his MAJESTY) it may likewiſe prepoſſeſs him in Favour of their Goodneſs; which though an indirect and collateral, is yet no inconſiderable Argument to recommend a Book of this Kind.

NINTHLY, I have delivered the Whole by way of LECTURES, and ANNOTATIONS thereto, (a Method of late Years the moſt uſed and approved of) not only becauſe the Reader is by this Means led through a SYSTEM of PHILOSOPHY, without being confined to the Irkſomeneſs of a preciſe and regular Path, and a ſimultaneous Proſpect or View of the Whole at once; but alſo becauſe that vaſt Variety of Things, which here preſent themſelves, may have the Pleaſure that attends Variety heightened and encreaſed by emerging freſh to his View as he paſſes along, and regaling him with ſomething

ſtill novel and unexpected. *Annotations* ſeem to anſwer the End of *Scenes* in a Play; they preſent the Whole moſt agreeably in Parts, which thus more immediately affects us, and gives us the greateſt Pleaſure and Entertainment.

THUS much for the Oeconomy of the Work; and as it is manifeſtly calculated for a general Good, ſo I believe none will find fault with the Deſign. As to the Manner of executing it, I have done it as well as I could; and if any one thinks me deficient in any thing, let him qualify his Criticiſm by conſidering that I pretend not to Infallibility, but only where I give a plain and clear Demonſtration. In this, if I am not underſtood ſometimes, I am not perhaps to be blamed; for I'll appeal to Envy itſelf, if the Manner of the Demonſtration, where moſt difficult, be not as plain and as obvious as the Nature of the Thing will admit. And here it will be proper to obſerve, that although I have made uſe of *Geometry*, *Algebra*, and *Fluxions*, to inveſtigate the Truth of all the principal Propoſitions in Philoſophy, yet as I cannot ſuppoſe Readers in general qualified to underſtand it in that Manner, I have been careful to expreſs the Reſult of every Proceſs in plain *Engliſh*; ſo that every one may know what is the Truth,

though he cannot be so properly said to understand it. For Instances of what I now speak of, see the several Articles of *Annotat.* XXVII, XXXIV, and XLVIII.

I do not know that I have advanced any thing here, which I have not one Way or another proved to be true; however, I desire that nothing but what is demonstrated may be accounted to deserve the Title of NEWTONIAN PHILOSOPHY. Our Great Author seemed quite averse to *Hypotheses*; and though he indulged them to others under proper Restrictions, yet did himself never make use of this *fallacious Way of Reasoning*, though he has been unjustly charged therewith by some, who, by *National Instinct*, scorn to subscribe to any System but of their own making. Hence it is we often hear them babbling about *Newton*'s *Hypotheses*, &c. and making unsufferable Comparisons between him and *Descartes* and *Leibnitz*. One would think they who knew but little of Philosophy would yet see no Reason for a Parallel between a System of Philosophy that is founded on nothing but *Demonstration*, and is every Way comportant with *Phænomena*, and one that is wholly *hypothetical*, and, instead of accounting for, runs counter to every thing that appears! I think it is little Glory, (not to say *Vain-Glory*) for a

Man

Man to boaſt that his Philoſophy *is not that of Newton* *, till he has firſt ſatisfied the World his Abilities and Diſcernment into the Nature of Things are greater than thoſe of Sir *Iſaac*. This I am well aſſured of, that the greater Share of *Mathematical Knowledge* any Man has, the more readily he embraces Sir *Iſaac*'s Principles; and I ſcarce ever found any that oppugn'd Sir *Iſaac*, but who were either groſsly ignorant of what they deſpiſed, or were actuated by ſome particular Views. In ſhort, nothing but *Pride*, *Prejudice*, and *Ignorance*, will ever ſtir up Enemies to the *Newtonian Philoſophy*.

THEY who depreciate this Philoſophy by the Term *Hypotheſis*, ſeem to me not to have read (at leaſt not conſidered) what Sir *Iſaac* has wrote, when he tells us, "That in Experi-
" mental Philoſophy, Hypotheſes are not to
" be regarded.—What I call ATTRACTION
" may be performed by IMPULSE, or by
" ſome OTHER MEANS unknown to me.
" I uſe that Word to ſignify *only in general*
" ANY FORCE *by which Bodies tend towards*
" *one another*, whatſoever be the Cauſe †."
After our illuſtrious Author had thus ex-
preſsly

* See the Preface of Abbot NOLLET's *Leçons de Phyſique*.

† OPTICS, *Qu*. 31. See alſo the Definitions of his *Principia*.

presly disclaimed all Hypotheses, and so fully explained his Sentiments about the Power which he calls *Attraction*, is it not very wonderful to hear a Set of People *charging him with founding his Philosophy on the Hypothesis of Gravity?* Will these very sagacious Gentlemen allow *any Power at all* is concerned in the *Tendency* of one Body to another? If they do, is it not because it is impossible to consider Bodies acting at a Distance upon each other without? If so, is not the Existence of such a Power a Reality in Nature? And can such a Force then be called an *Hypothesis?* Surely not. Again, their accusing Sir *Isaac* for accounting for Things *by Attraction*, is meer cavilling about Words, and begging him guilty; he declares he knows not the Manner in which the Force acts, whether by *Attraction* or *Impulsion*, or otherwise; but as some Name must be given it, why may not he call it by that of *Attraction* or *Gravity*, as well as by any other? Not that this great Man did even this without a Reason; for 'tis evident from a thousand Experiments and Phænomena, that this is much more like the Effect of one Body's *drawing* than *impelling* another towards it.

But so far is Sir *Isaac* from *supposing* that Bodies tend to each other by ATTRACTION, that

that he once condescended to form an *Hypothesis* to shew that it might be done by IMPULSION; and that of a very subtle Elastic Spirit or Æther. But as he was not able to prove the Existence of any such Thing, he seemed not at all delighted with the Thought, nor ever laid any Stress upon it; and accordingly we have not admitted it as a Principle of the *Newtonian* Philosophy. Besides, suppose this could be proved, we should still be at a loss to account for the *material Cause* of this very great Elastic Power between the Particles of this Æther; we should indeed advance three or four Steps higher on the infinite Ladder, but should be equally *nonplus'd* with the mechanical Cause of *Elasticity* then, as we are at present with that of *Gravity*. But after all, 'tis evident by Experiments that this Power which actuates Bodies, whatever it be, is not either *Attractive* or *Repulsive* solely; but it produces both the different Effects in *different Circumstances of Distances*. Thus the Loadstone at one Distance moves the Needle *towards itself*, at another Distance it causes it *to recede* or move *the contrary Way*. This Power in Iron is attractive, if touched upon the Stone in one Direction; but if in a contrary it becomes repulsive. The same ambiguous Properties of this Power obtain in *Electricity*, and undoubtedly

doubtedly in all other Sorts of Attractions. Therefore, before we set ourselves about finding out a *Power*, *Spirit*, or *Æther*, that shall move Bodies either by *Attraction* or *Repulsion*, it is in the first Place necessary to find out an Agent that shall do both, for such a Principle is at present the *Desideratum* to our further Advances in the Research of natural Causes.

It is sometimes surprizing, to observe, how very fond People seem of this *subtle Æther*; one accounts for the *Cause of Gravity* thereby, another for *Muscular Motion*, a third for *Electricity*, a fourth from hence derives *Animal Spirits*, a fifth *Elementary Fire:* And, in short, we want nothing but the *Proof of such an Æther* to account for almost every Thing by it. But is it not a preposterous Proceeding to account for any Thing by a Principle in itself unaccountable, and utterly unknown?

However, we are arrived at great Dexterity since Sir *Isaac*'s Time; for we can now almost prove the Existence of this *Æther* by the Phænomena of Electricity; and then we find it very easy to prove that Electricity is nothing but this very Æther condensed and made to shine. But, I believe, when we enquire into the Nature and Properties of this Æther and Electricity, we shall find them

them ſo very different and diſſimilar, that we cannot eaſily conceive how they ſhould thus mutually prove each other. For according to Sir *Iſaac*, this Æther is the moſt ſubtle of all Bodies, and conſiſts of Particles (of courſe) very much ſmaller than thoſe of Light, and which have a much greater Velocity; this Æther therefore pervades with the greateſt Freedom all Bodies, actuates their Parts, *&c.* But on the other hand, we find by Experiments that the Fire of Electricity is a very groſs Subſtance; for it daſhes againſt the Surface of Glaſs like ſo much Water, and does not enter the Pores, as Light does, that is ſo much groſſer than Æther. This is ſtill made more plain and demonſtrable by electrifying a Wire in an exhauſted Receiver, where the Electricity iſſues out in much greater Abundance, and in divers Jets, ſtriking againſt the Sides of the Glaſs, and appearing to run down by the Sides in all reſpects like *Liquid Phoſphorus*; and is no ways reflected or refracted like common Light, or even like the Light of a Candle: For if a Candle be viewed through a Priſm, the Light coming from every Part thereof is refracted, and the whole Flame is coloured and diſtorted; but it is quite otherwiſe with the *Flame of Electrical Fire*, for I often made that much

larger

larger than the Flame of a Candle, and viewed it through the Prifm, but no Colours were thereby produced, no different Form of Flame, but every way the fame as it appears to the naked Eye. Indeed the denfer Part, juft upon the Tip of the Iron whence the Blaze diverged, appeared a little tinged with Colours; which I take to proceed from fome common *Solar Light* intermixed with that of Electricity. From hence (and for many other Reafons I could name) I fee no Caufe to believe that the Matter of Electricity is any thing like the Idea we ought to have of the *Spiritus fubtiliffimus* of Sir *Ifaac*. Nor indeed can it be the fame with the Matter of common Light, which is differently reflexible and refrangible; whereas this appears not to be fo, but is a particular Sort of Light, and nearly the fame as *Phofphorus*, which I could never obferve to exhibit the leaft Appearance of Colours through the Prifm. The Smell alfo of *Electrical Fire* is fo very much like that of *Phofphorus*, that one may be eafily induced to believe a great Part of the Compofition of both is the fame.

As to the Doctrine of ELEMENTS and *Elementary Fire*, we leave that to others, and expatiate only on the noble *Theory of Light and Fire* left us by our incomparable

Author, who justly considers them as they are in Nature, illustrates all their Properties and Qualities, and applies them to the Explication of a great Number of the most interesting and important Phænomena of Nature. The *Newtonian* Doctrine of *Elements* is of the *Singular Number: One Sort of Matter only* makes the Substance of all the infinite Variety of Bodies we behold, Thus our Author speaks: "It seems probable to me, That God in the Beginning "formed Matter in solid, massy, hard, impenetrable, moveable Particles, of such "Sizes and Figures, and with such other "Properties, and in such Proportion to "Space, as most conduced to the End for "which he formed them.—Now by Virtue of certain active Principles or Powers, "all material Things seem to have been "composed of the above-mentioned Particles, variously associated in the first "Creation, by the Counsel of an intelligent "Agent."

To conclude: It was an Observation worthy of so great a Genius as Mr. *Cotes*, that the *Newtonian Philosophy* was our *(mutinissimum præsidium adversus Atheorum impetus)* best Defence against the Efforts and Assaults of Atheists.—That herein we more nearly view the Majesty of Nature, and enjoy it

in

in the most grateful Contemplation; at the same time we are excited more intensely to worship and revere the CREATOR and LORD of all Things. He ought to be reputed *blind*, who cannot immediately behold, in the best and wisest Structures of Things, the infinite Goodness and Wisdom of the Omnipotent Architect; and *mad*, who will not confess it.

A
CATALOGUE
OF THE
Principal BOOKS

Made use of in compiling the following Work.

1. PHILOS. *Nat. Principia Mathematica*, Auctore I. NEWTONO, *Eq. Aurato* Ed. Tertia.
2. *Eadem Principia illustrata Perpetuis Commentariis, communi Studio* PP. THOM. LE SEUR & FRAN JACQUIER. *Genevæ*, 1739.
3. *A View of Sir* I. NEWTON's *Philosophy*, by *Dr.* PEMBERTON. *Lond.* 1728.
4. *Physices Elementa Mathematica, Experimentis confirmata.* Auctore JACOBO s'GRAVESANDE. Tomis Duobus, Edit. Tertia. *Leidæ*, 1742.
5. *Essai de Physique*, par M. P. VAN MUSCHENBROEK, Tome i. & ii. *Leyden*, 1739.
6. *A Course of Experimental Philosophy*, by Dr. DESAGULIERS, 2 Vols. *London*, 1734 and 1744.
7. *Phoronomia, sive De Viribus & Motibus Corporum Libri Duo.* Auctore JACOBO HERMANNO. *Amstelædami*, 1716.

8. *The Philosophical Transactions abridged*, by Mr. LOWTHORP, JONES, MARTYN, EAMES, *&c.* 7 Vols.
9. *A Compleat System of Optics*, by Dr. SMITH, 2 Vols. *Cambridge*, 1738.
10. *Lectiones Opticæ.* Auctore I. NEWTONO, *Eq; Aurato.*
11. *Optics*, or a *Treatise of Reflections, Refractions, Inflections, and Colours of Light.* By Sir I. NEWTON, *Knt. Lond.* 1718.
12. *An Essay on Distinct and Indistinct Vision.* By Dr. JURIN.
13. *Architecture Hydraulique*, par M. BELIDOR, Tom. i. & ii. A *Paris.*
14. *Hydrostatical and Pneumatical Lectures*, by Mr. COTES, with Notes by Dr. SMITH. *London*, 1738.
15. *The Motion of Fluids Natural and Artificial*, particularly *Air* and *Water*. By Mr. CLARE. *London*, 1737.
16. *Astronomy*, in five Books. By Dr. LONG. *Cambridge*, 1742.
17. *An Introduction to the true Astronomy*, By Dr. JOHN KEILL. *London*, 1721.
18. *Astronomical Lectures.* By Mr. W. WHISTON. *London*, 1728.
19. *The Chronology of Ancient Kingdom amended.* By Sir I. NEWTON. *London*, 1728.

20. *The Elements of Algebra*, in Ten Books. By Dr. SAUNDERSON. *Cambridge*, 1740.

21. *Sir* I. NEWTON'*s Two Treatises of the Quadrature of Curves and Analysis, by Equations of an infinite Number of Terms explained.* By Mr. J. STEWART. *London*, 1745.

22. ROHAULT'*s Physics*, with Dr CLARKE's Annotations. *London*, 1710.

23. *A Compendious System of Natural Philosophy, with Notes.* By J. ROWNING, M. A. *Cambridge*, 1735.

24. *A System of the Conic-Sections, with the Doctrine of Fluxions.* By JOHN MULLER. *London*, 1736.

25. *A Treatise of Fluxions*, in Two Books. By COL. MACLAURIN. *Edinburgh*, 1742.

26. *A Treatise of Fluxions*, by the Marquis DE L'HOSPITAL, *with Additions* by Mr. STONE. *London*, 1730.

27. *A New Treatise of Fluxions.* By Mr. THOMAS SIMPSON. 2d Edition, *London*.

28. *An Introduction to Natural Philosophy.* By Dr. JOHN KEILL. *London* 1720.

29. *A Compendious and Methodical Account of the Principles of Natural Philosophy.* By Mr. BENJAMIN WORSTER. *London*, 1730.

30. *A Course of Lectures in Natural Philosophy.* By Dr. HELSHAM, with Additions by Dr. ROBINSON. *London*, 1739.

31. *Philosophiæ Mathematicæ Newtonianæ illustratæ Tomi Duo.* A GEOR. PETRO DOMCKIO. *Londini*, 1730.

32. *Commercium Epistolicum de varia re Mathematica*, *Londini*. 1725.

33. *The Figure of the Earth determined from Observations at the Polar Circle.* By M. DE MAUPERTUIS, *&c.* *London*, 1738.

34. *Sidereus Nuncius.* A GALILEO. *Londini*, 1653.

35. *Vegetable Statics.* By Dr. HALES. *London*, 1738.

36. *Hæmestatics.* By the same Author. *London*, 1733.

37. *The Elements of Astronomy, Physical and Geometrical.* By Dr. GREGORY, 2 Vols. *London*, 1715.

38. *An Institution of Fluxions.* By Mr. DITTON. *London.*

39. *A Synopsis of the Astronomy of Comets.* By Dr. HALLEY.

40. *Miscellanea Curiosa.* Three Vols. 8vo. *London.*

41. *Institutiones Medicæ.* Ab HERM. BOERHAAVE. *Lugd. Batavorum*, 1720.

42. *A Course of Lectures in Chemistry.* By Dr. SHAW.

A

TABLE OF CONTENTS.

LECTURE I.

CONTENTS.

CONTENTS.

ELECTRICITY.

MAGNETISM.

CONTENTS.

LECTURE II.

MECHANICS.

ATTRACTION *of* GRAVITATION.

MOTION *and* REST.

Of PENDULUMS.

DOCTRINE *of* PROJECTILES.

3

CONTENTS.

CIRCULAR MOTION.

CONTENTS.

LECTURE III.

MECHANICS.

MECHANIC POWERS.

CONTENTS.

WHEEL CARRIAGES.

LECTURE IV.

HYDROSTATICS.

LECTURE

LECTURE I.

Of EXPERIMENTAL PHILOSOPHY *in general. Of the* NEWTONIAN METHOD *and* RULES *of Philosophizing. Of* MATTER *in general, and its essential Properties. Of the* CHEMICAL ELEMENTS *or* PRINCIPLES *of Natural Bodies. Of* ATTRACTION *and* REPULSION. *The* Attraction *of* COHESION. *Of the* COHESION *of various Sorts of Bodies. The Phænomena of* CAPILLARY TUBES. *Of a* VACUUM. *The* Rationale *of various Processes of* CHEMISTRY, *and other Arts. Of the principal Properties of* ELECTRICAL Attraction *and* Repulsion. *Of* MAGNETISM *and its Laws; of the various Properties of the* MAGNET *in relation to the Needle; its* VARIATION *and* INCLINATION.

THE Business of Experimental Philosophy (the Subject of this Course of Lectures) is to enquire into and investigate the Reason and Causes of the various Appearances (or *Phænomena)* of Nature; and to make the Truth or Probability thereof obvious and evident to the

Senſes, by *plain, undeniable, and adequate Experiments*, repreſenting the ſeveral Parts of the grand Machinery and Agency of Nature.

IN our Enquiries into Nature we are to be conducted by thoſe Rules and Maxims which are found to be genuine, and conſonant to a juſt Method of Phyſical Reaſoning; and theſe Rules of Philoſophizing are, by the greateſt Maſter in this Science, (the incomparable Sir *Iſaac Newton)* reckon'd Four; which I ſhall give, from his *Principia*, as follows:

RULE I. *More Cauſes of natural Things are not to be admitted, than are both true and ſufficient to explain the Phænomena.* For Nature does nothing in vain, but is ſimple, and delights not in ſuperfluous Cauſes of Things.

RULE II. *And therefore of natural Effects of the ſame Kind the ſame Cauſes are to be aſſigned as far as it can be done.* As of Reſpiration in Man and Beaſts: Of the Deſcent of Stones in *Europe* and *America*: Of Light in a culinary Fire and in the Sun: And of the Reflection of Light in the Earth and in the Planets.

RULE III. *The Qualities of natural Bodies which cannot be increaſed or diminiſhed, and*

and agree to all Bodies in which Experiments can be made, are to be reckoned as the Qualities of all Bodies whatſoever. Thus, becauſe Extenſion, Diviſibility, Hardneſs, Impenetrability, Mobility, the *Vis Inertiæ*, and Gravity, are found in all Bodies which fall under our Cognizance or Inſpection, we may juſtly conclude they belong to all Bodies whatſoever; and are therefore to be eſteemed the Original and univerſal Properties of all natural Bodies.

RULE IV. *In Experimental Philoſophy, Propoſitions collected from the Phænomena by Induction are to be deemed (notwithſtanding contrary Hypotheſes) either exactly or very nearly true, till other Phænomena occur by which they may be rendered either more accurate, or liable to Exception.* This ought to be done, leſt Arguments of Induction ſhould be deſtroyed by Hypotheſes.

If according to theſe Rules we take a Survey of the viſible World, and ſtrictly examine the Nature of particular Bodies, we ſhall find Reaſon to conclude, that they all conſiſt of *one and the ſame Sort of Matter or Subſtance*; and that all the Diverſity or Difference we obſerve among them ariſes only from the various Modifications and

different Connection or Adhesion of the same primigenial Particles of Matter (I).

MATTER,

(I) These four Rules of Philosophizing are premised by Sir *Isaac Newton* to his third Book of the *Principia*; and more particularly explained by him in his *Optics*, where he exhibits the Method of proceeding in Philosophy in the following Words:

1. As in Mathematics, so in Natural Philosophy, the Investigation of difficult Things by Way of *Analysis*, ought always to precede the Method of *Composition*. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction (*i. e.* Reasoning from the Analogy of Things by natural Consequence) and admitting no Objections against the Conclusions but what are taken from Experiments, or other certain Truths. And altho' the arguing from Experiments and Observation by Induction be no Demonstration of general Conclusions; yet it is the best Way of arguing which the Nature of Things admits of, and may be looked upon as so much the stronger by how much the Induction is more general. And if no Exception occur from Phænomena, the Conclusion may be pronounced generally. But if at any Time afterwards, any Exceptions shall occur from Experiments, it may then be pronounced with such Exceptions. By this Way of *Analysis* we may proceed from Compounds to Ingredients, and from Motions to the Causes producing them; and in general from Effects to their Causes; and from particular Causes to more general ones, till the Argument ends in the most general. This is the Method of Analysis. And that of *Synthesis* (or Composition) consists in assuming Causes discovered and established as Principles, and by them explaining the *Phænomena* proceeding from them, and proving the Explanations.

2. That Matter or Substance is *one and the same* in all Bodies, and that all the Variety we observe arises from the various Forms and Shapes which it puts on, is, I think, very probable, and may be concluded from a general Observation of the Procedure of Nature in the Generation and Destruction of Bodies. Thus, for Instance,

Water

MATTER, thus variously modified and configurated, constitutes an infinite Variety of

Water rarified by Heat, becomes *Vapour*; great Collections of Vapours form *Clouds*; these condensed descend in Form of *Rain* or *Hail*; Part of this collected on the Earth constitutes *Rivers*; another Part mixing with the Earth enters into the Roots of Plants, and supplies Matter to, and expends itself into various Species of *Vegetables*. In each Vegetable it appears in one Shape in the *Root*, another in the *Stalk*, another in the *Flowers*, another in the *Seeds*, &c. From hence various Bodies proceed; from the Oak, *Houses*, *Ships*, &c. from Hemp and Flax, we have *Thread*; from thence our various Kinds of *Linen*; from thence *Garments*; these degenerate into *Rags*, which receive, from the Mill, the various Forms of *Paper*; hence our *Books*; which by Fire are converted partly into Water, partly into Oil, another Part into Air, a fourth Part into Salt, and a fifth into Earth; which are called the Elements of Bodies, and which mix'd with common Earth, are again resuscitated in various Forms of Bodies.

3. The ELEMENTS, or Principles to which all Bodies are ultimately reduced, are the five above mentioned, *viz.* (1.) WATER, or *Phlegm* (as 'tis call'd) which generally rises and goes off first, as in the Chemical Analysis of a Plant in a Retort by Fire. (2.) AIR, which escapes unseen in great Quantities from all Bodies; and tho' it has not till lately been known to make a Part of natural Bodies in a fix'd State (and therefore never taken Notice of as an Element of natural Bodies) yet that it is so in a very remarkable Degree, (even so far as to make half the Substance of some Bodies) I shall give sufficient Proof when I come to treat of artificial or factitious Air. (3.) OIL, which appears swimming on the Top of the Water. (4.) SALT, which is either *Volatile*, or rises in the Still, as that of Animal Substances; or *Fix'd*, as in Vegetables, and which is extracted by dissolving them in Water from a Lixivium of the Ashes, and afterwards by evaporating the Moisture to a Pellicle, and setting the Salt to shoot into *Chrystals*. (5.) EARTH, or what is

of Bodies, all which are found to have the following *common Properties*, viz.

EXTENSION, or that by which it possesses or takes up some Part of universal Space; which Space is call'd the PLACE of that Body. For all Bodies are *extended* either (1.) into *Length* only, and then it is called a LINE; or (2.) into *Length* and *Breadth*, which is called a SUPERFICIES; or (3.) into *Length*, *Breadth*, and *Depth*, which then is called a SOLID. These are the *three Dimensions*, according to the Quantity of which the *Magnitude* or Bulks of Bodies are estimated (II).

SOLIDITY,

call'd the *Caput-Mortuum*, being what remains of the Ashes after the Salt is extracted, which can be no farther alter'd by any Art whatsoever.

(II) The MAGNITUDE of Bodies is the Quantity of their Dimensions express'd in some common or *standard* Measure, as an *Inch*, a *Foot*, a *Yard*, &c. and it is thus estimated:

1. When Bodies have but one Dimension, as Lines, then it is express'd by the Number of Inches and Parts of an Inch contained in their Length: Thus the Dimension of the Line A B is 3 Inches; of CB 2 Inches; and the Comparison of the Length of AB to CB is call'd the Proportion or *Ratio*; thus the Ratio of AB to CB is that of 3 to 2, or (as it is usually express'd) AB : CB :: 3 : 2. And because this Comparison consists of *only one Ratio*, it is said to be a *simple Ratio*.

Plate I. Fig. 1.

2. Those Bodies which have two Dimensions, as Superficies, have their Magnitude express'd by the Rectangle under both, or the Product of their Length by their Breadth; thus if any Surface ABCD has its Length AB=4, and its Breadth BC=2; then its Dimension

Plate I. Fig. 2.

(which

SOLIDITY, sometimes called the IMPENETRABILITY of Matter, is that Property

(which in this Case is called the *Area*, or *Superficial Content)* is thus expressed, $AB \times BC = 4 \times 2 = 8$; that is, there are eight small Squares, which are square Inches, square Feet, &c. according to the Measure of the Sides.

3. If this Surface be compared with any other, as EG, whose Length is $EF = 3$, and Breadth $FG = 2$; then their Magnitudes will be to each other as $AB \times BC$ to $EF \times FG$, that is, as 8 to 6, or as 4 to 3. And because in this Comparison each Term consists of two Parts, or there is a twofold Ratio of Length to Length and Breadth to Breadth, therefore this is said to be a *Duplicate Ratio*; and so all Surfaces are to each other in the *Duplicate Ratio* of their Sides. Plate I. Fig. 3.

4. In like Manner all Solid Bodies, which have three Dimensions, have their Magnitudes express'd by the Product of their *Length*, *Breadth*, and *Depth* together. Thus if there be one Solid AG, whose Length $AB = 4$, its Breadth $AE = 2$, and Depth $AD = 3$; and another Solid HO, whose Length $HI = 3$, Breadth $HM = 1$, and Depth $HL = 2$; then will their Magnitudes be to each other as $AB \times BE \times AD$ to $HI \times HM \times HL$, that is $4 \times 2 \times 3 = 24$ to $3 \times 1 \times 2 = 6$, or as 24 to 6. And therefore Solids are said to be in a triplicate Ratio, *viz.* of their Length, Breadth, and Depth or Thickness; and the Standard Measure in this Case is call'd a *Cubic Inch*, *Foot*, &c. because of its being in the Form of a Dye, or *Cube*, which Figure is contained under 6 equal and rectangular Planes. Plate I. Fig. 4, 5.

5. And here it will be necessary to advertise the Reader, that any Quantity is generally denoted by a single Letter, as A, B, &c. and the Square of that Quantity by $A \times A$ or AA, or A^2, B^2, &c. and the Cube by A^3 or B^3. And when we express the Ratio of two Quantities A and B, it is usually thus, $A : B$; and when we compare this Ratio with any other, as of C to D, we express it thus, $A : B :: C : D$. If both Terms of the Ratio increase or decrease together, the Ratio is said to be direct, and express'd as before: But if one Quantity increases

ty by which a Body excludes all others from the Place which itself possesseth; for it would be absurd to suppose two Bodies could possess one and the same Place at the same Time. From this Definition it follows, that the *softest Bodies* are equally *solid* with the *hardest* (III).

DIVISI-

increases while the other decreases, and *vice versa*, in each Ratio; then they are expressed in a different Manner, *viz.* $A : \frac{1}{B} :: C : \frac{1}{D}$, or $A : C :: \frac{1}{B} : \frac{1}{D}$. And the Addition of one Ratio $A : B$ to another $C : D$ is performed by multiplying the first Terms in each Ratio together, and also the last; and the Ratio of those Products $AC : BD$ is the Sum of both the other. To square any Ratio is to multiply it by itself $A : B \times A : B = A^2 : B^2$, and so the Cube of any Ratio, $A : B$ is $A^3 : B^3$. If any would know more of the Nature of Ratios, they may consult Dr. *Sanderson*'s Algebra, or my *Logarithmologia*.

(III) FIGURABILITY is as necessary to Matter as any of the Properties abovementioned; for since Matter is not infinite, it must be circumscribed within certain Limits and Bounds on every Part, which constitute the Figure of the Body; and as the Particles of Matter may exist together in any Manner of Situation, so the Figures or Form of Bodies, which they compose, may be infinitely various and different from each other.

2. On this Property several Things of Moment depend; thus according to the several Figures of the Corpuscles, they will touch by a greater or lesser Quantity of Surface, and so will cohere more or less firmly together; from hence will arise various Qualities of Bodies, which are the Foundation of most of the considerable Phænomena of Nature, as will be hereafter taken Notice of.

3. Hence we may observe likewise, that Bodies of different Figures contain, under the same Quantity of

Surface,

DIVISIBILITY is that Property by which the Particles of Matter in all Bodies are capable of a Separation or Disunion from each other. Hence the *Resolution* or *Dissolution* of Bodies into their constituent Corpuscles, as in many Operations of Chemistry. How far this may actually obtain in Nature is not easy to say: But that Matter is infinitely divisible

Surface, different Magnitudes or Bulks; thus a Circle contains a greater Area under the same Length of Periphery, than any other figured Superficies; and a *Sphere*, under the same Quantity of Surfaces, contains a greater Bulk or Space, than any other Solid whatever.

4. Of all the infinite Variety of Forms or Figures, which Matter is liable to, there are only *five* which will admit the Particles of Matter, placed together, to fill the Space between them compleatly, or so as to leave no Pore, Vacuity, or Interstice between them: and Bodies of these Forms have been known by the general Name of the *five Platonic Bodies*, and are as follow:

1. *Tetrahedron*, which has *four equal* triangular Sides.
2. *Hexahedron*, of *six equal* square Sides, *viz*. a Cube.
3. *Octahedron*, of *eight equal* pentagonal Sides.
4. *Dodecahedron*, of *twelve equal* triangular Sides.
5. *Icosahedron*, of *twenty equal* triangular Sides.

5. Hence the Asserters of a *Plenum* are necessitated to prove, that tho' Matter is liable to an Infinity of Forms indifferently, yet all its Particles must have but some one of the five abovementioned; otherwise it would be as evident as an *Axiom*, that the Particles of Matter have a different Form, and must therefore admit Vacuities or Pores in the Composition of every Kind of Bodies. Tho' this Argument for a Vacuum has not been observed, yet perhaps there is none more evident, geometrical and conclusive.

divisible in a mathematical Consideration, is demonstrable various Ways (IV).

MOBILITY is that Property which all Bodies have, of being moveable or capable of changing their Situations or Places. This Property

(IV) 1. The easiest Way to demonstrate the infinite Divisibility of Matter, I think, is the following. Let AB be the Length of a Particle to be divided; through each Extreme, A and B, let there be drawn the Lines CD, EF parallel to each other: In the Line CD, between A and C, let there be taken any Point G; and in the Line EF, from B towards F, let there be taken any finite Number of Points, H, I, K, L, &c. If now from the Point G we draw the Lines GH, GI, GK, GL, &c. they will each of them cut off a Part of the given Particle AB; and yet after all, a Part, PA, will remain; and since this will be the Case for any finite Number, it is plain the Particle AB contains a Number of Parts greater than any *finite Number*, and therefore *infinite*.

Plate I. Fig. 6.

2. Another Demonstration equally obvious is the following, *viz.* Let AB be the Length of the given Particle; thro' A draw CD, and in the Point E, at right Angles, draw the indefinite right Line EF, in which let there be any finite Number of Points taken, as G, H, I, K, &c. on these, as Centers, describe the several Arches of Circles EL, EM, EN, EO, &c. to cut the Particle AB, each of these will take off a Part, and yet there will a Part, AP, remain: therefore, &c. as before.

Plate I. Fig. 7.

3. Tho' we have no Possibility of an Instance of this infinite Divisibility of Matter, yet Nature proceeds towards it to inconceivable Lengths in many of her wonderful (tho' common) Operations. How minutely is Matter divided by Fire, as a Piece of Tallow, for Instance, in a lighted Candle! What a prodigious Sphere or Space does the least Flame fill with Particles of Light, or a Grain of *Assa fœtida* with odorous Particles! How numerous and small the Particles or Vapours arising from the Surface of Fluids! Or the Particles of Copper, which from a single Grain shall be diffused thro' the Substance

Property of Matter is evident to all our Senſes; and what nobody has confeſſedly deny'd (V).

THE VIS INERTIÆ, (as Sir *Iſaac* call'd it) or the *Inactivity* of Matter, is that Property of it, by which it endeavours to continue in its State either of *Motion* or *Reſt*, or by which it reſiſts the Actions and Impreſſions of all other Bodies which tend to generate or deſtroy Motion therein (VI).

ATTRAC-

ſtance of many Millions of Times its own Quantity of Water, and tinge it of a different Colour! But Inſtances of this Kind are endleſs.

(V) *Des Cartes* and his Followers, who deny a *Vacuum*, do virtually deny Motion; for ſince Motion is the Tranſlation of a Body from one Part of Space to another, if there be no *void Space*, ſuch a Tranſlation muſt be made thro' a Space abſolutely filled with Matter, or not at all; if the firſt be aſſerted, it will follow, that two Bodies may be in one and the ſame Place at once, which is abſurd; therefore there can be no Motion at all upon their Principles. To ſay a Body may move in a *Plenum*, as a Bird moves thro' a Body of Air, or a Fiſh thro' Water, is ſaying nothing to the Purpoſe; becauſe both Air and Water abound with Vacuities, (as is well known from the Nature of each) whereas a *Plenum* admits of no vacuous Space, and therefore of no Motion of the Particles, nor conſequently of any Body between them. Beſides, neither the Bird nor Fiſh would be able to continue in the Air or Water, were they not ſpecifically heavier than thoſe Fluids; but upon the Suppoſition of a *Plenum*, all ſpecific Gravity is taken away (as will be ſhewn) and ſo the Caſe is not parallel, and therefore the Argument fallacious.

Plate I. Fig. 8.

(VI) This Power or Force of Matter ariſes wholly from its Sluggiſhneſs or Inactivity, and not at all from its Gravity, as ſome (of no ſmall Name) have miſtakenly

ATTRACTION is a Property that we find all Matter endued with in a greater or a lesser Degree: By this Property the Particles of Matter attract each other by a Power which causes them mutually to accede to, or approach each other. This Tendency of one Body to another is called GRAVITATION, or, in the Abstract, the GRAVI-

kenly conceived of it. To make this Matter plain, suppose a Body A at rest in the Point A, it would not of itself move out of that Point, unless by some external Force impressed. *First*, Suppose the Body absolutely light, and it be required to move it from A to B, in a certain Time; to this End some Force must be applied acting in the Direction AB, by which, at the End of that Time, the Body will be found in B; and this Force once impressed will continue in the Body, and in every such Particle of Time will carry it thro' a Space equal to AB. *Secondly*, Suppose now Gravity begins to act, and give the Body Motion, and that by the Power of Gravity the Body be carried from A to D in a Line, passing thro' the Earth's Centre, in the same Time as it before moved from A to B; draw BC and DC equal and parallel to AD and AB; and it is evident all the Effect of Gravity will be only to cause the Body during that Time, to descend through Space equal to AD from the Line AB, and so in the End of that Time instead of being at B it will be in C; but since the Point C is as far from D as B is from A, it follows, that whether Gravity acts or not, the same Space is described in the same Time in the Direction AB, and so the same Force is required to give it the Motion, which therefore has no Relation to or Dependance on Gravity; but arises solely from the Nature of inactive Matter, and is on that Account ever proportional to the Number of Particles or Quantity of Matter in Bodies.

GRAVITY or WEIGHT of Bodies (VII).

REPULSION ſeems to be a Property belonging to the ſmall Particles of Matter univerſally; for they do not more evidently *attract* in ſome Circumſtances than they *repel each other* in others, as will be evident by Experiments, hereafter to be exhibited (VIII.)

THESE

(VII) This Power, or Virtue, was originally communicated to all Matter by the Omnipotence of the Deity, and is no ways neceſſary to its Exiſtence; therefore I apprehend when Sir *Iſaac* ſays, he is ignorant of the Cauſe of Gravity, he can only mean the particular Manner in which it is exerted from one Body, and in which it acts upon another, and not that he was ignorant of the Source or Principle of Power whence it could be derived. But as the Manner in which it is exerted, ſo the Means by which it is propagated from one Body to another, is as yet unknown. To account for the Propagation of this Virtue, as of Light, *&c.* 'tis ſuppoſed that there is an exceeding fine imperceptible Medium, or ethereal Spirit, diffuſed thro' all the Syſtem, and pervading the Pores of all Bodies; that this Medium is extremely elaſtic, and that by its Tremours and Undulations, which are almoſt inſtantaneous, the Emanations of the attracting Virtue, Light, *&c.* are propagated with an immenſe Velocity, much after the ſame Manner as Sounds are conveyed by the undulatory Motion of the Air. By Means of this moſt ſubtle Spirit, 'tis ſuppoſed, likewiſe, that electric Bodies exert their Power of attracting and repelling light Bodies at greater Diſtances; that Light is emitted, reflected, refracted, inflected, and heats Bodies; that all Senſation is excited, and animal Motion performed at Will; namely, by the Vibrations of this Spirit mutually propagated along the ſolid Filaments of the Nerves from the outward Organs of Senſe to the Brain, and from the Brain to the Muſcles.

(VIII) *Attraction* and *Repulſion* differ in no other Reſpects

These are the general Properties of Matter, which we must regard in our Explications of the various Phænomena of Nature, in the Sense we have defined them. These are the several *Data*, or fundamental Principles on which the Science of Philosophy depends, and which will each of them afford an ample Field both in the *speculative and practical Part*. We shall (for the Sake of Method) begin with the *Attraction* of Bodies, consider its several Species, and prove their Existence and the Properties of each by Experiments.

The *Power of Attraction*, or Cause of *Gravity*, we presume not to define, or say what it is, but only that it is, or does exist; and the Laws of its Action we shall

spects than this, that the attracting Virtue in the first Case carries Bodies *towards* the attracting Body, and in the latter it carries them *from* it. In each Case, the Particles are moved in the same Manner among themselves, by the *Attracting*, *Electric* or *Magnetic Power*, Some very eminent Philosophers of late think different from Sir *Isaac Newton*, who always asserted, that Attraction and Repulsion were two different and opposite Powers, but that the latter begun where the former ended; but these Gentlemen assert, they both belong to or begin originally from the Particles of Matter themselves, and that those which once attract or repel, always do so; however this Principle requires to be supported by more Experiments, and better Reasons than we have yet seen in its Favour.

endea-

endeavour to aſſign by what may be diſcovered by Reaſon and Experiment. To this End we muſt conſider, that *any Kind of Power or Virtue, proceeding or propagated from a Body in Right Lines every Way as from a Centre, muſt decreaſe in its Energy or Strength as the Squares of the Diſtances from the Body increaſe*; for 'tis evident, the Force will be every where as the Number of Particles iſſuing from the central Body on a given Space, which Number of Particles will decreaſe as the Squares of the Diſtances increaſe. Thus the Number of Particles which at *one Diſtance* AB, from a Point in the Body at A, falls on a Square Inch BEFG, will be *four times* as great as the Number which falls on a Square Inch CHIK at *twice* that Diſtance AC; and *nine times* as great as the Number which falls on the ſquare Inch DLMN at *three times* that Diſtance AD; and ſo on, as is evident from the Diagram. Plate III. Fig. 1.

Hence, ſince we have no Reaſon to doubt but that all Kinds of Attraction conſiſt in fine imperceptible Particles or inviſible *Effluvia*, which proceed from every Point in the Surface of the attracting Body, in all right-lined Directions every way, which in their Progreſs lighting on other Bodies

Bodies urge and ſollicit them towards the ſuperior attracting Body; therefore the Force or Intenſity of the attracting Power in general muſt always decreaſe as the Squares of the Diſtances increaſe.

HENCE alſo we may obſerve, by the way, that *Light* and *Heat*, *Odours* and *Perfumes*, which conſiſt of Particles or *Effluvia* that proceed every way from luminous, heated, and odoriferous Bodies, as from a Centre, have always their Forces abated according to the above Law. The Force of *Sounds* alſo decreaſes in the ſame Proportion, for Reaſons that will be hereafter aſſigned.

BY virtue of attracting Power, the grand *Machinery* of the *Solar Syſtem*, and doubtleſs of all the others in the Univerſe, is effected, eſtabliſhed, and conſerved. It is therefore of the greateſt Conſequence, to be acquainted with the different Species of this univerſal Power or Agent, and to learn by Experiments the peculiar Nature, Laws, or Manner of Action in each. Naturaliſts generally reckon *Four* different Sorts of Attraction, *viz.*

I. THE *Attraction of* COHESION, which is peculiar to Corpuſcles or primigenial Particles of Matter, of which larger Bodies are compoſed, by the Accretion and firm Adhe-

Adhesion of these Particles, arising from their strong attractive Power.

II. THE *Attraction of* ELECTRICITY; which is peculiar to some Kinds of Bodies, as Glass, Amber, Sealing-wax, *&c.* which are therefore called *Electrical.*

III. THE *Attraction of* MAGNETISM, or of the Loadstone, which is peculiar to, and mutual between the Loadstone and Iron.

IV. THE *Attraction of* GRAVITATION, which is observable only in the larger Compositions and Systems of Matter; as in the *Earth* and *Moon*, and the *Sun* and *Planetary Bodies* which compose the solar System. Of each of these Species of Attraction in Order; and first,

OF the *Attraction of* COHESION: The Laws and Properties of this Attraction are the following. (1.) It is very discernible and most powerful in Corpuscles, or the smallest Particles of Matter. (2.) It is mutually exerted between those Particles; or they mutually attract, and are attracted by each other. (3.) The Sphere of Attraction, or Extent of this Power, is greater in some Particles of Matter than in others, but very small at the utmost: For (4.) This Power is insensible in solid Bodies in the least sensible Distance, acting as it were

only on Contact; and therefore, (5.) It must be nearly proportional to the Quantity of contiguous Surfaces; or the Parts of Bodies cohere most strongly, whose touching Surfaces are largest. (6.) This Power must decrease as the Squares of the Distances increase; because it must be supposed to issue from each Particle in right-lined Directions. (7.) Where the Sphere of Attraction ends, there a *Repelling* Power begins, by which the Particles, instead of *attracting*, *repel* and fly from each other. (8.) By this Power the small Portions or Drops of a Fluid conform themselves to a spherical Figure (IX).

THE

(IX) To illustrate this Matter of Corpuscular Attraction and Repulsion: Let A, B, (in *Fig.* 9, 10, 11, 12.) represent two small Bodies or Corpuscles of Matter; and their Spheres of Attraction, (or Extent of their Attracting Power every Way) be CDE, and EFG. Then it is evident, Plate I.

1. That the Force with which the Bodies attract each other will be greatest in the Case where they touch each other.

2. If the Bodies are of a spherical Figure (as *Fig.* 9.) they can touch but in a single Point; and therefore the Power with which they stick or cohere together will be the slightest or weakest possible. Plate I.

3. But if the Bodies are bounded by plain Surfaces (as *Fig.* 12.) the Power of Cohesion will be so much the greater, as the Number of Particles, or Area of the Surface by which they touch, is larger.

4. If the Corpuscles A, B, touch not, but are yet within the Sphere of each other's Attraction, (as *Fig.* 10.) they will then be easily moveable or voluble about each other,

THE first and second of these Properties are made manifest by various Experiments; as the sudden Union of two contiguous *Drops of Mercury, Water, &c.* The strong Adhesion of two *Leaden Balls*, which touch by polish'd Surfaces (X); as also of *Glass Planes*,

other, and yield to the least Impression of external Force; and this will be the Case of all Particles, whatever be their Figures or Forms.

5. If the Corpuscles A, B, are separated to such a Distance as to be without each other's Sphere of Attraction (as in *Fig.* 11.) then will they fly from, or repel each other, with a Power that extends to a considerable Distance, and is exerted every Way equally from each Particle.

6. If there be a great Number of these Particles thus disengaged from each other, but yet confined within certain Bounds, they will, by this repulsive Power, be all situated at equal Distances from each other through the whole Space, if the confining Power be on every Side equal.

7. This Power, while *attractive*, extends to but very small Distances from the Corpuscles, and then becomes *repulsive*, for strictly speaking, there are no Bodies in Nature which repel *absolutely*, but only at certain Distances; thus the Loadstone does not at all Distances repel the Needle, but only at a definite Distance; for either Pole will attract either End of the Needle at a very near Distance, or upon Contact, as is easy to experiment.

(X) This Adhesion of leaden Balls is so very considerable, that with two (not weighing above a Pound each, nor touching upon more than $\frac{1}{10}$ of a square Inch Surface) I have lifted above 200*lb.* weight. In order to this, the Surfaces by which they touch must be finely planed with the Edge of a sharp Pen-knife, and equally press'd together with a considerable Force, with a gentle Turn of the Hand at the same Time; and thus two com-

Planes, and *Crystal Buttons*: The Ascent of Water between *Glass Planes*, and in *Capillary Tubes*: The rising of Water by the Sides

common leaden Bullets will adhere so firmly together, as to require upwards of 80 *lb.* to separate them.

2. In polish'd Surfaces that are very hard, as Glass, Brass, &c. it is impossible to bring them into such close Contact as to cohere without the Interposition of Water, or something humid, to fill the Pores by expelling the Air contained therein, which prevents the Planes coming together while dry; the Humidity in this Case proves a Cement which holds the Planes together by all its Force of Attraction on either Side.

3. This Force of Attraction between the Brass Planes is greater with Oil than with Water; and greater with any Sort of Grease or Fat that will harden with Cold than with Oil. I never yet could meet with four Men strong enough to separate Planes thus put together, which are but $4\frac{1}{4}$ Inches Diameter; and therefore they cohere with a Force much superior to the Force or Pressure of Air on such a Surface, which contains about 14 square Inches, and allowing 15 *lb.* for Pressure to every Inch, it will amount to but 210 *lb.* which is not half the Strength of four Men pulling against each other to the best Advantage. I have often seen six Men endeavour to separate them in vain.

4. Professor *Muschenbroek* has made many Experiments to shew the Force of Cohesion between Planes of various Substances, and about 2 Inches Diameter, well polished, having first heated them in boiling Water, and then besmeared them first with a cold Tallow-Candle, and afterwards with boiling Grease, and the Weights to separate them were as in the following Table:

		Cold Grease.	*Hot Grease.*
Planes	of Glass	— 130 *lb.*	300 *lb.*
	of Brass	— 150	800
	of Copper	— 200	850
	of Marble	— 225	600
	of Silver	— 150	250
	of Iron	— 300	950

5. These

Sides of a Glaſs Veſſel, and into Tubes of *Sand*, *Aſhes*, *Sugar*, *Sponge*, and all porous Subſtances.

THE

5. Theſe Planes adhere by other Sorts of Matter with Forces as in the Table below, where the Weights neceſſary to ſeparate them are ſpecified:

With Water	— —	12 Ounces.
With Oil	— —	18 Ounces.
With *Venice* Turpentine	—	24 Ounces.
With Roſin	— —	850 Pounds.
With Tallow Candle	—	800 Pounds.
With Pitch	— —	1400 Pounds.

Though theſe Experiments would not always give the ſame Numbers, yet they ſufficiently ſhew the vaſt attractive Forces, and the very great Difference between them.

6. After this he gives us an Account of his Experiments made to find the Force with which Bodies cohere naturally, or what is the abſolute Force of Coheſion in various Bodies, which he eſtimates by the Weights required to pull them aſunder, drawing according to their Length; this he tried firſt in Wood, and afterwards in Metal. His Pieces of Wood were of a long ſquare Form, of which each Side was $\frac{26}{100}$ of an Inch, and by Weights ſuſpended they were drawn aſunder, according to the ſeveral Sorts, as mentioned below:

Wood of Linden-Tree	—	1000 *lb.*
of Alder	— —	1000
of Fir	— —	600
of Oak	— —	1150
of Elm	— —	950
of Beech	— —	1250
of Aſh	— —	1250

7. The Trial he made with Metals was of Weights ſuſpended to Wires of each Sort, whoſe Diameter was $\frac{1}{10}$ of a Rhinland Inch; or becauſe the Rhinland Foot is to ours as 139 to 135, the Wire was $\frac{139}{1350}$ Parts of an Inch *Engliſh*. The Metals and Weights were as follow:

THE third of these Properties is evident by the Experiments of Water rising above the common Level, and Mercury's sinking below it, in Capillary Tubes: By the sticking or adhering of Water to common Substances, which by Mercury are left dry.

Of Copper	—	—	$299\frac{1}{2}$ *lb.*
Of yellow Brass		—	360
Of Gold	—	—	500
Of Silver	—	—	370
Of Iron	—	—	450
Of Tin	—	—	$40\frac{1}{4}$
Of Lead	—	—	$29\frac{1}{4}$

8. These Experiments shew the *absolute Force of Cohesion* in Bodies. That which he calls the *Relative Force*, is that by which any Body resists the Force of any other Body acting upon it in a Direction perpendicular to its Length; and this he estimated in the same Pieces of Wood as before, by putting one End into a square Hole of a metal Plate, and hanging Weights towards the other End sufficient to break each Piece of Wood at the Hole. These Weights and Distances from the Hole were in his Experiments as follow:

		Distance.			*Weights.*
Fir	—	9 Inches		—	40 Ounces.
Oak	—	$8\frac{1}{2}$	—	—	48
Elm	—	9	—	—	44
Pine	—	$9\frac{1}{2}$	—	—	$36\frac{1}{2}$
Alder	—	$9\frac{1}{4}$	—	—	48
Beech	—	7	—	—	$56\frac{1}{2}$

Such as would see much more on this Subject, may consult the Author's admirable *Essai de Physique*; and his Treatise of the Cohesion of Bodies, altogether on the Subject.

THE fourth and fifth Properties are evinced by the Experiments of the different Heights to which Fluids aſcend between Glaſs Planes unequally inclined, and in Capillary Tubes of different Bores: Alſo, by the accelerated Motion of a Drop of Oil between two inclined Planes: And likewiſe by the *Hyperbolical Curve* formed by the Superficies of a Fluid aſcending between Glaſs Planes touching each other on one Side.

THE ſixth Property is evident. The ſeventh ſeems evident between *fat and oily Particles of Matter*, and thoſe of an *aqueous Nature:* But is moſt manifeſt from the Elaſtic Property of the Air, whoſe Particles compreſs'd together reſtore themſelves by this repellent Power to their firſt State: Alſo by the Aſcent of *Steam* or *Vapour* from humid or fluid Bodies.

THE eighth Property of this Attraction is manifeſt by Drops of Water falling on Duſt; by Drops of Dew gathering on the Tops of Graſs; and laſtly, by Quickſilver divided into ſmall Portions, which always form themſelves into perfect Spherules or Globules (XII).

FROM

(XII.) 1. In order to account for the Phænomena of Glaſs Capillary Tubes, *&c.* it will be neceſſary firſt to premiſe,

FROM this Account of the *Attraction of Cohesion* we learn a rational Solution of several very curious and surprizing Phænomena

premise, that there is a greater Attraction between the Particles of Glass and Water, than there is between the Particles of Water themselves: For if it were not so, the least Quantity or Drop of Water C applied to the under Side of a Glass Tube, in a Position parallel to the Horizon, as A B, would not adhere to it, but immediately fall down by its Gravity; but we see that it does not, till its Bulk and Gravity be so far increased as to overcome the Attraction of the Glass, and then it falls off.

Plate I. Fig. 13.

2. Since we find such a strong attractive Power in the Surface of Glass, it will be easy to conceive how sensibly such a Power must act on the Surface of a Fluid (not viscid) as Water, contained within the small Cavity or Bore of a Glass Tube; as also that it will be in Proportion stronger as the Diameter of the Bore is smaller; for that the Efficacy of the Power follows the inverse Proportion of the Diameter is evident from hence, that only such Particles as are in Contact with the Fluid, and those immediately above the Surface, can affect it.

3. Now those Particles form a Periphery, or rather a very narrow *Annulus* or Ring contiguous to the Surface, the upper Part of which attracts and raises the Surface, and the lower Part, which is in Contact with it, supports and holds it up; so that neither the Thickness nor Length of the Tube avails any thing, only the said Periphery of Particles, which is always proportional to the Diameter of the Bore.

4. The Quantity of the Fluid raised will therefore be as the Surface of the Bore which it fills; that is, as the Diameter, since the Effect would not be otherwise proportional to the Cause.

5. Since the Quantities follow the Ratio of the Diameters, the Heights to which the Fluid will rise in different Tubes will be inversely as the Diameters, which is thus demonstrated.

Let

mena of Nature: As why the Parts of Bodies adhere and ſtick ſo firmly together;

Let $\begin{cases} Q,\ q, \text{ repreſent the Quantities of Matter raiſed;} \\ D,\ d, \text{ the Diameters of the Tubes Bores;} \\ H,\ h, \text{ the Height to which the Fluid riſes in the Tubes.} \end{cases}$ Plate I. Fig. 15.

Then ſince Q, *q*, repreſent the Contents of two Cylinders of the Fluid (ſuppoſe GL and IM) it will be $Q : q :: DDH : ddh$, from the Nature of a Cylinder; and from the Nature of this Attraction, it is $Q : q :: D : d$ (as was juſt now ſhewn;) therefore we have, As $D^2 H : d^2 h :: D : d$; and ſo $D^2 Hd = d^2 Dh$, that is $DH = dh$; conſequently, $D : d :: h : H$.

6. The Velocity with which it begins to riſe in the Tube is exceeding great, but preſently abates, and ſo continues by increaſing the Weight of the Fluid that is raiſed; and it will continue to riſe till an Equilibrium be made between the Gravity and the attracting Force of the Glaſs.

7. The Preſſure of the Air neither helps nor hinders the riſe of the Fluids, for the Effect is the ſame *in Vacuo* as in the Air. But the more viſcid the Fluid, the leſs apt it is to riſe in the Tube.

9. By the ſame Power the Water riſes between Glaſs Planes, ſo as to form the celebrated HYPERBOLIC CURVE, when they are ſo placed in Water as to touch on one Side, and be ſeparate or opened to a ſmall Diſtance on the other. Thus ſuppoſe ABCD the Surface of Water, in which are placed the two Planes *fm*O and *em*O, ſo as to touch all along the Side *m*O, and yet open on the other Part, ſo as to contain a ſmall Angle *f*O*e*; then forming the Rectangles O*dhp*, O*cio*, O*bkn*, O*alr*, &c. and meaſuring very exactly, they will always be found equal to each other; and this is a well known Property of the Hyperbolic Space between the Curve and its Legs O*e* and O*m*. Plate I. Fig. 14.

10. If two long Glaſs Planes are firſt ſmeared over with Oil, and then ſet together at their Ends, and inclined to each other under a very ſmall Angle, and a Drop

ther; why ſome are *hard*, others *ſoft*; ſome *fixed*, others *fluid*; ſome *elaſtic*, others *void of*

Drop of the Oil ſo placed between them as to touch both Planes, it will immediately begin to move towards the touching Ends, or Angles of the Planes, and that Motion will be continued with an accelerated Velocity, by reaſon of the increaſing Attraction of the Planes, on account of the decreaſing Diſtance between them, and the larger Portion of touching Surface on each Side the Drop; concerning all which a great deal has been wrote to very little Purpoſe.

11. If Glaſs be applied to any other Fluid, whoſe Particles attract each other more ſtrongly than Glaſs attracts them, all the Phænomena of ſuch a Fluid in Capillary Tubes, and between Glaſs Planes, will be juſt the Reverſe of thoſe now mentioned of Water. Now *Quick-ſilver* is ſuch a Fluid, and therefore it will ſtand lower within a Capillary Tube than without; the Surface *convex* and not *concave*, as in Water; and between the Planes it will move a contrary Way, &c.

12. But if a Baſon or Diſh be made of Copper or Braſs, and poliſhed well within Side, and then tinned all over, Mercury put into ſuch a Veſſel will every where unite with the Tin, and may be properly ſaid to *wet* it, as Water does Glaſs; and the Mercury, if clean, put into this *Mercurial Baſon*, will be attracted and riſe all round the Sides, and have the ſame *Phænomena* with *Water* put into a *wet Glaſs*, Tranſparency only excepted.

13. Here I think proper to mention an Experiment, with which I have often entertained myſelf and others, and that is, pouring Mercury into the abovementioned Diſh or Baſon formed into a true Spherical Figure, and well planiſhed, it leaves the concave Surface ſo nicely ſilvered, that it makes a moſt delightful Mirrour, and reflects the Light ſo perfectly, as to produce the ſtrongeſt and moſt diſtinct Image of an Object that can poſſibly be effected by Art. How far the Nature and Applica-

 tion

of any elastic Quality; which all arise from the different Figure of the Particles, and the

tion of Mirrours may be improved by this Method, must be left to farther and future Experience: And also whether it be not practicable by this Means to make a very large and most effectual Burning-Glass, at a very easy and cheap Rate.

14. A late *French* Author (L'Abbé *Nollet)* has thought fit to call in question the Cause we have here assigned for the Rise of Fluids in Capillary Tubes, and has almost sneered Dr. *Jurin*'s learned Dissertations on that Subject; but I see nothing that could oblige him to do this, from what he has said to explode it; no Man can be convinced by his *ipse dixit*, that this is a mere Hypothesis; the *Phænomena* so plainly indicate the Cause to be Attraction, and agree in every Particular so nicely to the Theory, that 'tis not to be expected we should give it up till we see better Reason for it than this reverend Father has given in his prolix Digression on this Subject. See his *Leçons de Physique Experimentale*, Tom. II. Page 421, *&c.*

15. But though the Attraction of the Glass is the indubitable Cause of the Ascent of Fluids, yet it must be allowed that the Nature and Genius of the Fluid is to be regarded in most of the *Phænomena*, which are not proportioned to the attracting Power of the Glass only, but to that Power conjointly with the various Disposition of Fluids to yield thereto; nor is the Density of the Fluid of primary Consideration, it being evident by Experiment, that some lighter Fluids will rise to a less Height than others which are much heavier: Thus *Spirit of Wine* will rise but $4\frac{1}{2}$ Tenths of an Inch in a Tube, where Oil of Tartar will rise 9, as in the Table below. Nor does it at all depend on the Viscidity or Tenacity of Parts; for hard white Varnish (very thick and viscid) and Spirit of Wine ascend nearly to the same Height. I am inclined to think from the Experiments which I have made, that the Ascent of Fluids depend

the greater or lesser Degree of Attraction consequent thereupon. Hence the Reason why

depend greatly upon the Spirit they contain, and that not on the Quantity of it, but on some peculiar Action or Disposition thereof, relative to the attracting Power of the Glass. So from the Table it appears, that Spirituous Liquors rise to a small Height in general; but Water, which has none, to the greatest; yet Spirit of Hart's-horn and Spirit of Urine rise higher than Ale and Small-Beer, which contain but a small Proportion of Spirit.

16. But to set this Matter in a clear Light, I have very carefully made the Experiment on the following Fluids, which were each of them often repeated, and with the same Success; the Bore of the Tube was about $\frac{1}{30}$ of an Inch in Diameter.

	Altitude.
Common Spring Water —	12 Tenths of an Inch.
Spirit of Urine ——	11
Tincture of Galls —	11
Recent Urine —	10
Spirit of Salt —	9
Ol. Tar. per Deliquium —	9
Vinegar ——	$9\frac{1}{2}$
Small Beer ——	9
Strong Spirit of Nitre —	$8\frac{1}{2}$
Spirit of Hart's-horn —	$8\frac{1}{2}$
Cream —	8
Skimmed Milk ——	8
Aqua fortis ——	$7\frac{1}{2}$
Red Wine ——	$7\frac{1}{2}$
White Wine ——	$7\frac{1}{2}$
Ale ——	$7\frac{1}{2}$
Ol. Sulph. per Campanam —	$6\frac{1}{2}$
Oil of Vitriol —	$6\frac{1}{2}$
Sweet Oil —	6
Oil of Turpentine —	$5\frac{1}{2}$
Geneva ——	$5\frac{1}{2}$
Rum —	5

Brandy

why Flies walk on the Surface of the Water, and wet not their Feet. On this Principle

	Altitude.
Brandy —	5 Tenths of an Inch.
White hard Varnish —	5
Spirit of Wine ——	$4\frac{1}{2}$
Tincture of Mars —	$4\frac{1}{3}$

17. From other Experiments it appears that Heat and Cold are not concerned in the Ascent of Fluids, the hottest Water standing at the same Height as the Cold. Also Salts of any Kind dissolved in Water makes but little Alteration in the Heights, for when Water has taken up all it can in the Solution, it will rise 10 or $10\frac{1}{3}$, which small Deficiency is owing undoubtedly to the increased Gravity of the Fluid.

18. There is one Phænomenon of Capillary Tubes very extraordinary, which is this; if a large bored Tube NP be at one End P drawn out into a fine Capillary as at O, such a Tube filled with Water, and then inverted into a Vessel of Water, as ABCD, will retain all the Water suspended through its whole Length, if it does not exceed the Height to which a Capillary of the same Bore, as at O, will attract it. To see the Reason of this, we must consider the Motion of a small Quantity of Water CD in a tapering Capillary AB in an horizontal Position. And here we must assign two Principles of Motion in the Fluid CD; one is the *attracting Power* of the Glass at C and D, the other is the *Velocity* of the moving Fluid in these Places. For the *Momentum* of any Body, whether Solid or Fluid, is always proportioned to the Intensity of the moving Power, and the Velocity of Motion. (As is shewn fully hereafter.) But the moving Power or Attraction at C is to that at D as the Diameter of the Bore at C to the Diameter thereof at D: Or, putting a, A, for the two Attractions in D and C, and d, D, for the Diameters, we have $a : A :: d : D$.

Plate I. Fig. 15.

Fig. 16.

19. Again, the Velocities (V and v) of the Water in those Points are as the Squares of the Diameters inversely

Principle we account for the Manner how Plants imbibe the nutritive Juice or Moisture

versely (as will be shewn hereafter;) that is, $V : v :: DD : dd$. Therefore $V \times a : v \times A :: D^2 d : d^2 D :: D : d$. That is, *the whole Motion of the Water at* D *is to that at* C *as the Diameter at* C *to that at* D; *consequently the Water will move towards the End of the Tube* B.

20. Because when the Water moves in the Tube, the Velocity at D is greater than that at C, and therefore the Spaces described in equal Time by the Extremities of the Fluid unequal, it follows that the Water will move from D towards B with an accelerated Velocity, the Difference between the Diameters *d* and D continually increasing. This likewise shews the Reason of the accelerated Motion of the Drop of Oil between the two inclined Glass Planes, as mentioned *Art.* 10.

Plate I. Fig. 17. 21. The Motion of the Water once begun at D, will continue till it has with great Rapidity reached the End B; and if then the End B be raised above the horizontal Line AF, yet will the Water be drained at *B*, and not descend, till the perpendicular Altitude *B*F is equal to the Height to which the Water would rise in a Capillary of the same Bore as at B.

Plate I. Fig. 18. 22. Hence also if the Tube be held in an upright or perpendicular Position, as AB, the Water will not descend, unless its Weight be such as when added to the Power acting at C towards A, the Sum shall be greater than the Power acting upwards at B; which Power arises from the Attraction of the Tubes, the Velocity of the Fluid, and the Cohesion of its Particles; for unless we consider this Cohesion of the Particles, the Weight of the Water, when the Vessel is large, will be too great to suffer a Suspension of the Fluid by the other Forces in the Capillary Part.

23. Nor is this Cohesion of the Parts a *Petitio Principii*, but what we know by Experiment to be the Cause of such a *Phænomenon* when nothing else can be so; thus we find the Particles in a Drop of Water hang together

ture of the Earth by the Fibres of the Roots: Alſo for the Riſe of the Sap in Vegetables, and for the whole Oeconomy of Vegetation in general. Hence the *Rationale* of the various Secretions of Fluids by the Glands of an Animal Body, and their wonderful Circulation through the fine Capillary Veſſels. Hence alſo the Reaſon of Soldering and Gilding of Metals; alſo of Melting or Fuſion of Heat. Hence alſo the Exhalation of Vapours by the Heat of the Sun or Fire; the Aggregation of Aqueous Particles in the Air, forming the Drops of Rain. We hence ſee the Reaſon of *Diſtil-*

gether by Coheſion, till the Gravity of the Whole draws it from the Part to which it coheres. Without this Coheſion there could be formed no Drop at all.

24. Hence likewiſe it is, that a Column of Mercury well purged of Air, will be ſuſpended by Coheſion to the Height of 70 or 75 Inches, when the Air alone will ſuſtain it to no greater Altitude than that of 31 Inches at moſt. Dr. *Jurin* ſuppoſes the Preſſure of an elaſtic ſubtle Medium (which can penetrate the Pores of Glaſs and Water) to have ſome Part in producing this very odd Phænomenon. But if all we can learn from Experience be thought inſufficient, I ſhall leave the Reader to amuſe himſelf with what Conjecture he ſhall think fit; aſſuring him in the laſt Place, that this wonderful Suſpenſion of the Fluid in ever ſo large a Glaſs, as A C, whoſe Top terminates in the Capillary B, is not owing to the Preſſure of the Air, becauſe the ſame Thing will happen in an exhauſted Receiver on the *Air-Pump*. See Appendix, Numb. II. to *Cote's Lectures*. Plate I. Fig. 19.

Distillation, *Filtration*, *Dissolution*, *Digestion*, *Sublimation*, *Precipitation*, *Crystallization*, and all the other Operations of *Chemistry* and *Pharmacy*, which are no otherwise to be accounted for. Lastly, we find *Sir Isaac Newton* (at the End of his *Optics*) gives a beautiful and clear Solution to those wondrous Phænomena of subterranean Accensions and *Explosions*; of *Volcanos* and *Earthquakes*; of *Hot Springs*, *Damps*, and suffocating *Exhalations* in Mines, &c. on the Principles of this Sort of Attraction and Repulsion (XIII.)

(XIII.) 1. The Rationale of the several important Particulars here mentioned will easily appear from the preceding Principles or Properties of Matter. Thus HARDNESS of Bodies arise from such Figures of the Particles of Matter as permit them to touch by large Surfaces, and thus to have a great Power of Attraction; and at the same Time, so to fit and fall in with each other, that they prevent any Motion ensuing among themselves by any external Pressure; whereas in another Sort of Figure, the Particles may be admitted to touch by pretty large Surfaces, yet such as not confining each other, will permit the Motion impress'd on the external Parts to proceed among the interior Particles, which will thus easily move among themselves, and make what we call Softness in Bodies.

2. FIXITY is much the same as Hardness, and owing to the same Cause, and is opposed to FLUIDITY, which may in some Sense be esteemed the greatest Degree of Softness, because a fluid Body yields to the least Pressure, and therefore the attractive Power, by which the Particles of a Fluid cohere, must be the least possible; but this results from such a Figure as admits of the fewest Points of Contact possible, that is, a *Spherical Figure*. And

And hence 'tis evident that Bodies are more or leſs fluid as their Parts approach more or leſs to a globular Figure. All which follows from *Annotat.* III. 2.

3. ELASTICITY in Solids ariſes from the ſame Principle of corpuſcular Attraction; thus if a Steel Spring, or Wire, or Piece of very thin Glaſs, be bent out of its natural Poſition, the Particles on the convex Part are forced from that intimate Contact they before had; and on the concave Part they are forced nearer together, or harder upon each other, than in the natural State; in both Caſes there will be a conſiderable Reſiſtance to be overcome, and require a ſuperior Force. During this State of the Particles they may be ſaid to be under a Sort of Tenſion on one Side, and Compreſſion on the other; and ſince by this Force they are not drawn out of each other's Attraction, as ſoon as the Force is remitted or ceaſes to act, the attractive Power reduces the Particles, and unbends the Wire by reſtoring it to its *natural State and Site.* Now 'tis well known that many Subſtances are compoſed of ſuch fibrous Parts or Filaments, which reſemble fine Wire, and are interwoven and diſpoſed in ſuch a Manner (as in *Sponge*, for Inſtance) that they cannot be compreſſed without being bent or wreſted from their natural Poſition, whence all ſuch Bodies will in ſuch a Caſe exert a Force or Spring, and thus prove *elaſtic.* This Theory may be applied to Bodies of a cloſer Texture, as *Cork*, *Wood*, *Ivory*, *Glaſs*, &c. And ſuch Matter as wants this Texture and Diſpoſition of Parts, will not exert ſuch a Spring or Force, but take the Figure ariſing from the impreſſed Force, and thus prove what we call *Non-Elaſtic Bodies.*

4. SOLDERING is another notable Effect of this cohesive Power; for when the Surfaces of two Pieces of Metal are made clean, or freed from all intervening Matter beſides the Solder, the Solder being then fluxed, its Particles will freely enter the Pores, or come into Contact with thoſe of the poliſhed Surfaces of Metal, and by that Means adhere firmly to each, and therefore when the melted Solder becomes cold and fixed, it holds the Surfaces exceeding faſt together, in the Nature of a GLUE or CEMENT, which are all of the ſame

kind, but differ in the Degree of the cohesive or attracting Power of the Parts.

5. MELTING or FUSION of fixed Bodies depends upon the same Theory; for when the Particles of a fixed Body are separated by the Action of Heat so far as not to touch, or but very slightly, yet not so far as to be out of each other's Attraction, they then will easily move by each other, and put on all the Appearances of a fluid Body, (as plainly follows, from *Annotat.* X. 4.)

6. EVAPORATION proceeds likewise from hence, that when the Particles are so far separated by Heat as to be without each other's Attraction, they then begin to repel each other, and thus will seem to rise from the Surface of the Fluid in Form of a Vapour, or Body of Particles, which are at equal Distances from each other; and becoming thus specifically lighter than the same Bulk of airy Particles, they will rise in the fluid Body of Air till they come to that Part of it which has the same Gravity, and they will there make what we call CLOUDS, which will move this Way or that, according to the Current of Air in those Regions. See *Annot*. X. 5, 6.

7. The VAPOURS thus raised become the Original Matter of all METEORS, one Degree of Cold condensing them into larger Globules, which fall in Drops of RAIN; a greater Degree producing a Fixedness or Coagulation of the Particles, which shoot like Salts into various curious Forms, and make the *Fleaks* of SNOW; a third and still greater Degree of Cold congeals the Vapours into a harder Substance, greatly variegated in Form and Consistence, and thus produces HAIL. If the Cold condenses the Vapour so that it can't rise high above the Surface of the Earth, it will there hover about, and fill the lower Air with an obscuring FOG or MIST: Or, if the Cold be more intense, it freezes the Mist to every Twig and Blade of Grass in Form of a white Incrustation, which we call a RIME. If the Air be warm, so that the Vapour therein be too fine to be visible in the Day-time, it will yet be condensed by the Coolness of the Evening so far as to descend, and settle upon the Tops of Grass in the Form of DEW: But, lastly, if the Evenings of such a fine Day be cold enough to freeze,

freeze, then instead of a Dew there will appear a WHITE FROST all over the Surface of the Ground.

8. The CAPILLARY SYPHON is a most curious *Phænomenon*, arising from the same Principle; for the Water being raised to the Flexure or bended Part, by the attracting Force of the shorter Leg immersed into it, the same Force continuing to act carries it even into the longer Leg, and there conspiring with the Force of Gravity, the Water precipitately descends and drives the Air before it, and thus would keep running or dropping out till all the Water were exhausted to the Orifice of the immersed Leg, if the Operation were to be continued long enough.

9. The FILTER is to be considered as nothing more than a *Compound Capillary Syphon*; for the Threads or Filaments of which it consists, lying very near together, make long and slender Vacuities or Interstices, which represent so many Capillary Syphons, which attract, raise, and decant off the Fluid in a considerable Quantity, proportional to their Number or the Largeness of the *Filter*.

10. After the same Manner it is, undoubtedly, that the Humidity of the Earth is drawn into the Substance of the Roots of Plants and Vegetables, which we know consist of long and very minute Fibres so disposed as to form an infinite Number of tubular Interstices, which act in the Nature of a Filter, and imbibe the Juices and Moisture destined for the Nutriment and Growth of the Plant.

11. Thus also it is reasonable to suppose the fine Expansion or Ramification of the *Lacteal*, the *Lymphatic*, and *Sanguiferous Vessels* thro' all the Substance of the *Glands* and *Viscera* in the finest Capillary Tubes, does greatly assist, if not wholly promote, the Circulation of the Blood and Juices, in order to the various Secretions of each respective Gland; an Effect too great for the Pulsive Force of the Heart, or larger Vessels, when thus so infinitely and minutely divided, were it not to be joined by the strong Attraction of those Capillary Parts.

12. By this Means it is that Tallow and Oil rise into the Wicks of Candles and Lamps: That Sponge so readily sucks up and retains so great a Quantity of a

Fluid: That Tubes of Ashes or Sand will thus raise Water to the Height of several Feet. 'Tis by this Virtue that the Pen retains Ink, and the Paper draws it forth in Writing; or refuses to admit it, if oiled, by the Repellency of the Particles of Oil.

13. By this Power fixed Bodies are dissolved by proper Mediums; as *Sugar* and all Kinds of *Salts* by Water; *Silver*, *Copper*, *Tin*, &c. by *Aqua Fortis*; Gold by *Aqua Regia*; and other Bodies in other Menstruums. The Reason is, that when the Particles of the Solid and Fluid attract one another more strongly, than either those of the Solid or those of the Fluid attract each other separately, then a Separation of the Parts or *Dissolution* must ensue. For since, for Instance, if a Particle of the Fluid be attracted with a greater Force than a contiguous Particle of the Solid, that fluid Particle will rush upon the other with so great an Impetus, as instantly to displace it, and separate it from the Solid Particle to which it before adhered with a weaker Force; and thus we are to conceive of every other Particle, till the Whole be dissolved, which will then end when each Fluid Particle has filled its Sphere of Attraction with those of the Solid; and the Fluid is then said to be *saturated*, and will dissolve no more.

14. PRECIPITATION likewise is a Consequence from the same Principle; for if to the Solution of any Solid some other Solid or Fluid Body be added, whose Particles are attracted by the Fluid with a greater Force than those of the dissolved Solid, then will the former take Place of the latter, which, being disengaged, will fall to the Bottom, and settle in Form of a very fine Powder; thus is Silver precipitated by Copper, which the *Aqua-Fortis* attracts more powerfully than Silver. Thus a Solution of Copper is again precipitated by Iron, and other Solutions have their respective Precipitants.

15. FERMENTATION is also an Effect of the same Cause, inasmuch as it consists in the internal Agitation and Commotion of the Parts of the fermenting Body, arising from their mutual Action and Attraction so often mentioned. By this means Bodies are either wholly dissolved, as has been said; or have their Parts so far altered

tered and compounded with other Matter, that they seem to change their Natures, and acquire quite new Properties and Principles. Thus Malt by Fermentation yields a Spirit by Distillation, tho' not like it before; not to mention Putrefaction, and the many notable *Phænomena* thence arising. See *Shaw's Lectures on Chemistry*.

16. The HEAT, EBULLITION, and EXPLOSION of sundry Mixtures are owing to the violent Action and great Rapidity of Motion among the constituent Particles; for all Heat and Accension proceeds from an Attrition or Rubbing of the Particles of Matter one among another. Thus *Aqua-Fortis* and the Particles of Copper act so violently on each other as to cause a great Effervescence and Heat; and Steel-Filings produce a Blast or Explosion instantly. Again, Oil of Caraway-Seed, poured on compound Spirit of Nitre, kindles to Flame with such prodigious Force, that *in Vacuo* it has blown up and burst an exhausted Receiver six Inches wide and eight Inches deep.

17. In accounting for THUNDER and LIGHTNING we have recourse to the same Principles, that is, a Fermentation between the sulphureous Steams exhaled from the Earth, and the Nitrous Acids or Salts in the Air, which take Fire, and cause Explosions in the same Manner as in the Experiments just mentioned. And in some such Manner all other *fiery Meteors* are produced.

18. Also that Burning-Mountains, Mineral Coruscations and Suffocating Damps, Earthquakes, hot Springs, &c. are produced by the intestine Commotion arising from the Mixture of divers Nitro-acid and Sulphureous Matters in the Bowels of the Earth, is evident from a common and easy Experiment, which is by taking equal Parts of Sulphur and Steel-Filings, and working them together, with a little Water, to the Consistence of a Paste; then putting it under Ground, it will presently begin to heat, and in a few Hours take Flame and blow up the Earth about it with a considerable Shock, so as to represent an Earthquake, and a Sort of Volcano in Miniature.

19. That Metals dissolved in Fluids diffuse themselves equally through all the Substance, and keep an

equal Distance among themselves, seems owing to a repulsive Virtue. Also the *Reflection* and *Inflection* of the Rays of Light from Bodies before they come to touch them, as also the Emission of Light in luminous Bodies, seem effected by such a repulsive Power; which (as has been said) commences where the Sphere of Attraction ends.

20. The Spherical Figure, which the Drops of a Fluid affect being left to themselves, is another Consequence of Corpuscular Attraction; for considering one single Particle of the Fluid, its attractive Force being every Way equally exerted to an equal Distance, it must follow that other Fluid Particles are on every Side drawn to it, and will therefore take their Places at an equal Distance from it, and consequently form a spherical or perfectly round Superficies; in like Manner as the Earth and Sea affect a spherical Figure from the central Attraction being every Way equally directed.

21. The surprising *Phænomena* of Glass Drops (sometimes called Prince *Rupert's* Drops) are most probably supposed to arise from hence, *viz.* That while the Glass is in Fusion or melted State, the Particles are in a State of Repulsion; but being dropped into cold Water, it so condenses the Particles in the external Parts or Superficies, that they are thereby reduced within each other's Attraction, and by that Means form a Sort of hard Case which keeps confined the aforesaid Particles in a repulsive State; but when this Case is broke by breaking off the Tail of the Drop, the said confined Particles have then Liberty to exert their Force, which they do by bursting the Body of the Drop, and reducing it to a very peculiar Form of Powder. This Theory seems to be corroborated by making the Drop red hot, and letting it cool gently in the open Air, for then it has no such Effect. Yet another Experiment seems to impugn this Hypothesis; and that is, by grinding away any Part of the Drop on a Grind-stone, the remaining Part continues entire, and yet there appears no manifest Reason why it should not break or burst into Dust, if the internal Particles be the Cause of it, since by this Means they must needs be set at Liberty in the most ample Manner possible: Unless it be, that in Grinding, the Vacuities between the internal Particles are fill'd up with

THE *Second Species of Attraction* is that of ELECTRICAL BODIES, as *Glass*, *Amber*, *Sealing-Wax*, *Jet*, &c. the principal Properties of which are as follow. (1.) Those Bodies attract others which are very light, as Feathers, Hairs, Leaf-Brass, *&c.* (2.) The Sphere of Extent of this attracting Power is at the Distance of several Feet; but,

with the Matter of the Stone forced in by the hard Attrition, and by this Means, fixing the Particles of the Glass next the Stone; and thus destroying their repulsive Force, constituting, as it were, another Sort of hard external Case, which confines the Particles no less than the other. And this seems to be confirm'd by that Part which touch'd the Stone remaining entire, when all the Rest of the Stone is reduced to Dust by breaking off the Tail or small Part, as any one will find to be the Case on Trial.

SCHOLIUM.

By this wonderful and most extensive Power of Corpuscular *Attraction* and *Repulsion*, we account for most or all the *Phænomena* of Nature, arising from the heterogeneous Mixture and various Associations of Particles of different Bodies; but why, in some Cases, those Particles should readily attract, and in others as strangely repel each other, there has as yet been no demonstrative Reason assign'd. The Hypothesis of a *Polarity*, or *Magnetic Power*, in every Particle of Matter, by which they are disposed on one Part to attract, and on the other to repel, in Manner of the Load-stone, is as specious as it is ingenious; but since the *Newtonian Philosophy*, in fundamental Principles especially, is founded entirely on the Basis of experimental Proof, or Demonstration from Geometry, we ought to be careful how we introduce any Conjecture for Truth, unless all Circumstances conclude and urge the same, which I believe is not the Case of that above-mentioned.

(3.) It varies with the State of the Weather, being greateſt in hot and dry Weather, but weaker in warm and moiſt. (4.) It may be communicated to a great Diſtance, *viz.* ſeven or eight hundred Yards, by the Intervention of a proper Body, as Hempen Strings, *&c.* (5.) This Virtue is excited by Attrition, or hard Rubbing, by the Hand or with a Piece of Cloth, but will not be produced by the Warmth of Fire. (6.) It penetrates or pervades the Pores of Glaſs; and, (7.) It may be communicated to other Bodies, ſo as to render them electrical. (8.) If the Sphere of Attraction be interrupted on any Part, it deſtroys the Efficacy of the Whole. (9.) By this Virtue Bodies are not only attracted, but alſo repelled alternately to a very ſenſible Diſtance, and with a ſurpriſing Velocity. (10.) The Body once repell'd from the Tube will not be again attracted by it, till it has firſt touch'd ſome other Body. (11.) Bodies attracted, and ſticking to the Tube, will be then attracted by other Bodies not electrical; as the Finger, *&c.* (12.) This Virtue will exert itſelf *in Vacuo*, as well as in open Air. (13.) It appears lucid, and ſparkles like Fire, in a dark Room. (14.) It is alſo ſenſible to the

the Ear by a crackling Noise, like a green Leaf in the Fire (XIV).

THE

(XIV) 1. To the Properties of Electrical Bodies, here enumerated, we shall add the following.—This Quality is of two Sorts, *viz.* (1.) *Vitreous Electricity*, or that which belongs to *Glass*; and (2.) *Resinous Electricity*, or that which belongs to *Amber*, *Rosin*, *Wax*, *Gums*, and such like Substances. They are different Kinds, because their Effects are contrary; for the light Bodies repell'd by one will be attracted by the other, tho' excited, which cannot happen when both Bodies have the same attractive Power.

2. All Vegetable Substances differ from Animal ones in this Respect also; for an Hempen or Flaxen Thread or String will receive the Electricity of Glass, and carry it instantaneously to an incredible Distance, to some non-electric Body, but will not impart it to a Silk or Hair Line by which it is suspended. This is the Reason why several Men standing on Cakes of Rosin or Wax (which will not receive the *Vitreous Electricity*) convey that Virtue from one to the other in a most surprising Manner. Thus also it is that a Feather suspended by a Flaxen Thread will be alternately attracted and repell'd, but if hung by a Silken String will not be again attracted after it is once repell'd; and *vice versa*, with respect to Amber or Sealing-Wax.

3. Hitherto we have not discover'd the Uses of this wondrous Quality; but Mr. *Gray*, just before he died, hit upon an Experiment of this Kind, which seem'd to indicate that the Attractive Power, which regulates the Motions of the Heavenly Bodies, is of the Electric Kind. The Experiment was thus; he fixed a large round Iron Ball upon the Middle of a large Cake of Rosin and Wax, and exciting the Virtue strongly in the Cake, a fine Feather suspended by a Thread, and held near the Iron Ball, was carried round it by the Effluvia, in a circular Manner, and perform'd several Revolutions; it moved the same Way with the Planets, from West to East, and its Motion (like theirs) was not quite *circular*

THE *Third Species of Attraction* is that of the MAGNET or LOADSTONE; the primary Properties whereof are the following. (1.) Every Loadstone has two Points call'd *Poles*, which emit the Magnetic Virtue. (2.) One of those Poles attracts, the other repels *Iron*, but no other Body. (3.) This Virtue is communicated to Iron very copiously by the Touch, which renders it strongly Magnetic. (4.) A Piece of Iron so touch'd by the Loadstone, and nicely suspended on a sharp Point, will be determined to settle itself in a Direction nearly *North* and *South*. (5.) The End of the Needle touch'd by the *South* Pole of the Stone will point *Northwards*; and the contrary. (6.) Needles touch'd by the Stone will dip below the Horizon, or be directed on the touch'd Part to a Point within the Earth's

cular but a little *elliptical*. See the whole Account in *Philos. Trans.* N° 444.

Many other Experiments of this kind have been tried since; but when they are rightly considered they will be found of no Service in accounting for the Motions of the Heavenly Bodies, which depend on much more rational and certain Principles, as we shall shew in a proper Place.

Since the first Edition of this Work there have been a great Number of new Discoveries and Improvements in ELECTRICITY and MAGNETISM, which we must by no Means suffer to be wanting in a Compleat System of Philosophy, and therefore shall add an Appendix to this Lecture, giving an Account of the same.

Earth's Surface. This is call'd the *Dipping Needle.* (7.) This Virtue is also to be communicated to Iron, by a strong Attrition all one Way; whence Files, Drills, *&c.* are always found to be Magnetical. (8) Iron Rods or Bars acquire a Magnetic Virtue by standing long in one Position. (9.) Fire totally destroys this Virtue by making the Stone or Iron red hot. (10.) This Power is exerted sensibly to the Distance of several Feet. (11.) It is sensibly continued through the Substance of several contiguous Bodies or Pieces of Iron, as *Keys,* &c. (12.) It pervades the Pores of the hardest Bodies; and (13.) Equally attracts the Iron *in Vacuo*, as in open Air (XV).

THESE,

(XV) 1. To the above Properties of the Loadstone I shall add the following, viz. *That the same Pole of the Stone will communicate to a Piece of Iron the Power of attracting or repelling the same End of a touch'd Needle, by drawing it different Ways thereon.* Or thus, If a Piece of Iron be drawn to the Right Hand and attracts, it will repel if drawn to the Left, which is not a little wonderful.

2. I have oftentimes, by a smart Stroke of a Hammer on the untouch'd End of the Dipping-Needle, caused the Magnetic Virtue to come all to that End from the other, so as to make it dip on that Side as much as it did on the other Side before. And on the contrary, I have by such a Stroke sometimes made it dip much more on the touch'd End than before. Again, sometimes by striking it, the Needle, which dipp'd before,

THESE, and many others are the Properties of a Body, not more wonderful than useful to Mankind.

before, will be restor'd to its Equilibrium, as if the Virtue had made its Escape, or were uniformly diffused over all the Needle. So capricious are the Phænomena of this amazing Power.

3. There is a very curious Method of rendering visible the Directions which the Magnetic Effluvia take in going out of the Stone: Thus, let A B, C D, be the Poles of the Stone; about every Side gently strew some Steel-Filings on a Sheet of white Paper; these small Particles will be affected by the Effluvia of the Stone, and so posited as to shew the Course and Direction of the Magnetic Particles on every Part. Thus in the Middle of each Pole between A B, and D C, it appears to go nearly strait on; towards the Sides it proceeds in Lines more and more curved, till at last the Curve-Lines from both Poles exactly meeting and coinciding, form numberless Curves on each Side, nearly of a circular Figure, as represented in the Diagram. This seems to shew that the Magnetic Virtue emitted from one Pole circulates to and enters the other.

Plate II. Fig. 1.

4. The Cause of the Variation of the Needle has remain'd hitherto without any demonstrative Discovery; yet since its Declination, and Inclination, (or Dipping) do both of them manifestly indicate the Cause to be somewhere in the Earth, it has given Occasion to Philosophers to frame Hypotheses for a Solution, which make the Earth a large and general *Magnet* or Loadstone, of which all the lesser ones are but so many Parts or Fragments, and, being possess'd of the same Virtue, will, when left to move freely, have the same Disposition, and Similarity of Position, and other Circumstances.

5. The most considerable of these Hypotheses is that of the late sagacious Dr. *Halley*, which is this; *The Globe of the Earth is one great Magnet, having four Magnetical Poles or Points of Attraction, near each Pole of the Equator two; and that in those Parts of the World which lie*

lie near adjacent to any one of those Magnetical Poles, the Needle is chiefly governed thereby, the nearest Pole being always predominant over the more remote one. Of the North Poles, that which is nearest to us, he supposes to be in the Meridian of the Land's End, and about seven Degrees from the North Pole of the World, which governs the Variations in *Europe*, *Tartary*, and the *North* Sea; the other he places in a Meridian passing thro' *California*, about fifteen Degrees from the North Pole of the World, which governs the Needle in North *America*, and the Oceans on either Side. In like Manner he accounts for the Variations in the Southern Hemisphere. See *Philos. Trans.* N° 148.

6. The Variation of the Needle from the North and South Points of the Horizon not being the same, but variable in different Years and in a diverse Manner in different Parts of the Earth, made the *Doctor* farther conjecture, that two of the Magnetic Poles were fixed, and two moveable; and in order to make this out, he supposes the external Part of the Earth to be a Shell or *Cortex*, containing within it a Magnetic moveable *Nucleus*, of a globular Form, whose Centre of Gravity is the same with that of the Earth, and moveable about the same Axis.

7. Now if the Motions of both the Shell and the *Nucleus* were the same, the Poles of each would always have the same Position to each other; but he supposes the Motion of the *Nucleus* to be a very small Matter less than that of the Shell, which yet is scarce sensible in 365 Revolutions; and if so, the Magnetic Poles of the *Nucleus* will by slow Degrees change their Distance from the Magnetic Poles of the Shell, and thus cause a Variation in that Needle's Variation, which is govern'd by the moveable Pole of the *Nucleus*; while that Variation which respects the fix'd Poles of the Magnetic Shell remains more constant; as in *Hudson*'s Bay the Change is not observed to be near so fast as in these Parts of *Europe*. And it appears from some late Accounts, that in some of the northernmost Parts of the Bay the Needle loses its Magnetism, and which has been recovered again by laying the Needle by a Fire, and keeping it in a warm Room.

8. What

8. What seems a little strange is, that the *Doctor* has no where, that I know of, undertook to account for the Dipping of the Needle by this Hypothesis; tho' the Invention of this (by Mr. *Blagrave*) was before that of the Change of the *Needle's Variation* (by Mr. *Gellibrand*;) nor do I see at present which Way this *Phænomenon* is explicable by it. But we have not yet so many accurate Observations of the *Needle's Inclination* as we have of its *Variation*, which is its only useful Property. By several Experiments Mr. *Graham* very accurately made it appear, that the Quantity of the Needle's Inclination to the horizontal Line was an Angle of about 74 or 75 Degrees: That is, suppose AB a touch'd Needle, supported on the Point C of the Pin CD, it will contain an Angle ACH or BCO with the horizontal Line HO of 74 or 75 Degrees. (Plate I. Fig. 20.)

9. The Variation of the Needle has within a Century past undergone a remarkable Alteration; for at *London* it was observed by Mr. *Burrows* in the Year 1580 to be 11°. 15'. East; that is, if N, S, represent the North and South Points of the Horizon, and E, W, the East and West Points, the Needle then had the Situation AB, so that the Arch NB = 11°. 15'. (Plate II. Fig. 2.) After that, in the Year 1622, it was observed by Mr. *Gunter* to be but 6° East. In the Year 1634 Mr. *Gellibrand* observed it to be 4°. 5'. East. In 1657, it was observed by Mr. *Bond* to be nothing at all, that is, the Needle placed itself then in the Situation SN, and pointed directly to the North. After this, in the Year 1672, Dr. *Halley* observed it to be 2°. 30'. Westward; and again in the Year 1692, he found it 6°. West. Since then, in the Year 1722, Mr. *Graham*, by most accurate Experiments, found it to be 14°. 13'. And at present it is very near 22 Degrees; by some accurate Observations this Summer it was found to be 21° 50'.

10. The *Variation* of the *Declination* and *Inclination* of the Needle is variable and subject to no regular Computation. What the Quantity of both Sorts of Variation is in the several Parts of the World, will be shewn in Dr. *Halley*'s Map of the World, improved from the Obser-

Plate I.

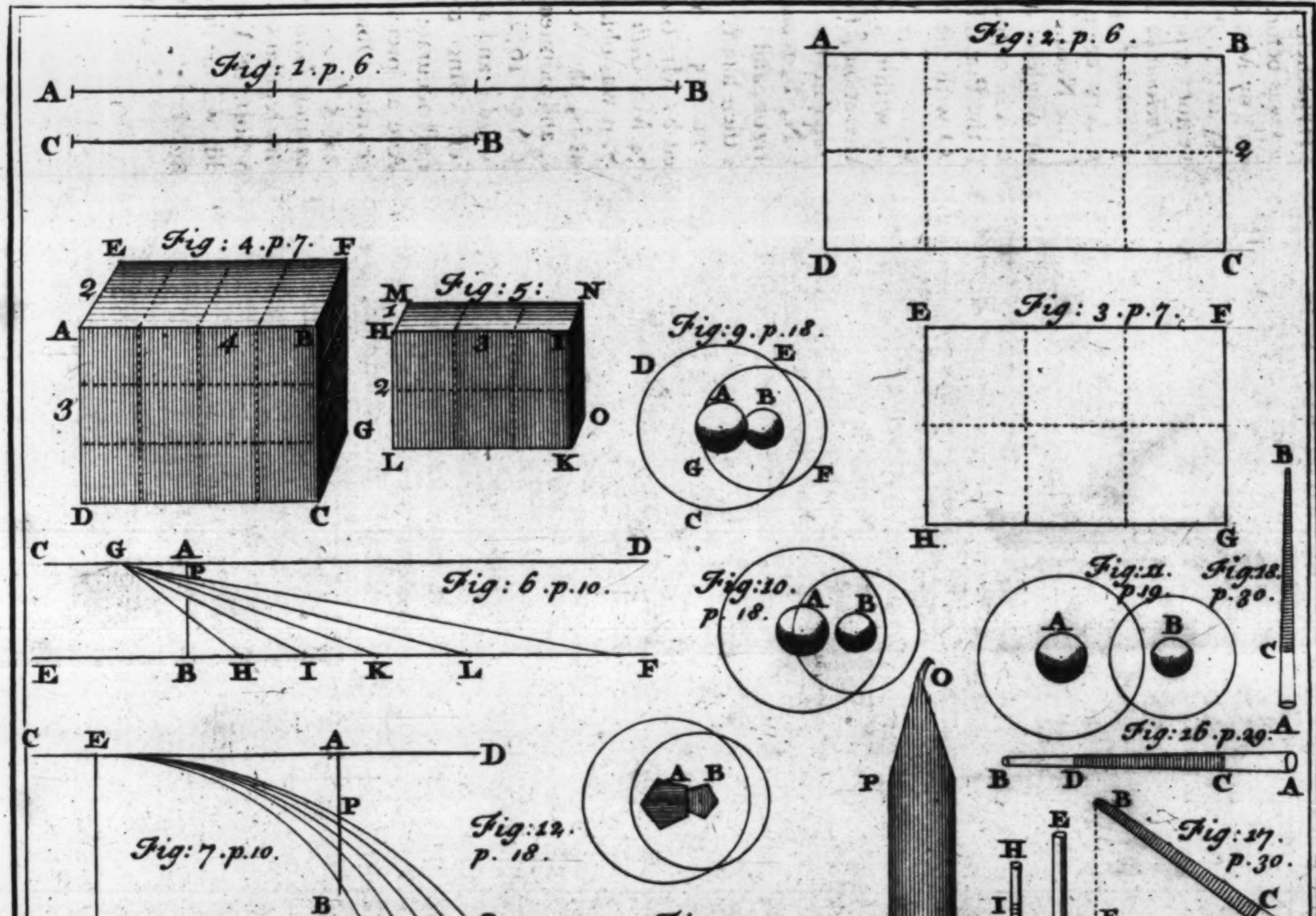


Fig: 1. p. 6.
A
B
C
B
Fig: 2. p. 6.
A
B
D
C
Fig: 3. p. 7.
E
F
H.
G
Fig: 4. p. 7.
E
F
A
B
D
C
G
Fig: 5:
M
N
H
I
L
K
O
Fig: 9. p. 18.
D
E
A
B
G
F
C
Fig: 6. p. 10.
C
G
A
P
D
E
B
H
I
K
L
F
Fig: 10. p. 18.
Fig: 11. p. 19.
Fig: 18. p. 30.
Fig: 26. p. 29.
Fig: 7. p. 10.
Fig: 12. p. 18.
Fig: 27. p. 30.

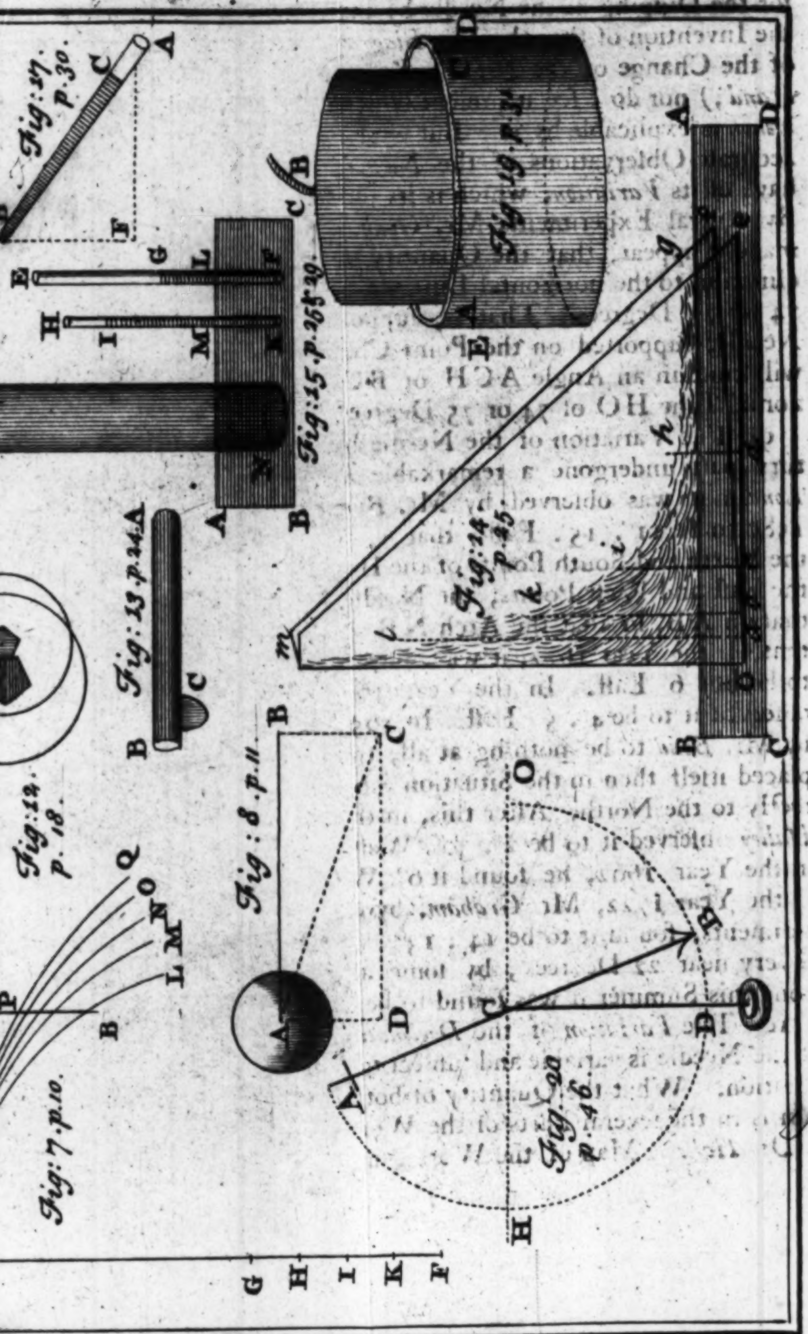


Fig: 17. p. 30.
Fig: 15. p. 26 & 29.
Fig: 19.
Fig: 14
Fig: 13. p. 24.
Fig: 12. p. 18.
Fig: 8. p. 11.
Fig: 7. p. 10.
Fig: 20 p. 46.

Observations of Mr. *Pound*, which is inserted at the End of the LECTURE on WIND and SOUND.

11. I think the Law of Magnetic Attraction is not yet ascertained; Sir *Isaac Newton* supposes it to decrease nearly in the *Triplicate Ratio* of the Distance: But Dr. *Helsham*, trying the Experiment with his Loadstone, found it to be as the *Squares of the Distances* inversely. And the Power of my Loadstone decreases in a different Manner from either, it being in the *Sesquiplicate Ratio* of the Distances inversely. For Exactness, I made a square Bar of Iron just $\frac{1}{4}$ of an Inch thick, and then provided three Pieces of Wood of the same Form and Thickness precisely. Then poising the Loadstone very nicely at the End of a Balance which would turn with less than a Grain, I placed under it the Iron with first one Piece of Wood, then two Pieces, and lastly all three Pieces upon it, by which means the Steel Points of the Pole were kept at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of an Inch from the Iron, and in those Distances the Weights put into the opposite Scale to raise the Loadstone from the Wood (which it touch'd while the Beam was horizontal) were as follows:

		Grains.	*Ratio of the Squares.*	*Ratio of the Cubes.*	*Sesquip. Ratio.*
Distances	$\frac{1}{4}$	156	156	156	156
	$\frac{1}{2}$	58	39	19	56
	$\frac{3}{4}$	28	17	6	30

12. Whence it appears, that the Numbers of Grains to counter-act the Power of the Loadstone in these Distances approach very near, and are almost the same with those which are in the *Sesquiplicate Ratio*, but are widely different from those which are in the *Duplicate* or *Triplicate Ratio*; and this Experiment I tried several Times for each Distance, and with scarce any Variation in the Success.

13. The Power or Force of Magnets is generally greater in small than in large ones, in Proportion to their Bulk. It is very rare that very large ones will take up more than 3 or 4 Times their own Weight; but a small one is but tolerably good that will take up no more than 8, 10, or 12 Times its Weight. The Honourable Mr. *Berkley*, at *Bruton*, has one whose

Weight

Weight (with the Armature) is but 43 Grains, which will take up 1032 Grains, which is 24 Times its Weight. But that of Mr. *Newton* (which he wears in his Ring, instead of a Diamond of less Value) which weighs scarce THREE *Grains*, will take up 746 *Grains*, which is 250 Times its Weight, which is by far the strongest and best of any I have seen; and therefore, as a great
Fig. 3. Curiosity, I have given a Print of the RING and STONE with the *Iron* hanging to it.

14. Iron Bars by standing long in the same Position will acquire the Magnetic Virtue to a surprizing Degree. I have seen one about 10 Feet long and 3 Inches thick, (supporting the *Summer Beam* of a Room) which has been able to turn the Needle at 8 or 10 Feet Distance, and exceeded a Loadstone of $3\frac{1}{2}$ Pounds Weight. From the middle Point upwards it was a North Pole, and downwards a South Pole; and I have by me an Iron Rod which has several Poles or Parts which change the Position of the Magnetic Needle: It is the Beam of a
Fig. 7. large Steelyard, which let AE represent; then at A, and from thence to B, the North End of the Needle is attracted; at B, and from thence to C, the North End is repell'd and the South End attracted; from C to D, the North End is again attracted and the South End repell'd, and from D to E the South End is attracted, and the North End repell'd; lastly, at the End E begins another Pole, and there the Needle stands nearly parallel with the Beam, with neither End attracted.

15. It will be proper here to acquaint the Reader with the several Methods of communicating the Magnetic Virtue to Iron, in so great a Degree, that such a Piece of Iron or Steel is deservedly call'd an *Artificial Magnet* or *Loadstone*; and as it possesses all the Properties, so it is used to all the Purposes of the Natural Loadstone.

16. The first Way of making an Artificial Magnet is by preparing several Steel *Laminæ*, or thin Plates, about 6 or 8 Inches long, and half an Inch broad; these well polished, and properly touch'd on a very strong Loadstone, at each End, and then put together and arm'd with Steel Points like the common Stone, will become an Artificial Magnet.

Vol I. p. 48. Plate II.

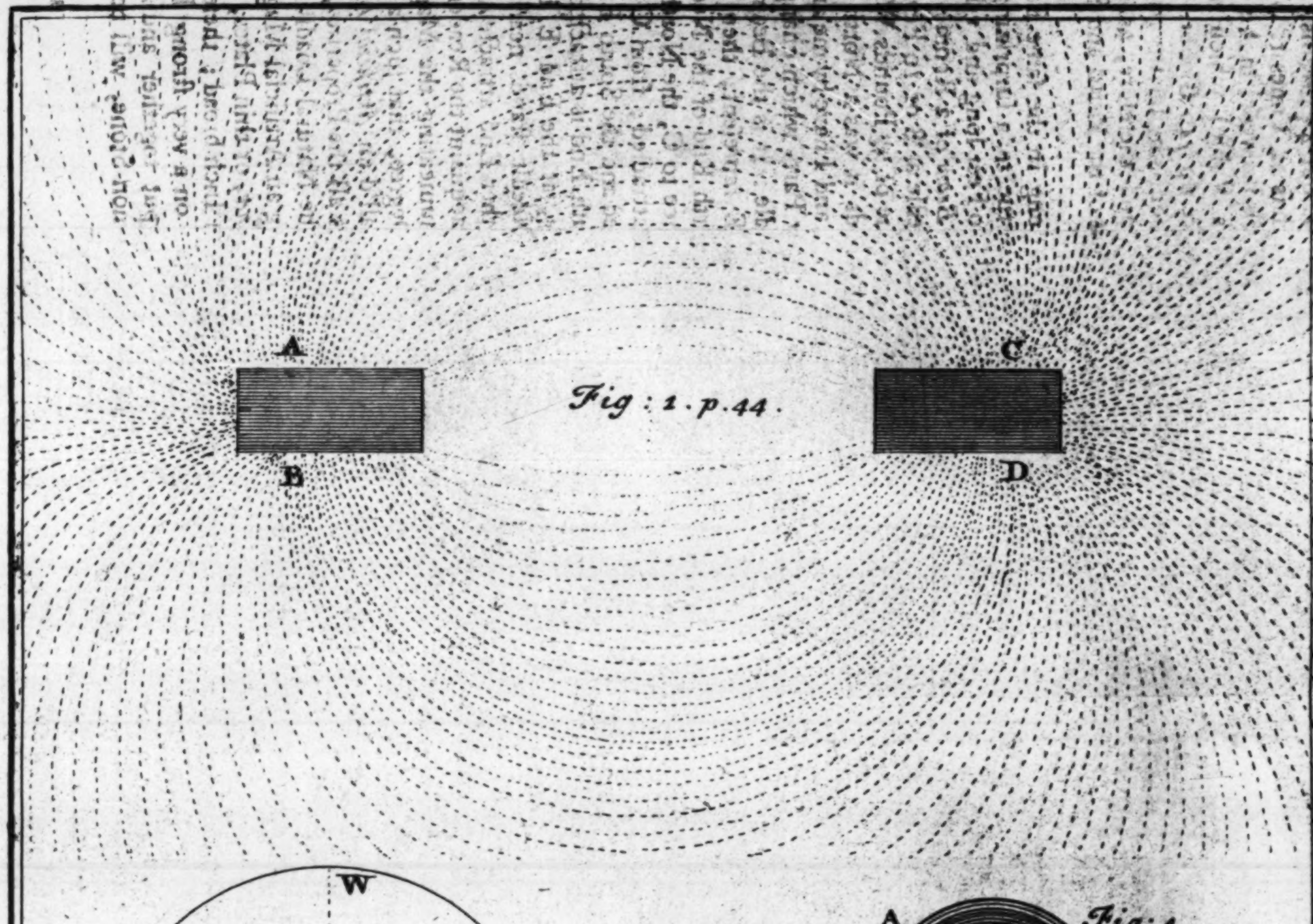


Fig : 1. p. 44.

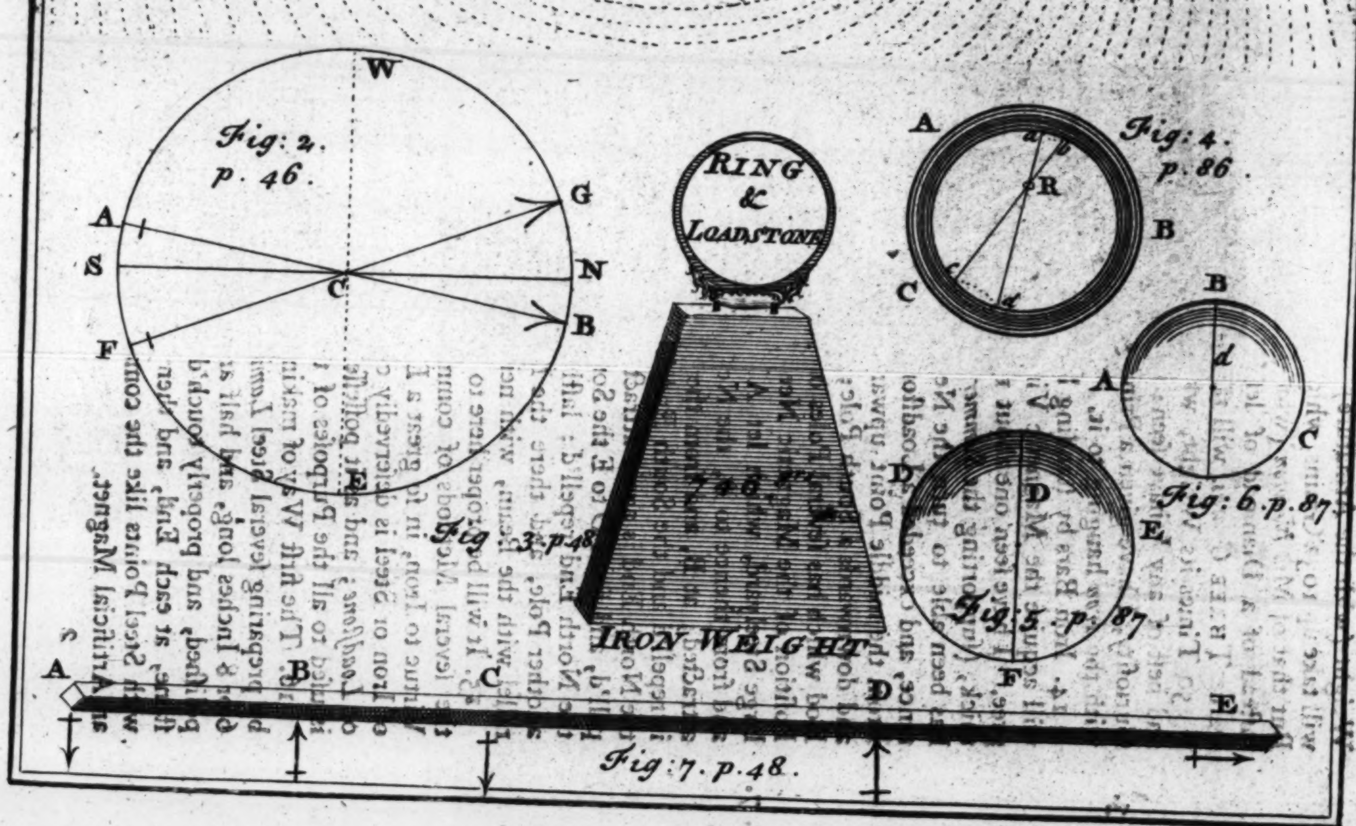


Fig: 2.
p. 46.
W
A
S
F
C
E
G
N
B
RING
&
LOADSTONE
Fig. 3. p.48.
IRON WEIGHT
Fig: 4.
p. 86.
R
Fig: 5. p. 87.
Fig: 6. p. 87.
D
E
Fig: 7. p. 48.

17. A ſecond Way is by preparing ſolid Pieces of Steel, and having wrought them to a proper Figure, poliſhed, and armed them with Steel Armour; if then one of them be properly held in the Magnetic Meridian Line, and the other which is armed be drawn ſeveral Times along it, it will conceive a great Degree of Magnetiſm; if then the Armour be removed from this and placed on the other, and that in like Manner touched on this, it will become magnetical. But to deſcribe the particular Manner, with all its Circumſtances, would be too tedious; I ſhall therefore refer the Reader to the Account given by the reputed Inventor, Mr. *Savery* *, who was the firſt that publiſhed any conſiderable Diſcoveries relative to *Artificial Magnetiſm*; but I have heard Mr. *Lovelace*, ſenior, affirm, that he was, before that Time, poſſeſſed of the Secret, and communicated it to Mr. *Savery*.

18. A third Way is to provide a Piece of Steel, and continue rubbing it *very hard all one Way* with a poliſhed Steel Inſtrument, which will, by this Sort of Friction, conceive ſo great a Degree of Virtue, as to become an *Artificial Magnet*, of greater Force, very often, than the natural ones. A ſmall Piece of Wire thus ordered, will be as tender, and point as nicely Northwards, as any one touched by the Natural Magnet.

19. A fourth Way is to heat a Piece of Steel red hot, and then quench it in Water, being held therein in ſuch a Poſition as the Dipping-Needle takes when ſuſpended on a ſharp Point. This Method will ſucceed only in the Hand of thoſe who by frequent Trials have acquired a peculiar Sagacity this Way; and indeed the ſame may be ſaid of all the others. It is Experience only that gives a Perſon the Ingenuity neceſſary for ſucceeding in all ſuch Affairs.

20. A fifth Way is by ſetting a Bar of Iron or Steel in a proper Place, and there letting it ſtand for a long Time in a Poſition unaltered, and it will acquire a very great Magnetic Virtue, as has been already taken Notice of; a perpendicular Poſition is beſt; and the larger the Bar, and the longer it ſtands, the better or ſtronger it will be.

* *Philoſ. Tranſ.* N° 414.

21. A ſixth Method of giving this Virtue to Iron in a ſurpriſing Degree, and alſo to the Loadſtone itſelf, yea of altering or reverſing its Poles, and diſpoſſeſſing it of its Virtue, almoſt in any Manner, was long a Secret in the Hands of a few, but is now well known; and for the Reader's practical Entertainment, as well as Inſtruction, we ſhall give a Plate in the following Appendix, repreſenting a particular Method of communicating this Virtue in the greateſt Degree, as invented by a Gentleman well known for a peculiar *Enchireſis* and great Succeſs in *Magnetical* and *Electrical Experiments.*

N. B. Since the following APPENDIX ON ELECTRICITY was printed off, I have been appriſed of a *New Diſcovery* highly conducing to the Illuſtration of that hitherto almoſt unintelligible Subject. A Stone (ſaid to be brought from the Iſle of *Ceylon*, and is called *Tour Malin)* has lately been found to be poſſeſſed of the Electrical Property in a very ſingular and ſurpriſing Manner. It is the *firſt Stone*, or hard and unelaſtic Body, I believe, that was ever experienced to be electrical. It is the *firſt Subject* that has been found to have the electric Virtue excited *without Friction*; for *Heat* or Warmth alone is ſufficient to render it very ſenſibly electrical. It is the *firſt Subſtance* found to poſſeſs a twofold Species of Electricity; *Glaſs* has only one Sort, and Amber only one other, contrary to the former; but this wonderful Stone poſſeſſes *both Sorts*, the Vitreous on one Side, and the Reſinous on the other; in ſhort, all Kinds of electrical Experiments are ſhewn by *it alone*; which can be ſaid of no other Body. It is ſuppoſed this Diſcovery will tend greatly to illuſtrate, if not wholly confirm, the new Doctrine of *Electrical Thunder and Lightning.*

APPENDIX

To LECTURE I.

PART I.

Concerning the Improvements and new Experiments in ELECTRICITY.

THE Improvements in Electricity have been ſo great and numerous, that it would be almoſt in vain to attempt even the epitomizing of them all. We muſt therefore refer our inquiſitive Readers to the ſeveral Authors who have wrote on this Subject; but ſtill we think it neceſſary in a Treatiſe of this Kind, that there ſhould be a larger Account of this wonderful Power than what was given in the firſt Edition of this Work; and therefore we have added by Way of Appendix, two Papers of Experiments by Mr. CANTON, as we find them in the Tranſactions of the Royal Society. The firſt of which is contained in N° LIII. of Vol. XLVIII. Part I. The Title of which is,

Electrical Experiments, with an Attempt to account for their several Phænomena; together with some Observations on Thunder-Clouds, *by* John Canton, *M. A. and F. R. S.*

Experiment 1.

FROM the Cieling, or any convenient Part of a Room, let two Cord-balls, each about the Bigness of a small Pea, be suspended by Linen Threads of eight or nine Inches in Length, so as to be in contact with each other. Bring the excited glass Tube under the Balls, and they will be separated by it, when held at the Distance of three or four Feet; let it be brought nearer, and they will stand farther apart; intirely withdraw it, and they will immediately come together. This Experiment may be made with very small Brass Balls hung by Silver Wire; and will succeed as well with Sealing-wax made electrical, as with Glass.

Experiment 2.

IF two Cork-Balls be suspended by dry Silk Threads, the excited Tube must be brought within eighteen Inches before they will

will repel each other; which they will continue to do, for ſome Time, after the Tube is taken away.

As the Balls in the firſt Experiment are not inſulated, they cannot properly be ſaid to be electrified: But when they hang within the Atmoſphere of the excited Tube, they may attract and condenſe the electrical Fluid round about them, and be ſeparated by the Repulſion of its Particles. It is conjectured alſo, that the Balls at this Time contain leſs than their common Share of the electrical Fluid, on Account of the repelling Power of that which ſurrounds them; though ſome, perhaps, is continually entering and paſſing through the Threads. And if that be the Caſe, the Reaſon is plain, why the Balls hung by Silk, in the ſecond Experiment, muſt be in a much more denſe Part of the Atmoſphere of the Tube, before they will repel each other. At the Approach of an excited Stick of Wax to the Balls, in the firſt Experiment, the electrical Matter is ſuppoſed to come through the Threads into the Balls, and be condenſed there, in its Paſſage towards the Wax: For, according to Mr. *Franklin*, excited Glaſs *emits* the electrical Fluid, but excited Wax *receives* it.

Experiment 3.

LET a Tin Tube, of four or five Feet in Length, and about two Inches in Diameter, be inſulated by Silk; and from one End of it let the Cork-balls be ſuſpended by Linen Threads. Electrify it, by bringing the excited Glaſs Tube near the other End, ſo as that the Balls may ſtand an Inch and a Half, or two Inches apart: Then, at the Approach of the excited Tube, they will by Degrees loſe their repelling Power, and come into Contact; and as the Tube is brought ſtill nearer, they will ſeparate again to as great a Diſtance as before: In the Return of the Tube they will approach each other till they touch, and then repel as at firſt. If the Tin Tube be electrified by Wax, or the Wire of a charged Phial, the Balls will be affected in the ſame Manner at the Approach of excited Wax, or the Wire of the Phial.

Experiment 4.

ELECTRIFY the Cork-Balls as in the laſt Experiment by Glaſs; and at the Approach of an excited Stick of Wax their Repulſion will be increaſed. The Effect will be the ſame,

ſame, if the excited Glaſs be brought towards them, when they have been electrified by Wax.

THE bringing the excited Glaſs to the End, or Edge of the Tin Tube, in the third Experiment, is ſuppoſed to electrify it poſitively, or to add to the electrical Matter it before contained; and therefore ſome will be running off through the Balls, and they will repel each other. But at the Approach of excited Glaſs, which likewiſe *emits* the electrical Fluid, the Diſcharge of it from the Balls will be diminiſhed; or Part will be driven back, by a Force acting in a contrary Direction; and they will come nearer together. If the Tube be held at ſuch a Diſtance from the Balls, that the Exceſs of the Denſity of the Fluid round about them, above the common Quantity in Air, be equal to the Exceſs of the Denſity of that within them, above the common Quantity contained in Cork; their Repulſion will be quite deſtroyed. But if the Tube be brought nearer; the Fluid without, being more denſe than that within the Balls, it will be attracted by them, and they will recede from each other again.

WHEN the Apparatus has loſt Part of its natural Share of this Fluid, by the Approach of excited Wax to one End of it, or is electrified negatively; the electrical Fire is attracted and imbibed by the Balls to ſupply the Deficiency; and that more plentifully at the Approach of excited Glaſs, or a Body poſitively electrified, than before; whence the Diſtance between the Balls will be increaſed, as the Fluid ſurrounding them is augmented. And in general, whether by the Approach or Receſs of any Body; if the Difference between the Denſity of the internal and external Fluid be increaſed or diminiſhed; the Repulſion of the Balls will be increaſed, or diminiſhed accordingly.

Experiment 5.

WHEN the inſulated Tin Tube is not electrified, bring the excited Glaſs Tube towards the Middle of it, ſo as to be nearly at Right Angles with it, and the Balls at the End will repel each other; and the more ſo, as the excited Tube is brought nearer. When it has been held a few Seconds, at the Diſtance of about ſix Inches, withdraw it, and the Balls will approach

each

each other till they touch; and then ſeparating again, as the Tube is moved farther off, will continue to repel when it is taken quite away. And this Repulſion between the Balls will be increaſed by the Approach of excited Glaſs, but diminiſhed by excited Wax; juſt as if the Apparatus had been electrified by Wax, after the Manner deſcribed in the third Experiment.

Experiment 6.

INSULATE two Tin Tubes, diſtinguiſhed by *A* and *B*, ſo as to be in a Line with each other, and about half an Inch apart; and at the remote End of each, let a Pair of Cork-Balls be ſuſpended. Towards the Middle of *A*, bring the excited Glaſs Tube; and holding it a ſhort Time at the Diſtance of a few Inches, each Pair of Balls will be obſerved to ſeparate: Withdraw the Tube, and the Balls of *A* will come together, and then repel each other again; but thoſe of *B* will hardly be affected. By the Approach of the excited Glaſs Tube, held under the Balls of *A*, their Repulſion will be increaſed: But if the Tube be brought, in the ſame Manner, towards the Balls of *B*, their Repulſion will be diminiſhed.

IN

In the fifth Experiment, the common Stock of electrical Matter in the Tin Tube, is supposed to be attenuated about the Middle, and to be condensed at the Ends, by the repelling Power of the Atmosphere of the excited Glass Tube, when held near it. And perhaps the Tin Tube may lose some of its natural Quantity of the electrical Fluid, before it receives any from the Glass; as that Fluid will more readily run off from the Ends or Edges of it, than enter at the Middle: And accordingly, when the Glass Tube is withdrawn, and the Fluid is again equally diffused through the Apparatus, it is found to be electrified negatively: For excited Glass brought under the Balls will increase their Repulsion.

In the sixth Experiment, Part of the Fluid driven out of one Tin Tube enters the other; which is found to be electrified positively, by the decreasing of the Repulsion of its Balls, at the Approach of excited Glass.

Experiment 7.

Let the Tin Tube, with a Pair of Balls at one End, be placed three Feet at least from any Part of the Room, and the Air rendered very dry by Means of a Fire: Electrify

Electrify the Apparatus to a conſiderable Degree: then touch the Tin Tube with a Finger, or any other Conductor, and the Balls will, notwithſtanding, continue to repel each other; though not at ſo great a Diſtance as before.

THE Air ſurrounding the Apparatus to the Diſtance of two or three Feet, is ſuppoſed to contain more or leſs of the electrical Fire than its common Share, as the Tin Tube is electrified poſitively, or negatively; and when very dry, may not part with its Overplus, or have its Deficiency ſupplied ſo ſuddenly, as the Tin; but may continue to be electrified, after that has been touched for a conſiderable Time.

Experiment 8.

HAVING made the Torricellian Vacuum about five Feet long, after the Manner deſcribed in the *Philoſophical Tranſactions*, Vol. xlvii. p. 370. if the excited Tube be brought within a ſmall Diſtance of it, a Light will be ſeen through more than half its Length: Which ſoon vaniſhes, if the Tube be not brought nearer; but will appear again, as that is moved farther off. This may be repeated ſeveral Times, without exciting the Tube afreſh.

THIS

THIS Experiment may be conſidered as a Kind of ocular Demonſtration of the Truth of Mr. *Franklin*'s Hypotheſis; that when the electrical Fluid is condenſed on one Side of thin Glaſs, it will be repelled from the other, if it meets with no Reſiſtance. According to which, at the Approach of the excited Tube, the Fire is ſuppoſed to be repelled from the Inſide of the Glaſs ſurrounding the Vacuum, and to be carried off through the Columns of Mercury; but, as the Tube is withdrawn, the Fire is ſuppoſed to return.

Experiment 9.

LET an excited Stick of Wax, of two Feet and an Half in Length, and about an Inch in Diameter, be held near its Middle. Excite the Glaſs Tube, and draw it over one Half of it; then, turning it a little about its Axis, let the Tube be excited again, and drawn over the ſame Half; and let this Operation be repeated ſeveral Times: Then will that Half deſtroy the repelling Power of Balls electrified by Glaſs, and the other Half will increaſe it.

BY this Experiment it appears, that Wax alſo may be electrified poſitively and negatively. And it is probable, that all Bodies whatſoever

whatsoever may have the Quantity they contain of the electrical Fluid, increased, or diminished. The Clouds, I have observed, by a great Number of Experiments, to be some in a positive, and others in a negative State of Electricity. For the Cork-Balls, electrified by them, will sometimes close at the Approach of excited Glass; and at other Times be separated to a greater Distance. And this Change I have known to happen five or six Times in less than Half an Hour; the Balls coming together each Time, and remaining in Contact a few Seconds, before they repel each other again. It may likewise easily be discovered, by a charged Phial, whether the electrical Fire be drawn out of the Apparatus by a negative Cloud, or forced into it by a positive one: And by whichsoever it be electrified, should that Cloud either part with its Overplus, or have its Deficiency supplied suddenly, the Apparatus will lose its Electricity: Which is frequently observed to be the Case, immediately after a Flash of Lightning. Yet when the Air is very dry, the Apparatus will continue to be electrified for ten Minutes, or a Quarter of an Hour, after the Clouds have passed the Zenith; and some-

times till they appear more than Half-way towards the Horizon. Rain, eſpecially when the Drops are large, generally brings down the electrical Fire: And Hail, in Summer, I believe never fails. When the Apparatus was laſt electrified, it was by the Fall of thawing Snow; which happened ſo lately, as on the 12th of November; that being the twenty-ſixth Day, and ſixty-firſt Time, it has been electrified, ſince it was firſt ſet up; which was about the Middle of May. And as *Fahrenheit's* Thermometer was but ſeven Degrees above freezing, it is ſuppoſed the Winter will not entirely put a Stop to Obſervations of this Sort. At *London*, no more than two Thunder-Storms have happened during the whole Summer: And the Apparatus was ſometimes ſo ſtrongly electrified in one of them, that the Bells, which have been frequently rung by the Clouds, ſo loud as to be heard in every Room of the Houſe (the Doors being open), were ſilenced by the almoſt conſtant Stream of denſe electrical Fire, between each Bell and the Braſs Ball, which would not ſuffer it to ſtrike.

I SHALL conclude this Paper, already too long, with the following Queries:

1. MAY

1. MAY not Air, ſuddenly rarified, give electrical Fire to, and Air ſuddenly condenſed, receive electrical Fire from, Clouds and Vapours paſſing through it?

2. Is not the *Aurora Borealis*, the Flaſhing of electrical Fire from poſitive, towards negative Clouds at a great Diſtance, through the upper Part of the Atmoſphere, where the Reſiſtance is leaſt?

As the ingenious Author of the foregoing Paper has given no Plate to illuſtrate thoſe Experiments with Figures; and as he has ſince contrived to ſhew them in a more neat and elegant Manner by Means of *Drinking-Glaſſes*, we ſhall therefore repreſent the Experiments this Way, as it may be eaſily practiſed by any private Perſon for his own Amuſement. Therefore, in *Fig.* 1. let AB and CD repreſent two Pieces of Wood about ſix or eight Inches long, half an Inch wide, and $\frac{1}{10}$ of an Inch thick; and at the End of each, let two fine Flaxen Threads be fixed, of about ſix Inches in Length, with two round Balls (about the Size of a white Pea, made of the Pith of Elder) be ſuſpended; then if the excited Tube be brought under the Balls at B, they will ſeparate, and repel each other, while the

Plate I. to this Appendix.

the Balls hanging from CD remain in Contact, placed at a Diſtance AC from the other; but if, while the Balls at B are electrified, the Glaſs be moved ſo that the Piece of Wood AB come in Contact with the Piece CD, then the Balls at B will loſe half the electric Fluid, which will be communicated to the Balls at D, and they will now repel each other; and the Diſtance between the Balls at B and D will be but half what it was at firſt between the two Balls at B. This Caſe is repreſented by the two Pieces of Wood at EF and GH.

THE Pieces of Wood at IK and LM repreſent the Caſe of the ſixth Experiment, where the Balls of LM are electrified negatively, and thoſe of IK poſitively.

THIS Method of ſupporting the Pieces of Wood with the Balls on Glaſſes, may be applied to a great Variety of curious Experiments, with regard to poſitive and negative Electrification, under all the different Circumſtances which have been mentioned in this Paper, and in thoſe that follow in the next by the ſame Author.

THE Torricellian *Vacuum*, mentioned in the eighth Experiment, is here repreſented in *Figure* the 2d, where *a c b* repreſents

ſents a hollow incurved Tube of Glaſs, whoſe Heighth *a c* is about three Feet. This Tube is firſt filled with pure Mercury, and then the Orifice of each Leg is inverted in two ſmall Baſons of Mercury *a b*. The Mercury in the Tube will ſubſide from the upper Part *c* to the equal Heighths *n o* in each Leg; then will there be a *Vacuum*, as in the common Barometer, in all the upper Part of the Tube. This Tube is then fixed in the Frame A B C D, either hapſed or tied on with Strings, as repreſented at *d g e f c*, which Frame is faſtened to the Side of the Room. If now, from the Prime Conductor *i k*, a Wire or Flaxen String *h* carry the electric Matter to the Baſon *b*, it will run through the Quick-Silver into the vacuous Part of the Tube, and produce the ſurpriſing Appearance of a Torrent of electrical Fire in the darkened Room, which will continue as long as the electrical Machine is in Motion.

THE 3d *Figure* repreſents a tall Glaſs Receiver, exhauſted by the Air-Pump; and a pointed Wire paſſing through a Cork in the Top conducts the electrical Fluid from the Machine to this *Vacuum*. The Machine being put in Motion, and the Room darkened, the Spectator will obſerve the

Electricity deſcending from the Point of the Iron, in the Appearance of a liquid Stream of Fire, of a pale whitiſh Hue, like *Phoſphorus*, and of a thick Conſiſtence, like Cream; which plainly ſhew that the electrical Matter, as it is produced from common Bodies, is of a very different Nature from the Matter of common Light or Fire, as it hath a very different Colour, Smell, and Conſiſtence; and that it is not the pure Matter of Light, or indeed any Thing ſimilar to it, is evident from hence, that it has no different Refrangibility in its Rays, which is the well known Property of Light; and therefore upon the Whole we may conclude, that if Lightning, the *Aurora Borealis*, &c. be the Effects of Electricity, the Matter is ſtill ſupplied to the Clouds and the upper Regions of the Air from the Earth below, in the manner as Sir *Iſaac Newton* has taught in his Book of Optics, and which we have already taken Notice of in the firſt Part of this Lecture.

ANOTHER of Mr. Canton's Papers is contained in Number XCIII. Part II. of the ſame Vol. under the following Title:

A Letter to the Right Honourable the Earl of Macclesfield, *President of the Royal Society, concerning some new electrical Experiments, by* John Canton, *M. A. and F. R. S.*

My Lord,

AS Electricity, since the Discovery of it in the Clouds and Atmosphere, is become an interesting Subject to Mankind; your Lordship will not be displeased with any new Experiments or Observations that lead to a farther Acquaintance with its Nature and Properties.

THE resinous and vitreous Electricity of Mr. *Du Fay*, which arose from his observing Bodies of the one Class to attract, what those of the other would repel, when each were excited by Attrition, received no Light till the Publication of the second Part of Mr. *Franklin*'s Experiments; wherein it appears, that the one Kind of Bodies electrify positively, and the other negatively; that excited Glass throws out the electric Fire, and excited Sulphur drinks it in. But no Reason has yet been assigned, why vitreous Bodies should receive, and resinous Bodies part with this Fire, by rubbing them. Some Persons indeed, of confide-

rable Knowledge in these Matters, have supposed the Expansion of Glass, when heated by Friction, to be the Cause of its receiving more of the electric Fluid than its natural Share; but this Supposition cannot be made with regard to Bodies of the other Sort, such as Sulphur, Sealing-Wax, &c. which part with it when treated in the same Manner. The following Experiments, first made at the latter End of *December* 1753, and often repeated since, may perhaps cast new Light on this difficult Subject.

HAVING rubbed a Glass Tube with a Piece of thin Sheet-lead and Flower of Emery mixt with Water, till its Transparency was entirely destroyed; after making it perfectly clean and dry, I excited it with new Flannel, and found it act in all Respects like excited Sulphur or Sealing-Wax. The electric Fire seems to issue from the Knuckle, or End of the Finger, and to spread itself on the Surface of this Tube, in the beautiful Manner represented at *A* and *B* in *Fig.* 1.

Plate II.

IF this rough or unpolished Tube be excited by a Piece of dry oiled Silk (especially when rubbed over with a little Chalk or Whiting), it will act like a Glass Tube with

with its natural Polish. And in this Case, the Fire appears only at the Knuckle, or End of the Finger; where it is very much condensed before it enters; as at *A* and *B* in *Fig.* 2. Plate II.

But if the rough Tube be greased all over with Tallow from a Candle, and as much as possible of it wiped off with a Napkin, then the oiled Silk will receive a Kind of Polish by rubbing it, and after a few Strokes, will make the Tube act in the same Manner as when excited at first by Flannel.

The oiled Silk, when covered with Chalk or Whiting, will make the greased rough Tube act again like a polished one: But if the Friction be continued till the Rubber is become very smooth, the electric Power will be changed to that of Sulphur, Sealing-Wax, *&c.*

Thus may the positive and negative Powers of Electricity be produced at Pleasure, by altering the Surfaces of the Tube and Rubber; according as the one or other is most affected by the Friction between them: For if the Polish be taken off one Half of a Tube, the different Powers may be excited with the same Rubber at a single Stroke. And the Rubber is

found to move much eaſier over the rough, than over the poliſhed Part of it.

THAT poliſhed Glaſs electriſes poſitively, and rough Glaſs rubbed with Flannel negatively, ſeems plain, from the Appearance of the Light between the Knuckle, or End of the Finger, and the reſpective Tubes; but yet may be farther confirmed by obſerving, that a poliſhed Glaſs Tube, when excited by ſmooth oiled Silk, if the Hand be kept at leaſt three Inches from the Top of the Rubber, will at every Stroke appear to throw out a great Number of diverging Pencils of electric Fire, as
Plate II, in *Fig.* 3; but not one was ever ſeen to accompany the rubbing of Sulphur, Sealing-Wax, *&c.* nor was I ever able to make any ſenſible Alteration in the Air of a Room, merely by the Friction of thoſe Bodies: Whereas the Glaſs Tube, when excited ſo as to emit Pencils, will, in a few Minutes, electrify the Air to ſuch a Degree, that (after the Tube is carried away) a Pair of Balls, about the Size of the ſmalleſt Peas, turned out of Cork, or the Pith of Elder, and hung to a Wire by Linen Threads of ſix Inches long, will repel each other to the Diſtance of an Inch and an Half, when held at Arm's-length in the

Middle

Middle of the Room. But their Repulſion will decreaſe as they are moved toward the Floor, Wainſcot, or any of the Furniture; and they will touch each other when brought within a ſmall Diſtance of any Conductor. Some Degree of this electric Power I have known to continue in the Air above an Hour after the Rubbing of the Tube, when the Weather has been very dry.

THE Electricity from the Clouds, in the open Air, may be diſcovered in the ſame Manner, if the Balls are held at a ſufficient Diſtance from Buildings, Trees, &c. as I have ſeveral Times experienced, by a Pair which I carry in a ſmall narrow Box, with a ſliding Cover, (*Fig.* 4.) ſo contrived as to keep their Threads ſtraight, and that they may be properly ſuſpended, when let fall out of it: and theſe Balls determine whether the Electricity of the Clouds or Air be poſitive, by the Decreaſe; or negative, by the Increaſe of their Repulſion, at the Approach of excited Amber or Sealing-Wax. Plate I.

To electrify the Air, or Moiſture contained in it, negatively; I ſupport by Silk, between two Chairs placed Back to Back at the Diſtance of about three Feet, a Tin

Tube with a fine Sewing-needle at one End of it; and rub Sulphur, Sealing-Wax, or the rough Glaſs Tube, as near as I can to the other End, for three or four Minutes. Then will the Air be found to be negatively electrical; and will continue ſo a conſiderable Time after the Apparatus is removed into another Room.

THE Air without-doors I have ſometimes known to be electrical in clear Weather; but never at Night, except when there has appeared an *Aurora Borealis*, and then but to a ſmall Degree, which I have had ſeveral Opportunities of obſerving this Year. How far poſitive and Negative Electricity in the Air, with a proper Quantity of Moiſture between, to ſerve as a Conductor, will account for this, and other Meteors ſometimes ſeen in a ſerene Sky, I ſhall leave to the Curious in this Part of Natural Philoſophy to determine. That dry Air at a great Diſtance from the Earth, if in an electric State, will continue ſo till it meets with a Conductor, ſeems probable from this Experiment: An excited Glaſs Tube with its natural Poliſh, being placed upright in the Middle of a Room, by putting one End of it in a Hole made for that Purpoſe in a Block of Wood, will generally

rally loſe its Electricity in leſs than five Minutes, by attracting to it a ſufficient Quantity of Moiſture, to conduct the electric Fluid from all Parts of its Surface to the Floor. But if, immediately after it is excited, it be placed in the ſame Manner before a good Fire, at the Diſtance of about two Feet, where no Moiſture will adhere to its Surface, it will continue electrical a whole Day; and how much longer I know not. It may not be improper to mention here, that if a ſolid Cylinder of Glaſs be ſet before the Fire till quite dry, it may as eaſily be excited as a Glaſs Tube, and will act like one in every Reſpect: The firſt Stroke will make it ſtrongly Electrical.

IN a Paper I laid before the *Royal Society*, on the 6th of *December* laſt, I conjectured, that the Electricity of the Atmoſphere might be obſerved even in the Winter; which I have ſince found to be true: For in the ſucceeding Months of *January*, *February*, and *March*, my Apparatus was electrified no leſs than twenty-five Times, both poſitively and negatively, by Snow, as well as by Hail and Rain; and to almoſt as great a Degree when *Fahrenheit*'s Thermometer was between 28 and 34, as I ever knew it in the Summer, except in a Thunder-Storm.

I SHALL

I SHALL be glad, if these Observations and Experiments may engage Persons of more Leisure and superior Abilities to pursue this Inquiry; as it is highly probable their Researches would be rewarded by many useful Discoveries. I have the Honour to be,

My Lord, &c.

PART II.

The Method of making ARTIFICIAL MAGNETS.

WE have already intimated, that Magnetism may be communicated by many different Ways to Bars of Iron, without the Touch or Use of real Magnets; and this has been done in an extraordinary Manner by Mr. *Savory*, Mr. *Lovelace*, Dr. *Knight*, Mr. *Mitchell*, Mr. *Canton*, and others at home; also the Messrs. *Du Hamel* and *Antheaume* at *France*. But though the Methods taken by those Gentlemen are different, yet the Force communicated to Steel Bars, properly tempered, is nearly the same in all; and the Method of

of ſucceeding beſt in theſe Caſes is generally deduced from a great Number of Experiments. For we are yet too little acquainted with Nature, and the Laws of Magnetiſm, to reaſon directly, or *a priori*, concerning its Properties; and as the ſtrongeſt artificial Magnets I have yet ſeen are thoſe made by Mr. *Canton*, I ſhall give his Paper directing the Method of making them, as it is contained in N° VI. Vol. XLVII, of the *Tranſactions*, under the following Title:

A Method of making artificial Magnets *without the Uſe of* natural *ones*; *communicated to the* Royal Society, *by* John Canton, *M. A. and F. R. S.*

PRocure a Dozen Bars; ſix of ſoft Steel, each three Inches long, one Quarter of an Inch broad, and one twentieth of an Inch thick, with two Pieces of Iron, each Half the Length of one of the Bars, but of the ſame Breadth and Thickneſs; and ſix of hard Steel, each five Inches and an Half long, Half an Inch broad, and three-twentieths of an Inch thick, with two Pieces of Iron of half the Length, but the whole Breadth and Thickneſs of one of the

the hard Bars: And let all the Bars be marked with a Line quite round them at one End.

Plate II. THEN take an Iron Poker and Tongs* *(Fig.* 1.) the larger they are, and the longer they have been uſed, the better; and fixing the Poker upright between the Knees, hold to it near the Top one of the ſoft Bars, having its marked End downward, by a Piece of ſewing Silk, which muſt be pulled tight with the left Hand, that the Bar may not ſlide: Then graſping the Tongs with the right Hand a little below the Middle, and holding them nearly in a vertical Poſition, let the Bar be ſtroked by the lower End, from the Bottom to the Top, about ten Times on each Side, which will give it a magnetic Power ſufficient to lift a ſmall Key at the marked End: Which End, if the Bar was ſuſpended on a Point, would turn toward the North, and is therefore called the North Pole, and the unmarked End is, for the ſame Reaſon, called the South Pole of the Bar.

FOUR of the ſoft Bars being impregnated after this Manner, lay the other two *(Fig.* 2.) parallel to each other, at the Diſtance of about one-fourth of an Inch, between

* Or two Bars of Iron.

the

the two Pieces of Iron belonging to them, and a North and South Pole againſt each Piece of Iron: Then take two of the four Bars already made magnetical, and place them together, ſo as to make a double Bar in Thickneſs, the North Pole of one, even with the South Pole of the other; and the remaining two being put to theſe, one on each Side, ſo as to have two North and two South Poles together, ſeparate the North from the South Poles at one End by a large Pin, and place them perpendicularly with that End downward, on the Middle of one of the parallel Bars, the two North Poles towards its South, and the two South Poles towards its North End: Slide them backward and forward three or four Times the whole Length of the Bar, and removing them from the Middle of this, place them on the Middle of the other Bar, as before directed, and go over that in the ſame Manner; then turn both the Bars the other Side upward, and repeat the former Operation: This being done, take the two from between the Pieces of Iron, and placing the two outermoſt of the touching Bars in their Room, let the other two be the outermoſt of the four to touch theſe with: And this Proceſs being

being repeated till each Pair of Bars have been touched three or four Times over, which will give them a confiderable magnetic Power, put the half dozen together after the Manner of the four (*Fig.* 3.) and touch with them two Pair of the hard Bars, placed between their Irons, at the Diftance of about half an Inch from each other: Then lay the foft Bars afide; and with the four hard ones let the other two be impregnated (*Fig.* 4.) holding the touching Bars apart at the lower End near two tenths of an Inch, to which Diftance let them be feparated after they are fet on the parallel Bar, and brought together again before they are taken off: This being obferved, proceed according to the Method defcribed above, till each Pair have been touched two or three Times over. But as this vertical Way of touching a Bar will not give it quite fo much of the magnetic Virtue as it will receive, let each Pair be now touched once or twice over in their parallel Pofition between the Irons (*Fig.* 5.) with two of the Bars held horizontally, or nearly fo, by drawing at the fame Time the North of one from the Middle over the South End, and the South of the other from the Middle over the North End of a parallel

parallel Bar; them bringing them to the Middle again without touching the parallel Bar, give three or four of theſe horizontal Strokes to each Side. The horizontal Touch, after the vertical, will make the Bars as ſtrong as they can poſſibly be made: As appears by their not receiving any additional Strength, when the vertical Touch is given by a greater Number of Bars, and the horizontal by thoſe of a ſuperior magnetic Power. This whole Proceſs may be gone through in about half an Hour, and each of the larger Bars, if well-hardened *, may be made to lift twenty-eight Troy Ounces, and ſometimes more. And when theſe Bars are thus impregnated, they will give to an hard Bar of the ſame Size, its full Virtue in leſs than

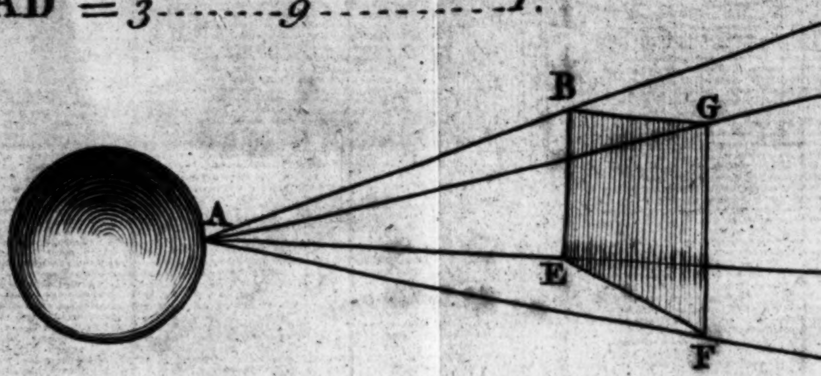
* The Smith's Manner of hardening Steel, whom I have chiefly employed, and whoſe Bars have conſtantly proved better than any I could meet with beſide, is as follows: Having cut a ſufficient Quantity of the Leather of old Shoes into very ſmall Pieces, he provides an Iron Pan, a little exceeding the Length of a Bar, wide enough to lay two Side by Side without touching each other or the Pan, and at leaſt an Inch deep. This Pan he nearly half-fills with the Bits of Leather, upon which he lays the two Bars, having faſtened to the End of each a ſmall Wire to take them out by: He then quite fills the Pan with the Leather, and places it on a gentle flat Fire, covering and ſurrounding it with Charcoal. The Pan being brought to ſomewhat more than a red heat, he keeps it ſo about half an Hour, and then ſuddenly quenches the Bars in a large Quantity of cold Water.

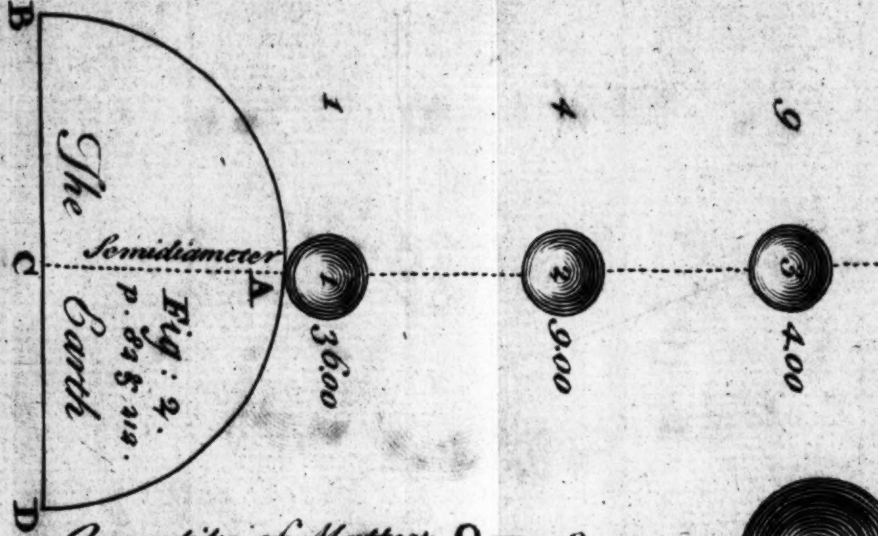
two

two Minutes: And therefore will anſwer all the Purpoſes of Magnetiſm in Navigation and experimental Philoſophy, much better than the Loadſtone, which is well known not to have ſufficient Power to impregnate hard Bars. The half dozen being put into a Caſe (*Fig.* 6.) in ſuch a Manner, as that two Poles of the ſame Denomination may not be together, and their Irons with them as one Bar, they will retain the Virtue they have received: But if their Power ſhould, by making Experiments, be ever ſo far impaired, it may be reſtored without any foreign Aſſiſtance in few Minutes. And if, out of Curioſity, a much larger Set of Bars ſhould be required, theſe will communicate to them a ſufficient Power to proceed with, and they may in a ſhort Time, by the ſame Method, be brought to their full Strength.

ver
ra-
ch
ell
n-
ng
er,
on
th
ir-
ver
ver
h-
es.
ver
ill
to
e,
eir

Distances		Squares	Forces
AB	= 1	1	9.
AC	= 2	4	$2\frac{1}{4}$
AD	= 3	9	1.





Quantity of Matter.. Q = 3
Celerity .... C = 9
27
Momentum .... M =

Quantity of Matter.. Q = 5
Celerity .... C = 7
35
Momentum .... M =

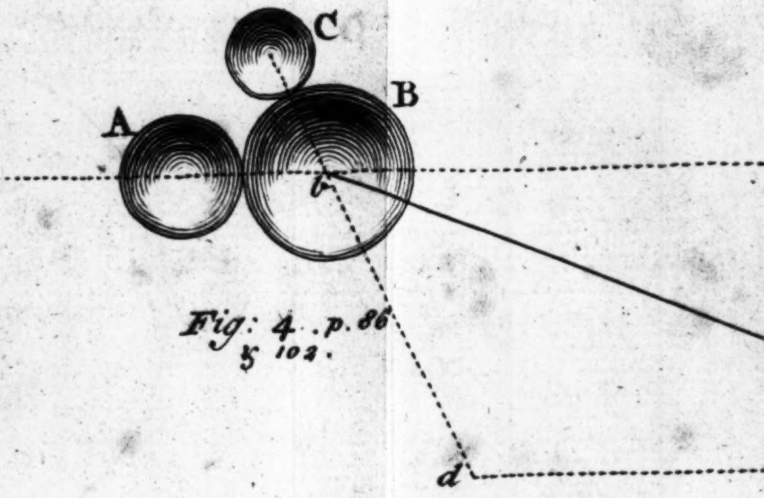


Fig: 4. p. 86 & 102.

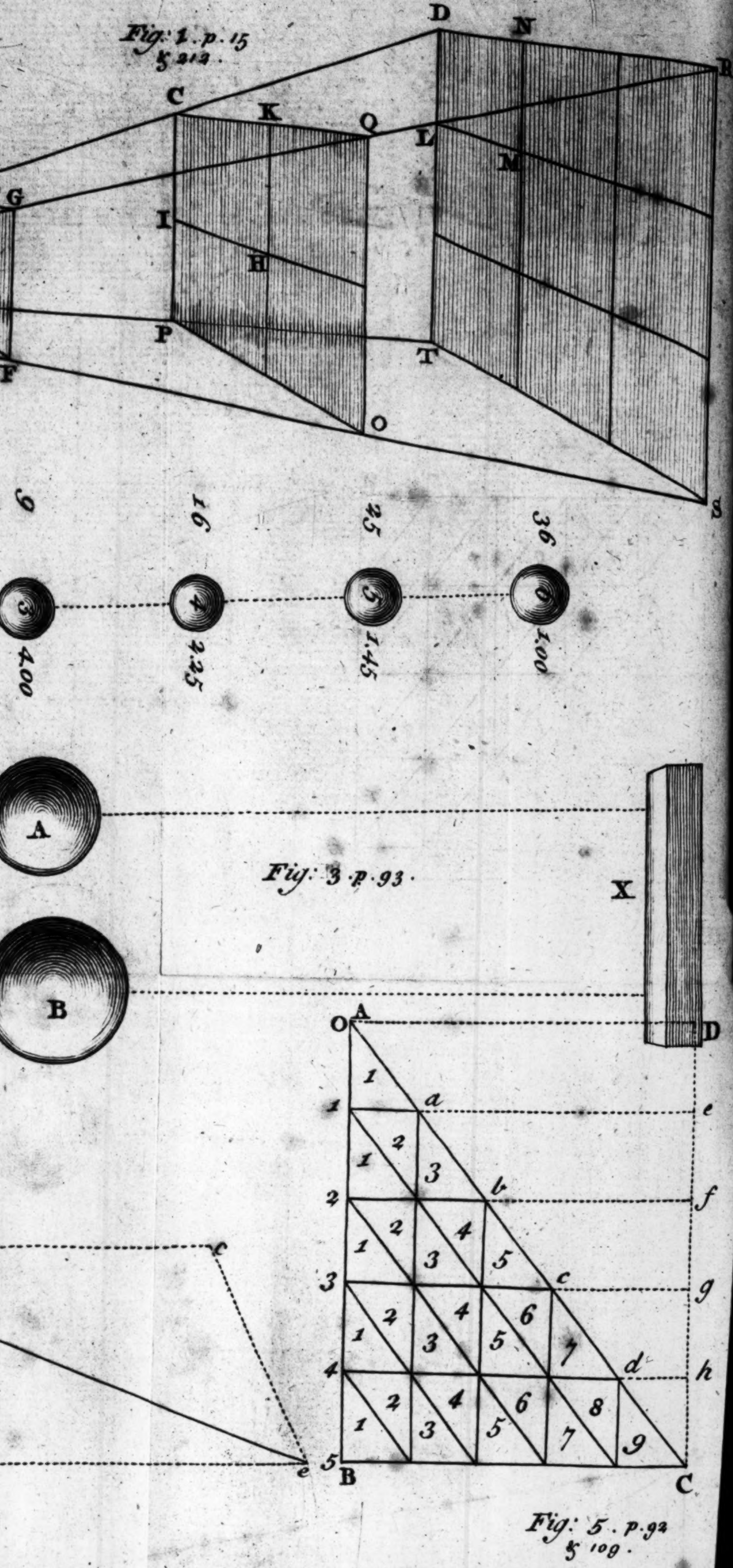


Fig: 1. p. 15 & 212.

Fig: 3. p. 93.

Fig: 5. p. 92 & 109.

E. Bowen sc.

LECTURE II.

Of the ATTRACTION *of* GRAVITATION, *and its Laws.* *Of* MOTION *and* REST, *absolute and relative*; *equable, accelerated, and retarded* MOTION. *Of the Spaces, Times, and Velocities of* BODIES *in* MOTION. *Of the* COLLISION *of* BODIES *Elastic and Non-Elastic.* *Of the* MOMENTUM *or* FORCE *of* STRIKING BODIES. *Of the* LAWS *of* MOTION. *Of the* COMPOSITION *and* RESOLUTION *of* MOTION. *Of* ACTION *and* RE-ACTION. *Of the* DESCENT *of heavy Bodies.* *Of the* DESCENT *of Bodies on the inclined Planes.* *Of the* DOCTRINE *of* PENDULUMS. *Of the* CYCLOID, *its* PROPERTIES *and* USE; *the Curve of quickest Descent.* *Of the Centre of* OSCILLATION *and* PERCUSSION. *The various Uses of the* PENDULUM, *as a* CHRONOMETER, *&c.* *Of a new Constructed* PYROMETER, *and the* EXPANSION *of* METALINE BODIES. *Of the* DOCTRINE *of* PROJECTILES, *or* ART *of* GUNNERY. *Of* CENTRAL FORCES, *and the various Laws and Properties of that*

Sort of Motion. Of the General Law of the PLANETARY MOTIONS. *Of the* FIGURE *of the* EARTH, *and how determined.*

THE *Fourth and last Species of Attraction* is that of GRAVITATION, which is evident only between large Bodies, as the Earth and Moon, the Sun and its Planetary Attendants. The Laws of this Attraction are as follows, *viz.* (1.) It is common to all Bodies, and mutual between them. (2.) It is proportional to the Quantity of Matter in Bodies. (3.) It is exerted every Way from the Centre of the attracting Body in Right-lined Directions. (4.) It decreases as the Squares of the Distances increase: That is, if a Body at A, on the Earth's Surface, distant *one Semidiameter* from the Centre C, weighs 3600 Pounds, it will at the Distance of 2, 3, 4, 5, 6, Semidiameters weigh 900, 400, 225, 145, 100 Pounds; which Numbers decrease as the Squares of the Distances 4, 9, 16, 25, 36, increase (XVI).

Pl. III. Fig. 2.

HENCE

(XVI) 1. Though we reckon this Species of Attraction different from that of Cohesion, yet when well consider'd, it may be found perhaps to differ no otherwise than as the *Whole from the Parts*; for the Gravity of large Bodies may be only the Result or Aggregate of the particular Powers of the constituent Particles, which singly act only upon Contact, and in small Distances;

HENCE we learn that all Bodies have WEIGHT, or are *heavy*; and that there is no ſuch Thing as *abſolute* LEVITY in Nature; and by the ſecond Law, the Weight of all Bodies is proportional to the Quantity of Matter they contain; and hence, ſince Bodies of equal Bulks are found to have unequal Quantities of Matter, it evidently follows, that a VACUUM, or Space void of Matter, muſt neceſſarily exiſt, and that an abſolute PLENUM is a

ſtances; but with their joint Forces, in vaſt Quantities, produce a mighty Power, whoſe Efficacy extends to very great Diſtances, proportional to the Magnitude of Bodies.

2. This Force of Gravity is, to Senſe, the ſame for any Diſtances near the Earth's Surface; ſince a ſmall Diſtance from the Surface of the Earth does not ſenſibly alter the Diſtance from the Centre, which is near 4000 Miles: At the Height therefore of one Mile, for Inſtance, the Diſtances from the Centre will be as the Numbers 4000 to 4001, and the Powers of Gravity inverſely as their Squares, *viz.* 16008001 to 16000000, or as 16008 to 16000, which are ſo near an Equality as not to be ſenſibly different from each other.

3. But when the Diſtance is ſo great as to cauſe a Diſproportion between that and the Semidiameter of the Earth, then will the Power of Gravity decreaſe very ſenſibly, according to the Law above laid down: Thus at the Diſtance of the Moon, which is, at a Mean, about 60 Semidiameters of the Earth, the Power of Gravity is to that on the Earth's Surface as 1 to 3600, as will be ſhewn under the Article of *Central Forces* with ſufficient Demonſtration.

Doctrine unphilosophical, and equally false and absurd (XVII.)

ALSO,

(XVII) 1. Since there is no Sort of Matter that we have any Knowledge of, but what will gravitate, to suppose there is any Sort that will not, is arbitrary, and contrary to the Rules of Philosophizing, and all sound Reasoning; by which we are confined to the Phænomena of real, and not imaginary Existence.

2. The Weight or Quantity of Matter in Bodies may be expreſs'd by their Bulks multiplied by their Densities. By the DENSITY of a Body is meant the greater or lesser Quantity of Matter under the same Bulk; and therefore the Density (D) of any Body A, is to the Density (D) of any other Body B of equal Bulk, as the Quantity of Matter (M) in the former is to the Quantity of Matter (m) in the latter. That is, $D : D :: M : m$; $\because Dm = DM$.

3. But if the Bulks are unequal, and the Quantities of Matter the same, or equal in any two Bodies B and C; then will the Density (D) of the Body B be to the Density (d) of the Body C, as the Bulk (b) of the latter, to the Bulk (B) of the former; *viz.* $D : d :: b : B$; $\therefore DB = db$.

4. Hence $D = \frac{db}{B} = \frac{Dm}{M}$; and so $Mdb = mBD$; and consequently $M : m :: BD : bd$; that is, *The Quantity of Matter in any two Bodies* A *and* C, *which differ in Bulk and Weight, are to each other as the Products of the Densities by their Bulks, as was above asserted.*

5. Hence also in such Bodies, $D : d :: Mb : mB$; that is, *the Densities of the two Bodies* A *and* C *are directly as the Quantities of Matter, and inversely as their Bulks.*

6. Lastly, *The Bulks or Magnitudes of two Bodies* A *and* C *are directly as their Quantities of Matter or Weights, and inversely as their Densities*; for we have $B : b :: Md : mD$.

7. What has been hitherto said relates to the absolute Weight of Bodies; but when the Weight or Gravities of

ALSO, from the third Law it follows, that all Bodies descending freely by their Gravity tend towards the Earth in Right Lines perpendicular to its Surface; and with equal Velocities, (abating for the Resistance of the Air) as is evident by the second Law above, and what will immediately follow in *Mechanics* (XVIII.)

AGAIN:

of Bodies of equal Bulk are considered and compared together, they are then called the Relative Weights or Specific Gravities of those Bodies, and since they are likewise proportional to the Quantities of Matter, it is evident, *that the Densities and Specific Gravities are proportional to each other*, in all Bodies; and therefore what has been said of the one is equally applicable to the other.

8. As a Corollary to this Article, we may observe with Sir *Isaac Newton*, that all Space cannot be filled with Matter, as the *Plenists* assert, but that there must necessarily be Vacuities or Interstices void of Matter in the Composition of natural Bodies; for if there were not, then Bodies of equal Bulk would ever be of the same Weight and Density, or, in other Words, the Density and Specific Gravity of *Cork* or *Air* would be the same with that of *Quicksilver* or *Gold*. And it is surprizing to think, that such Absurdities are not gross enough to be heeded by those who hesitate about a *Vacuum*.

(XVIII) The Reason why all Bodies descend with an equal Velocity towards the Centre of the Earth *(in Vacuo)* is because the Force of Gravity in each is proportional to the Quantity of Matter, or Number of Particles only, independent of any other Circumstance or Consideration whatsoever. Thus suppose the Body A has only one Particle of Matter, and the Body B consists of one hundred, then will the Force of Gravity in A be to that in B as 1 to 100; and therefore the Force acting on each single Particle in B is but $\frac{1}{100}$ of the whole

AGAIN: Since the Attraction is always directly as the Quantity of Matter, and inversely as the Square of the Distance, it follows, that were the internal Parts of the Earth a perfect Void, or hollow Concavity, a Body placed any where therein would be absolutely light, or void of Gravity; but supposing the Earth a solid Body throughout, the Gravity from the Surface to the Centre will decrease with the Distance, or it will be directly proportional to the Distance from the Centre (XIX).

HAVING

Force, and consequently is but equal to the Force acting on the single Particle A; since then each single Particle in B is acted upon with the same Force as the Particle A, the Velocities in each must be equal; that is, each Particle in the Body B, (and therefore the whole Body) must descend with an equal Velocity with that of A. This would be easy to conceive, if the Particles in the Body B were to fall separate; and I think it is as easy to understand that their Connection or Cohesion can no ways contribute to the Acceleration of their Motion, which is entirely the Effect of the attracting Force of the Earth.

Plate II. Fig. 4. (XIX) 1. For Illustration, suppose ABC be the outward Shell or Crust, and all the internal Parts of the Earth an hollow Space or Concavity, in which suppose any Body as R, to be placed in any Part whatsoever; thro' the Centre of the said Body let there be drawn the Lines *ad* and *bc*, which by their Revolution would describe Cones, whose Bases *ab*, and *cd*, in the spherical Superficies, are in Proportion to each other as the Squares of the Distances from R respectively. Now the Particle R is attracted by the Particles in those two Bases with an equal Force; for the Force is as the Square of the Distance of the Bases inversely, and as the Number of Particles

in

HAVING premised the necessary *Præcognita* to the Science of MECHANICS, which

in each Base directly, which Number of Particles is as the Square of the Distance from the Particle directly. But this *direct* and *inverse Ratio* of the Forces makes a Ratio of Equality; that is, the Forces on each Side are equal, and being in contrary Directions they destroy each other; and this being the Case in any other Part of the Concave Superficies, 'tis evident the Body R cannot be impell'd towards any Part, but will retain its Position as if not affected by Gravity at all.

2. The Forces with which a Body will be attracted on the Surface of two Spheres of the same Density, but unequal Bulks, *will be directly as the Diameters of the Spheres.* For the Force (f) of the lesser Sphere ABC will be to the Force (F) of the greater DEF inversely as the Squares of the Diameter (d^2) of the lesser to the Square of the Diameter (D^2) of the greater; that is, $f : F :: \frac{1}{dd} : \frac{1}{DD}$. Note, *This is so because the Forces are inversely as the Squares of the Distances from the Centres, which are equal to half the Diameters.*

Plate II. Fig. 5, 6.

3. Again: The Forces are as the Number of attracting Particles, which are as the Bulks or Magnitudes of the Spheres, which are as the Cubes of the Diameter; therefore also $f : F :: d^3 : D^3$; whence both these Ratio's compounded give $f : F :: \frac{d^3}{d^2} : \frac{D^3}{D^2} :: d : D$; or, *the Forces of Attraction are in proportion to the Diameters of the Spheres directly.*

4. Hence it follows, that were there a Perforation made, and a Body to descend therein from the Surface of the Earth to the Centre, its Gravity would always decrease with the Distance from the Centre; because in every Place it might be consider'd as on that Surface of a Sphere whose Diameter is equal to twice the Distance from the Centre, and which is proportional to the Sphere's Attraction; the Parts of the Earth above the Body being only Part of a Spherical Shell, which has no Effect on the Body, as was shewn above.

entirely depends on the Principle of Gravitation, we come immediately to consider the Object thereof, *viz.* the *Nature*, *Kinds*, and various *Affections* of *Motion*, and moving Bodies; and the Structure and Mechanism of all Kinds of Machines, commonly call'd *Mechanical Powers*, whether Simple or Compound.

MOTION is the continual and successive Change of Space, and is either *Absolute* or *Relative*. *Absolute Motion* is the Change of *Absolute Space* or Place of Bodies, as the Flight of a Bird, the Motion of a Projectile, *&c.* But *Relative Motion* is the Change of *Relative Space*, or that which has Reference to some other Bodies: As of two Ships under Sail, the Difference of their Velocities is the *Relative Motion* of the Ship sailing fastest; and is that alone which is discernible by us. The same is to be understood of *Absolute and Relative Rest* (XX).

AGAIN:

(XX) To make this Matter yet plainer, we are to consider, that SPACE is nothing else *but an absolute and infinite Void*; and that the PLACE of a Body is that Part of the immense Void which it takes up or possesses; and that Place may be considered either absolutely, or in itself alone, and then it is called the *Absolute Place* of the Body; or else with Regard to the Place of some other Body, and then it is call'd the *Relative* or *Apparent Place* of Bodies.

2. Now

AGAIN: Motion is either *Equable* or *Accelerated*. *Equable Motion* is that by which a Body

2. Now as Motion is only the Change of Place in Bodies, 'tis evident that will come under the same Distinction of *Absolute*, and *Relative* or *Apparent*. All Motion is in itself *Absolute*, or the Change of Absolute Space; but when the Motions of Bodies are consider'd and compar'd with each other, then are they *Relative* and *Apparent* only; they are *Relative*, as they are compar'd with each other; and they are *Apparent* only, inasmuch as not their *true or absolute Motion*, but the Sum or *Difference of the Motions* only is perceivable by us.

3. In comparing the Motions of Bodies, we may consider them as moving both the same Way, or towards contrary Parts; in the first Case the *Difference of the Motions* is only perceived by us, in the latter the *Sum of the Motions*. Thus for Example; suppose two Ships, A and B, set sail from the same Port upon the same Rhumb, and that A sails at the Rate of five Miles *per* Hour, and B at the Rate of three; here the Difference of the Velocities (*viz.* two Miles *per* Hour) is that by which the Ship A will appear to go from the Ship B forwards, or the Ship B will appear at A to go with the same Velocity backwards, to a Spectator in either respectively.

4. If the two Ships A and B move with the same Degree of Velocity, then will the Difference be nothing, and so neither Ship will appear to the other to move at all. Hence it is, that tho' the Earth is continually revolving about its Axis, yet as all Objects on its Surface partake of the same common Motion, they appear not to move at all, but are relatively at Rest.

5. If the two Ships A and B, with the Degrees of Velocity as above, meet each other; the one will appear to a Spectator in the other to move with the Sum of both Velocities, *viz.* at the Rate of eight Miles *per* Hour; so that in this Case the *Apparent Motion* exceeds *the True*, as in the other it fell short of it. Hence the Reason why a Person riding *against* the Wind finds the Force of it

a Body passes over equal Spaces in equal Times. *Accelerated Motion* is that which is continually augmented or increased; as *Retarded Motion* is that which continually decreases: And if the Increase or Decrease of Motion be equal in equal Times, the Motion is then said to be *equably accelerated or retarded* (XXI).

THE

it much greater than it really is, whereas if he rides *with it*, he finds it less.

SCHOLIUM.

6. The Reason of all these different Phænomena of Motion will be evident if we only consider, that we must be absolutely at Rest, if we would discern the true or real Motion of Bodies about us. Thus a Person on the Strand will observe the Ships sailing with their real Velocity; a Person standing still will experience the true Strength and Velocity of the Wind; and a Person placed in the Regions between the Planets will view all their true Motions; which he cannot otherwise do, because in all other Cases the Spectator's own Motion must be added to, or subducted from, that of the moving Body; and the Sum or Difference is therefore the *Apparent or Relative Motion*, and not the *True*.

(XXI) 1. *Equable Motion* is generated by a single *Impetus* or Stroke; thus the Motion of a Ball from a Cannon is produced by the single Action of the Powder in the first Moment, and therefore the Velocity it first sets out with would always continue the same, were it void of Gravity, and to move in an unresisting Medium, which therefore would be always equable, or such as would carry it through the same Length of Space in every equal Part of Time.

2. *Accelerated Motion* is produced by a constant Impulse, or Power which keeps continually acting upon the Body, as that of Gravity, which produces the Motion of Falling Bodies, which Sort of Motion is constantly

2 accelerated,

THE *Celerity* or *Velocity* of Motion is that Affection by which a Body passes over a *given Space in a given Time*, or what we com-

accelerated, because Gravity every Moment adds a new Impulse, which generates a new Degree of Velocity; and the Velocity thus increasing the Motion must be quicken'd each Moment, or fall faster and faster the longer it falls.

3. In like Manner, a Body thrown perpendicularly upward, as a Ball from a Cannon, will have its Motion *continually retarded*, because Gravity acts constantly upon it in a Direction contrary to that given it by the Powder, so that its Velocity upwards must be continually diminished, and so its Motion is continually retarded, till at last it be all destroy'd. The Body has then attain'd its utmost Height, and is for a Moment motionless, after which it begins to descend with a Velocity in the same Manner accelerated, till it comes to the Earth's Surface.

4. And because at a small Distance from the Earth's Surface, the Power of Gravity is every where the same, it follows, that in equal Moments of Time the Impulses on the Body will be equal, which will therefore generate an equal Increase of Velocity each Moment, and of Consequence the Motion resulting from thence will be equally quicken'd if downwards, or retarded if upwards, in Mediums without Resistance.

5. In this Case we may observe in Bodies that are projected upwards, (1.) That the Time of the Ascent is equal to that of the Descent. (2.) That at equal Heights above the Earth, the Velocity in the Ascent and Descent is equal, or the same; and therefore, (3.) That the Velocity acquired (by falling) at the Earth's Surface, is equal to that which is generated by the Powder, or other *Impetus* (which threw it up) in the first Moment of its Motion. What Difference will arise from a resisting Medium (as to Projectiles thrown up in the Air, &c.) will be hereafter more fully consider'd and specified.

commonly call the *Swiftneſs* or *Slowneſs* of Motion (XXII).

THE MOMENTUM or *Quantity of Motion* is all that Power or Force which a moving Body has to affect or ſtrike any Obſtacle or Impediment which oppoſes its Motion, and is equal to that impreſſed Force

(XXII) 1. From the above Definitions we may determine the Theorems, for the Expreſſions of the Time (T,) the Velocity (V), and the Space (S) paſſed over in Equable or Uniform Motion, very eaſily thus:

2. If the Time be given, or the ſame; the *Space paſſed over will be as the Velocity*, viz. S : V; that is, with twice the Velocity, twice the Space; with three Times the Velocity, three Times the Space, will be paſs'd over in the ſame Time; and ſo on.

3. If the Velocity be given, or remain the ſame; *then the Space paſs'd over will be as the Time* (viz. S : T) that is, it will be the greater or leſſer as the Time is ſo.

4. But if neither the Time nor Velocity be given, or known; *then will the Space be in the compound Ratio of both*, viz. S : TV.

5. Hence, in general, ſince S : TV, we have $V : \frac{S}{T}$; that is, *the Velocity is always directly as the Space, and inverſely as the Time.*

6. And alſo $T : \frac{S}{V}$; that is, *the Time is as the Space directly, and as the Velocity inverſely*; or, in other Words, it *increaſes with the Space*, and *decreaſes with the Velocity.*

7. If therefore in any Rectangle ABCD one Side AB repreſent the Time, and the other Side BC the Velocity, 'tis evident the Area of the ſaid Rectangle will repreſent the Space paſs'd over by an uniform Motion in the Time AB and with the Velocity BC.

Pl. III. Fig. 5.

Force by which the Body is compell'd to change its Place.

SINCE the Force of a Body is greater or leſſer as the Quantity of Matter is ſo, when the Velocity of its Motion is the ſame; alſo ſince the ſaid Force in the ſame Body is proportional to its Velocity: Therefore it follows, that the *Momentum* or Force of *Bodies* in Motion is in the compound Ratio of their Quantities of Matter and their Velocities conjointly. Thus if one Body A ſtrike an Obſtacle X, with 3 Parts of Matter and 9 Degrees of Celerity; and another Body B ſtrike it with 5 Parts of Matter, and 7 Degrees of Celerity; the *Momenta* of the reſpective Strokes will be as 27 to 35 (XXIII). Plate III. Fig. 3.

THE

(XXIII) Since the *Momentum* (M) of a Body is compounded of the Quantity of Matter (Q), and the Velocity (V), we have this general Expreſſion $M=QV$, for the Force of any Body A; and ſuppoſe the Force of another Body B be repreſented by the ſame Letters in *Italicks*, viz. $M=\mathcal{Q}V$.

2. Let the two Bodies A and B in Motion impinge on each other directly; if they tend both the ſame Way, the Sum of their Motions towards the ſame Part will be $QV+\mathcal{Q}V$. But if they tend towards contrary Parts, or meet, then the Sum of their Motions toward the ſame Part will be $QV-\mathcal{Q}V$; for ſince the Motion of one of the Bodies is contrary to what it was before, it muſt be connected by a contrary Sign. Or thus; becauſe when the Motion of B conſpires with that of A, it is *added to it*; ſo when it is contrary, it is *ſubducted from it*, and the *Sum or Difference* of the *abſolute Motions* is

THE general Laws of Motion which all Bodies observe, are the three following, *viz.*

LAW

is the whole *Relative Motion*, or that which is made towards the same Part.

3. Again, this total Motion towards the same Parts is the same both before and after the Stroke, in case the two Bodies A and B impinge on each other; because whatever Change of Motion is made in one of these Bodies by the Stroke, the same is produced in the other Body towards the same Part; that is, as much as the Motion of B is increased or decreased towards the same Part by the Action of A, just so much is the Motion of A diminished or augmented towards the same Part by the equal Re-action of B, by the *third Law* of Motion.

4. In Bodies *not elastic*, let x be the Velocity of the Bodies after the Stroke (for since we suppose them not elastic, there can be nothing to separate them after Collision, they must therefore both go on together, or with the same Celerity). Then the Sum of the Motions after Collision will be $\mathrm{Q}x+\mathcal{Q}x$; whence if the Bodies tend the same Way, we have $\mathrm{QV}+\mathcal{Q}V=\mathrm{Q}x+\mathcal{Q}x$, or if they meet, $\mathrm{QV}-\mathcal{Q}V=\mathcal{Q}x+\mathcal{Q}x$; and accordingly $\frac{\mathrm{QV}+\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}=x$, or $\frac{\mathrm{QV}-\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}=x$.

Pl. IV. Fig. 1. 5. If the Body (B) be at rest, then $V=0$, and the Velocity of the Bodies after the Stroke will be $\frac{\mathrm{QV}}{\mathrm{Q}+\mathcal{Q}}=x$. Thus if the Bodies be equal (*viz.* $\mathrm{Q}=\mathcal{Q}$) and A with 10 Degrees of Velocity impinge on B at rest;

Pl. IV. Fig. 2. then $\frac{\mathrm{QV}}{\mathrm{Q}+\mathcal{Q}}=\frac{10}{2}=5=x$. If $\mathrm{Q}=\mathcal{Q}$, and $\mathrm{V} : V :: 10 : 6$; we have $\frac{\mathrm{QV}+\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}=\frac{16}{2}=8=x$, the Velocity after the Stroke.

6. If

LAW I. *Every Body will continue in its State of Rest, or moving uniformly in a Right Line,*

6. If the Bodies are both in Motion, and tend towards opposite Parts, or meet; then when $\mathrm{Q}=\mathcal{Q}$, and $\mathrm{V}=\mathcal{V}$, 'tis plain $\frac{\mathrm{QV}-\mathcal{QV}}{\mathrm{Q}+\mathcal{Q}}=o=x$; that is, *the Bodies which meet with equal Bulks and Velocities will destroy each other's Motion after the Stroke, and remain at rest.* If $\mathrm{Q}=\mathcal{Q}$, but $\mathrm{V}:\mathcal{V}::6:14$, then $\frac{\mathrm{QV}-\mathcal{QV}}{\mathrm{Q}+\mathcal{Q}}=-\frac{8}{2}=-4=-x$; which shews that *equal Bodies meeting with unequal Velocities, they will after the Stroke both go on the same Way which the most prevalent Body moved before.* Plate IV. Fig. 3. Fig. 4.

7. If the Velocity $\frac{\mathrm{QV}\pm\mathcal{QV}}{\mathrm{Q}+\mathcal{Q}}$ be multiplied by the Quantities of Matter Q and $\mathcal{Q}$, we shall have $\frac{\mathrm{Q}^2\mathrm{V}\pm\mathrm{Q}\mathcal{QV}}{\mathrm{Q}+\mathcal{Q}}$ = the *Momentum* of A after the Stroke; and $\frac{\mathrm{QV}\mathcal{Q}\pm\mathcal{Q}^2\mathcal{V}}{\mathrm{Q}+\mathcal{Q}}$ = the *Momentum* of B. Therefore $\mathrm{QV}-\frac{\mathrm{Q}^2\mathrm{V}\pm\mathrm{Q}\mathcal{QV}}{\mathrm{Q}+\mathcal{Q}}=\frac{\mathrm{Q}\mathcal{Q}\mathrm{V}\pm\mathcal{Q}\mathrm{Q}\mathcal{V}}{\mathrm{Q}+\mathcal{Q}}=\frac{\mathrm{Q}\mathcal{Q}}{\mathrm{Q}+\mathcal{Q}}\times\overline{\mathrm{V}\pm\mathcal{V}}$ = the Quantity of Motion lost in A after the Stroke, and consequently is equal to what is gained in B, as may be shewn in the same Manner.

8. But since a Part of this Expression (*viz.* $\frac{\mathrm{Q}\mathcal{Q}}{\mathrm{Q}+\mathcal{Q}}$) is constant, the Loss of Motion will be ever proportional to the other Part $\mathrm{V}\pm\mathcal{V}$. But this Loss or Change of Motion in either Body is the whole Effect, and so measures the Magnitude or Energy of the Stroke. Wherefore

Line, except so far as it is compell'd to change that State by Forces impressed.

LAW

fore any two Bodies, not elastic, strike each other with a Stroke always proportional to the Sum of their Velocities ($\mathrm{V}+V$) if they meet, or to the Difference of their Velocities ($\mathrm{V}-V$) if they tend the same Way.

9. Hence if one Body (B) be at rest before the Stroke, then $V=o$; and the Magnitude of the Stroke will be as V, that is, as the Velocity of the moving Body A; and not as the *Square of its Velocity*, as many Philosophers (*viz.* the *Dutch* and *Italians*) maintain.

10. In Bodies perfectly elastic, the restituent Power or Spring, by which the Parts displaced by the Stroke restore themselves to their first Situation, is equal to the Force impress'd, because it produces an equal Effect; therefore in this Sort of Bodies, there is a Power of Action twice as great as in the former Non-elastic Bodies, for these Bodies not only strike each other by *Impulse*, but likewise by *Repulse*, they always repelling each other after the Stroke.

11. But we have shewn that the Force with which Non-elastic Bodies strike each other is as $\mathrm{V}\mp V$; therefore the Re-action of Elastic Bodies is the same; that is, the Velocity with which Elastic Bodies recede from each other after the Stroke is equal to the Velocity with which they approach'd each other before the Stroke. Whence if x and y be the Velocities of two Bodies A and B, tending the same Way, after the Stroke, since $\mathrm{V}-V=y-x$, we have $x+\overline{\mathrm{V}-V}=y$; whence the Motion of A after the Stroke will be Qx, and that of B will be $\mathcal{Q}x+\mathcal{Q}\mathrm{V}-\mathcal{Q}V$; and the Sum of these Motions will be equal to the Sum of the Motions before the Stroke, *viz.* $Qx+\mathcal{Q}x+\mathcal{Q}\mathrm{V}-\mathcal{Q}V=Q\mathrm{V}+\mathcal{Q}V$. Whence by reducing the Equation, it will be $Qx+\mathcal{Q}x=Q\mathrm{V}-\mathcal{Q}\mathrm{V}+2\mathcal{Q}V$; and $x=\dfrac{Q\mathrm{V}-\mathcal{Q}\mathrm{V}+2\mathcal{Q}V}{Q+\mathcal{Q}}=$ the Velocity of the Body A.

12. Again

LAW II. *The Change of Motion is always proportional to the moving Force impress'd.*

12. Again, the Velocity of B is $x + \mathrm{V} - V = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} + 2\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} + \mathrm{V} - V = \frac{2\mathrm{QV} - \mathrm{Q}V + \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}}$. Here we have supposed the Bodies tend the same Way before the Stroke; and it is evident from the Equation above, that so long as $\mathrm{QV} + 2\mathcal{Q}V$ is greater than $\mathcal{Q}\mathrm{V}$, the Velocity (x) of A after the Stroke will be affirmative, or the Body A will move the same Way after the Stroke as before; but when $\mathcal{Q}\mathrm{V}$ is greater than $\mathrm{QV} + 2V\mathcal{Q}$, the Velocity ($x$) will be negative, or the Body A will be reflected back.

13. If the Body B be at rest, then $V = o$; and $x = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V}}{\mathrm{Q}+\mathcal{Q}}$; which shews the Body A will go forwards or backwards, as QV is greater or lesser than $\mathcal{Q}\mathrm{V}$, or A greater or lesser than B.

14. If $\mathrm{Q} = 3$, $\mathcal{Q} = 2$, and $\mathrm{V} = 10$, and $V = o$; then after the Stroke the Velocity of A will be $\frac{\mathrm{QV} - \mathcal{Q}\mathrm{V}}{\mathrm{Q}+\mathcal{Q}} = \frac{30-20}{5} = \frac{10}{5} = 2$, and the Velocity of B will be $y = \frac{2\,\mathrm{QV}}{\mathrm{Q}+\mathcal{Q}} = \frac{60}{5} = 12$. If the Bodies are both in Motion, and $V = 5$, the rest the same as before; then $\frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} + 2\mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = 6 =$ Velocity of A after the Stroke, and $\frac{2\mathrm{QV} - \mathrm{Q}V + \mathcal{Q}V}{\mathrm{Q}+\mathcal{Q}} = 11 =$ Velocity of B after the Stroke. Plate IV. Fig. 5. Fig. 6.

15. If the Bodies A and B move towards contrary Parts, or meet each other, then will the Relative Velocity, to which the Force of the Stroke is proportional, be $\mathrm{V} + V$, and so the Velocities of A and B after the Stroke will be x and $x + \mathrm{V} + V$; and so the Motion

press'd, and is always made according to the Right Line in which that Force is impress'd.

LAW

of A will be $\mathrm{Q}x$ and $\mathcal{Q}x + \mathcal{Q}\mathrm{V} + \mathcal{Q}V$; the Sum of these Motions is $\mathrm{Q}x + \mathcal{Q}x + \mathcal{Q}\mathrm{V} = \mathcal{Q}V = \mathrm{QV} - \mathcal{Q}V =$ the Motion towards the same Part before the Stroke. Whence we have $x = \frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} - 2\mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}}$; and the Velocity of B will be $\frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} - 2\mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}} + \mathrm{V} + V = \frac{2\mathrm{QV} + \mathrm{Q}V - \mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}}$.

16. If $\mathrm{QV} + 2\mathcal{Q}V$ be greater than QV, the Motion of the Body A will be backwards; otherwise it will go on forwards as before. If $\mathrm{Q} = 3$, $\mathcal{Q} = 2$, $\mathrm{V} = 10$, and $V = 5$; then will the Velocity of A be

Pl. IV. Fig. 7. $\frac{\mathrm{QV} - \mathcal{Q}\mathrm{V} - 2\mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}} = \frac{-10}{5} = 2$, and so the Body A will go back with two Degrees of Velocity. The Velocity of B, after the Stroke, will be $\frac{2\mathrm{QV} + \mathrm{Q}V - \mathcal{Q}V}{\mathrm{Q} + \mathcal{Q}} = 13$.

17. If the Bodies are equal, that is, if $\mathrm{Q} = \mathcal{Q}$, then $x = \frac{-2\mathcal{Q}V}{2\mathcal{Q}} = -V$; which shews, that when equal

Fig. 8. Bodies meet each other, they are reflected back with interchanged Velocities; for in that Case also the Velocity of B becomes $\frac{2\mathrm{QV}}{2\mathrm{Q}} = \mathrm{V}$. An Example of this you have in *Fig.* 8, of *Plate* IV.

Fig. 9. 18. If the Bodies are equal, and one of them at rest, as B; then since $\mathrm{Q} = \mathcal{Q}$, and $V = o$ we have the Velocity of A after the Stroke $x = o$; or the Body A will abide at rest, and the Velocity of B will be $= \mathrm{V}$, the Velocity of A before the Impulse, as appears by the Example of *Fig.* 9.

Fig. 10. 19. If several Bodies, B, C, D, E, F, are contiguous in a Right Line, if another equal Body A strike B with any

any given Velocity, it shall lose all its Motion, or be quiescent after the Stroke; the Body B which receives it will communicate it to C, and C to D, and D to E, and E to F; and because Action and Re-action between the Bodies B, C, D, E, are equal, as they were quiescent before, they must continue so; but the Body F having no other Body to re-act upon it, has nothing to obstruct its Motion, it will therefore move on with the same Velocity which A had at first, because it has all the Motion of A, and the same Quantity of Matter by Hypothesis.

20. Let there be three Bodies A, B, C; and let A strike B at rest; the Velocity generated in B by the Stroke will be $y = \frac{2QV}{Q+\mathcal{Q}}$, and so the *Momentum* of B will be $\frac{2QV\mathcal{Q}}{Q+\mathcal{Q}} = \mathcal{Q}y$. With this *Momentum* B will strike C at rest and contiguous to it; the Velocity generated in C will be $\frac{2\mathcal{Q}y}{\mathcal{Q}+C}$; and its *Momentum* will be $\frac{2\mathcal{Q}yC}{\mathcal{Q}+C} =$ $\frac{2QC}{\mathcal{Q}+C} \times \frac{2QV}{Q+\mathcal{Q}} = \frac{4QV\mathcal{Q}C}{Q\mathcal{Q}+QC+\mathcal{Q}^2+\mathcal{Q}C}$. Fig. 11.

21. If now we suppose B a variable Quantity, while A and C remain the same, we shall find what Proportion it must have to each of them in order that the *Momentum* of C may be a *Maximum*, or the greatest possible, by putting the Fluxion thereof equal to nothing; that is, $\frac{4Q^2C^2V\dot{\mathcal{Q}} - 4QC\mathcal{Q}^2\dot{\mathcal{Q}}}{\overline{QC+Q\mathcal{Q}+\mathcal{Q}C+\mathcal{Q}^2}|^2} = 0$; whence we get $QC - \mathcal{Q}\mathcal{Q} = 0$, and so $QC = \mathcal{Q}\mathcal{Q}$; consequently $Q : \mathcal{Q} :: \mathcal{Q} : C$, or $A : B :: B : C$; that is, *the Body* B *is a Geometrical Mean between* A *and* C.

22. Hence if there be any Number (n) of Bodies in a Geometrical Ratio (r) to each other; and the first be

Bodies upon each other are always equal, and in contrary Directions: That is, by Action and Re-

A, the second will be r A, the third r^2 A, and so on to the last, which will be r^{n-1} A.

23. Also, the Velocity of the first being V, that of the second will be $\frac{2V}{1+r}$, (for $\frac{2QV}{Q+\mathfrak{Q}}$ is here $= \frac{2AV}{A+rA} = \frac{2V}{1+r}$) that of the third $\frac{4V}{\overline{1+r}^2}$, that of the fourth $\frac{8V}{\overline{1+r}^3}$, and so on to the last, which will be $\overline{\frac{2}{1+r}}\Big|^{n-1}$ V.

24. The *Momentum* of the first will be A V, that of the second $\frac{2rAV}{1+r}$, that of the third $\frac{4r^2AV}{\overline{1+r}^2}$, that of the fourth $\frac{8r^3AV}{\overline{1+r}^3}$, and so on to the last, which will be $\overline{\frac{2r}{1+r}}\Big|^{n-1}$ A V.

25. To give an Example; if $n = 100$, and $r = 2$; then will the first Body A be to the last r^{n-1}A, as 1 to 633825300000000000000000000000, nearly; and its Velocity to that of the last nearly as 271022000000000000 to 1: Lastly, the *Momentum* of the first to the last will be nearly as 1 to 2338480000000.

26. If the Number (n) of Bodies be required, and the Ratio of the *Momenta* of the first and last be given, as 1 to M, and the Ratio of the Series r given also; then putting $\frac{2r}{1+r} = R$, we have the *Momentum* of the last Body express'd by $\overline{\frac{2r}{1+r}}\Big|^{n-1} = M = R^{n-1}$; therefore the Logarithm of M is equal to the Logarithm of R multiplied

Re-action equal Changes of Motion are produced in Bodies acting upon each other; and these Changes are impress'd towards contrary Parts.

THE first of these Laws is founded on the *Vis Inertiæ* of Matter, whereby it is indifferently disposed to persevere in its State of Motion or Rest. It is not more evident that Matter at Rest requires an extrinsic Power to give it Motion, than that, when in Motion, the Force of some other Body resisting it is necessary to bring it to a State of Rest. For want of such Resistance we see the Planets and Comets long conserve their Motions undiminished; while moving Bowls, and Wheels, are gradually reduced to a State of rest, by the Friction or Rubbing of the Parts on which they move, against contiguous resisting Bodies; as is evident by the Experiment of the *Axis in Peritrochio*, moving first on fixed Parts, and afterwards on *Friction-Wheels.*

FROM this Law, and what will be farther demonstrated hereafter, it follows, that

multiplied by the Power $n-1$; that is, $l.\,M = \overline{n-1} \times l.\,R$; consequently $\frac{l.\,M}{l.\,R} + 1 = n$, the Number of Bodies required.

that no *perpetual Motion* can be effected, at leaſt by any human Power, with Bodies in a reſiſting Medium.

BY the *ſecond Law* we are inſtructed how to eſtimate the Sum of the Motions of Bodies moving the ſame or contrary Ways, when they directly ſtrike, or impinge on each other. Alſo we hence learn the *Compoſition and Reſolution of Motion* ariſing from Forces impreſs'd in oblique Directions; a Doctrine of the utmoſt Uſe in Philoſophy, and the Foundation of all *Mechanics*. To illuſtrate this: Let the Body

PlateIII. Fig. 4. B at reſt be impell'd by the Body A in the Direction *b c*, with a Force that would, in a given Time, cauſe it to move from *b* to *c*; at the ſame Inſtant, let another Body C ſtrike it in the Direction *b d*, with a Force that will carry it from *b* to *d* in the ſame Time; then compleat the parallelogram *b c e d*, and draw the Diagonal *b e*, that will repreſent the Direction and Diſtance through which the Body will move in the ſame Time by both the Forces conjointly (XXIV).

THE

(XXIV.) This is evident if we conſider that the Force impreſs'd by the Body C does no ways diminiſh the Velocity of a Body approaching to the Line *c e*, at the End of the given Time, and therefore it will then be found ſome-

THE *third general Law* is founded on Reason and Experience: We know from the

somewhere in the said Line *ce*. For the same Reason it will at the End of the said Time be carried to a Distance from *bc* equal to *bd*, and therefore it must also at the same Moment be found somewhere in the Line *de*; but it cannot be in the Line *ce* and *de* at the same Time; unless in that Point *e*, where they intersect each other, as the Proposition asserts.

2. We may now conceive the Body B moving by the single Impulse of some Power in the Direction *be*, such as will carry it thro' the Space *be* in a given Time: then this may be resolved into any other two Forces acting in the Directions *be* or *de*, and *bd* or *ce*, which Lines will also represent the Efficacy of the said Forces in the same Time.

3. This Doctrine of the *Composition and Resolution of Forces* will be found of very frequent Use; and will be farther illustrated in an Application to the Doctrine of *Oblique Percussion*, as follows. Let the Body A im- Plate IV.
pinge on B at rest in the Direction A C, which be- Fig. 12.
cause it does not go through the Center of the Body B, the Stroke will not be *Direct*, but *Oblique*; and let the Force of the Stroke be represented by the said Line A C.

4. This Force A C may be resolved into the two Forces A I, and I C; the former of which being parallel to the horizontal Line G F, cannot affect the Body B at all; for the Motion in that Line would only pass by the Superficies of B, and touch the uppermost Point without any Stroke or Force to remove it.

5. But that Part of the Stroke represented by I C, passes thro' the Center of B, and therefore expresses the whole Force with which the Body A strikes it in the Direction A C. *Therefore the Force of an oblique Stroke is to the Force of a direct Stroke, as* I C *to* A C, *passing through the Center of the Body* B.

the Nature of Attraction or Gravity, that if a Stone fall towards the Earth, the Quantity

6. Now since IC=AG, we have this Analogy, *The direct Stroke is to the oblique Stroke, as* A C to A G, *that is, as Radius to the Sine of the Angle of Inclination* ACG. Whence, it is evident, the less this Angle is, the more oblique the Stroke, and the less its Force, because the less must AG (=IC) be in respect of A C.

7. If the Bodies A, B, are *Non-elastic*, in the Line GF, take CF=CG=AI, and this will be the horizontal Velocity of A after the Stroke, which will be the same as before; and since the whole Force I C of the Striking Body A is known, we can find (by *Annot.* XXIII. 7.) what Part thereof remains after the Stroke, which let be express'd by FD, (taken from F to the Right) then drawing CD it shall represent the Motion and Direction of A after the Stroke, while B will go on towards E with the Motion BE generated by the Stroke, (which also may be found as before) and will be equal to F D, if the Bodies A and B are equal.

8. If the Bodies are *Elastic*, and B less than A, the Direction of A after the Stroke, (*viz.* CD) will make an *Acute Angle* with BE the Direction of B, or lie on the Right of CF. If B be greater than A, it will be reflected so that CD will make an obtuse Angle with CE, or lie on the left of it. But if both the Bodies are equal, the whole Motion of A in the Direction CE will be destroyed (by *Annot.* XXIII. 19.) and it will proceed with only the horizontal Velocity in the Direction CF, making a right Angle with CE, the Direction of B.

9. If two Bodies A, B, in Motion, impinge on each other in the Point C in the oblique Directions AC, BC, these are each resolvable into two, *viz.* AE and EC,
Plate IV. Fig. 13. and BD and DC. Now since EC and DC are parallel, they cannot obstruct each other, so will be the same after the Stroke as before it. But AE and BD, being opposite, will express the Forces with which the Bodies strike each other directly, and may be found by the Rules above delivered.

Quantity of Motion both in the Earth and Stone is the ſame. That the Iron attracts the Loadſtone with an equal Power of Magnetiſm, is evident by Experiment. That *Action* and *Re-action* are equal between impinging Bodies, or that the ſame Quantity of Motion that is generated in one Body is deſtroy'd in the other by the Stroke, whether the Bodies be *elaſtic* or *non-elaſtic*, will alſo be made apparent to the Senſes by Experiment. Whence alſo it will appear, that the Action or Effect of *elaſtic Bodies* is twice as great, as that of Bodies void of Elaſticity (XXV).

FROM

10. If then we make $CE = CE$, and $CD = CD$: Alſo CG = Motion of A after the Stroke, and CF = Motion of B, and compleat the Parallelograms *EG*, and *DF*, their Diagonals CA and CB will be the Directions, and expreſs the Velocities A and B after the Stroke. This Conſtruction is general, and may be accommodated to particular Caſes, varying with the Magnitudes and Velocities, &c. of the impinging Bodies.

(XXV) Let $Q =$ Quantity of Matter in the Earth, and $V =$ the Velocity with which it moves by Attraction; and let q and v denote the ſame Things in the falling Stone; then ſince the Earth and Stone act mutually on each other by Attraction, the Velocity of each, and conſequently the Spaces they deſcribe in the ſame Time, will be as the Power acting on each Body, which therefore will be inverſely as the Quantities of Matter in each: Conſequently $Q : q :: v : V$; and ſo $QV = qv$, or the *Momentum* of the Earth is equal to that of the falling Body.

FROM this Law we have a Solution of divers *Phænomena* otherwise not to be accounted

2. In like Manner, when a Horſe draws a Stone, the Cord being equally ſtretch'd between both, acts equally upon both, and that which has leaſt Reſiſtance yields, and is drawn along. This is uſually the Caſe of the Stone; but if its Weight be increaſed, and therewith its Reſiſtance, till it be equal or greater than that of the Horſe, then neither Horſe nor Stone will move; unleſs the Stone be laid on a Deſcent, and then it will move the contrary Way, and draw the Horſe after it.

3. In the ſame Manner we may underſtand how Rowing, Swimming, Flying, &c. is performed; for the Boat, the Fiſh, and the Bird, are Bodies eaſily moveable with the leaſt *Impetus*, but the Water and Air in which they move, tho' fluid Bodies, yet give great Reſiſtance to others which ſtrike them, or re-act with an equal Force, in a contrary Direction; and by this Means impel the Boat, the Fiſh, and Bird in a Direction nearly contrary to that in which they ſtrike it, and with an equal Force.

4. *Merſennus* tells us a Cannon 12 Feet in Length, weighing 6400 *lb.* gives a Ball of 24 *lb.* an uniform Velocity at the Rate of 640 Feet *per* Second. Put $\mathrm{w}=6400$, $w=24$, $v=640$, $\mathrm{v}=$Velocity, with which the Cannon recoils. Now ſince the *Momentum* of the Cannon and Ball are equal, we have $\mathrm{wv}=wv$, and ſo $\mathrm{v}=\frac{wv}{\mathrm{w}}=\frac{640\times 24}{6400}=2{,}4$, the Velocity of the Cannon; that is, it would recoil at the Rate of $2\frac{4}{10}$ Feet *per Second*, if free to move.

5. But if the Cannon be fix'd, it will receive a Shock or Preſſure from the expanſive Force of the Powder, equal to the Preſſure of a certain Weight, which Weight may thus be found. As the Powder conſtantly acts on the Ball while in the Cannon, it will drive it along with

Plate IV.

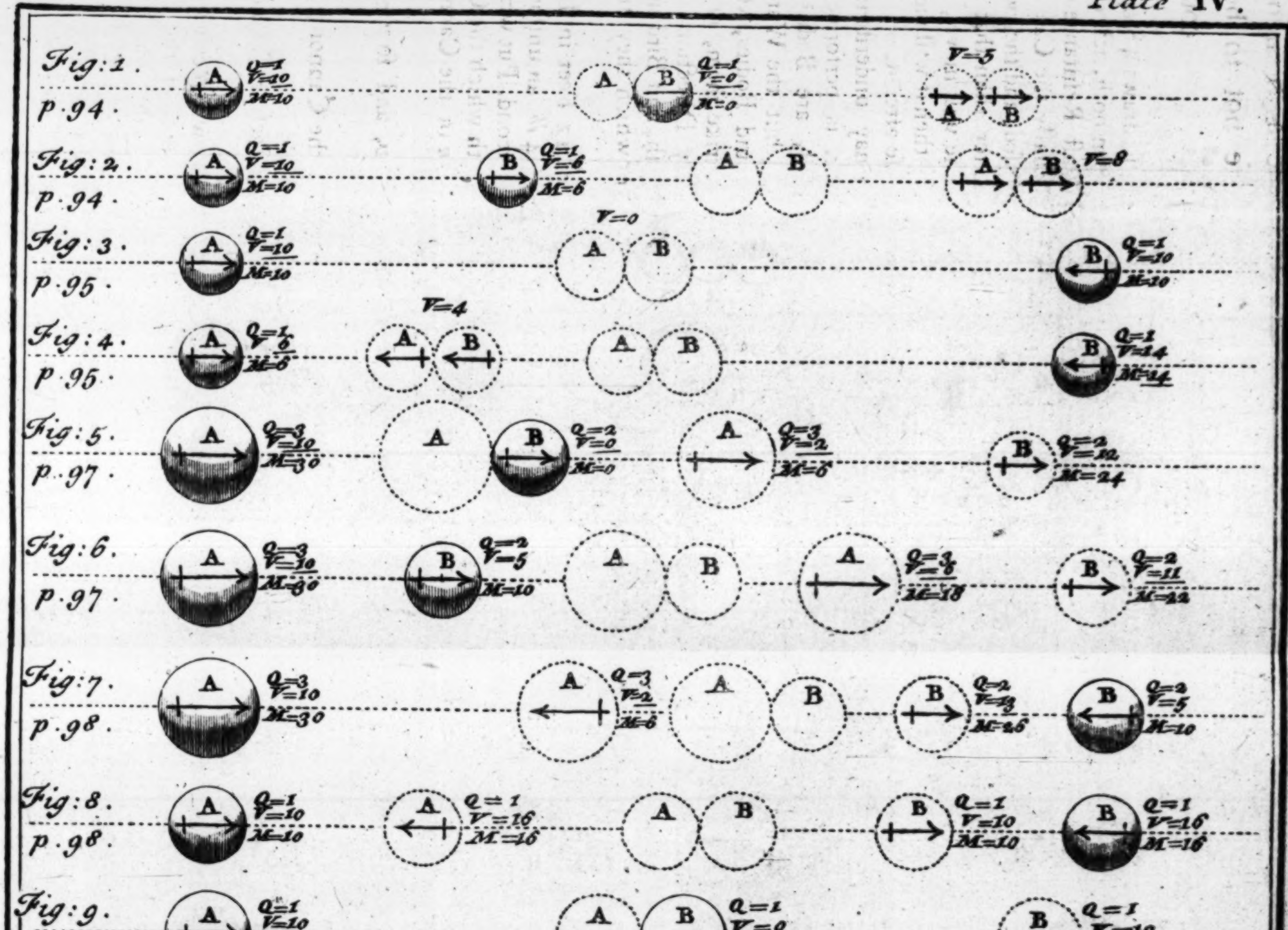
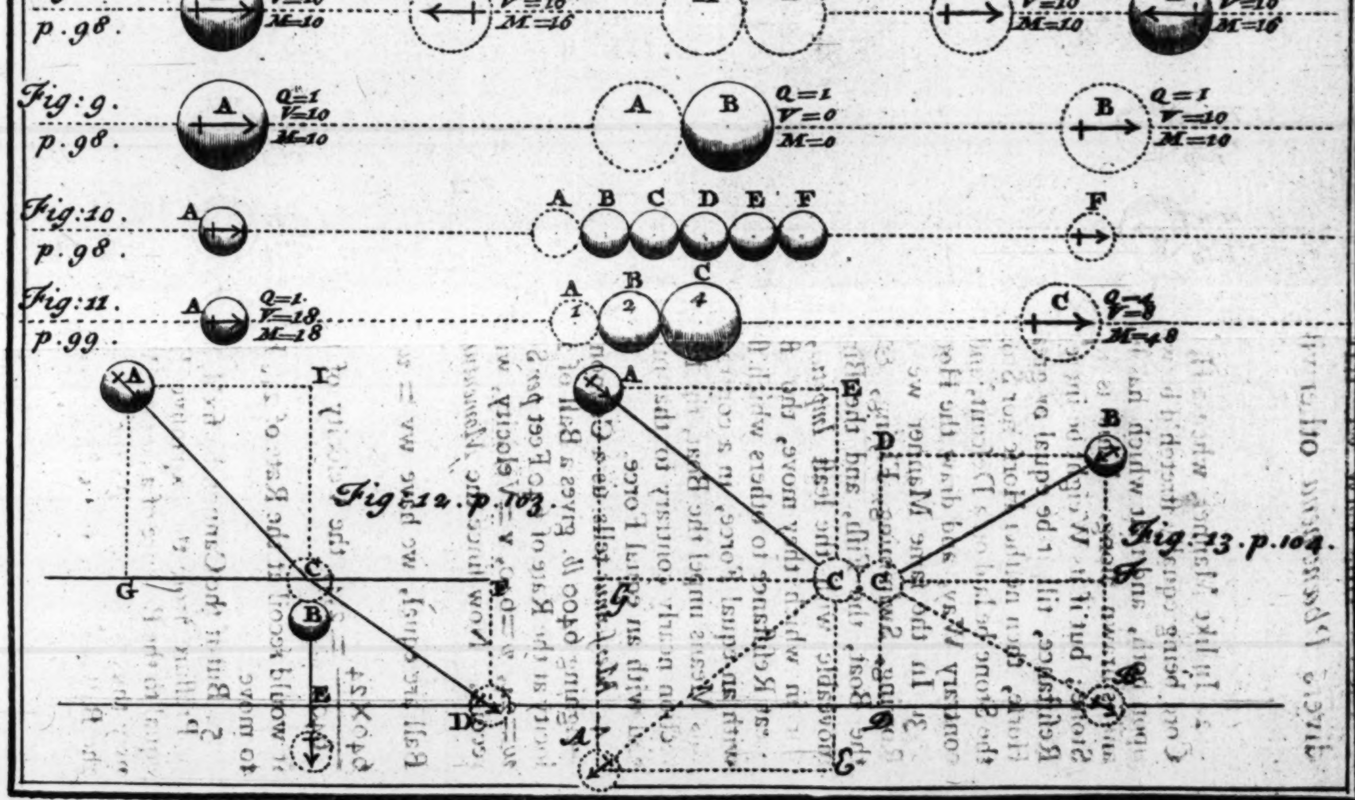


Fig:1.
p.94.
Fig:2.
p.94.
Fig:3.
p.95.
Fig:4.
p.95.
Fig:5.
p.97.
Fig:6.
p.97.
Fig:7.
p.98.
Fig:8.
p.98.
Fig:9.


p.98.
Fig:9.
p.98.
Fig:10.
p.98.
Fig:11.
p.99.
Fig:12.p.103.
Fig:13.p.104.

counted for: As, why soft and unelastic Bodies *retain the Impressions* of others, while

with an accelerated Velocity, which accelerated Velocity will produce an uniform Velocity of 24 Feet in the same Time (as we shall shew hereafter) when free from the Cannon. And since the Ball goes at the Rate of 640 Feet *per* Second, it will pass over 24 Feet in $\frac{1}{26\frac{2}{3}}$ of a Second, which therefore is the Time of its passing thro' the Cannon.

6. Now since in accelerated Velocities the Spaces passed over are as the Squares of the Times, therefore as $\overline{\frac{1}{26\frac{2}{3}}}|^2 : \overline{1''}|^2$, or as $1''$ is to $\overline{26\frac{2}{3}''}|^2$ $(=711\frac{1}{4})$ so is 12 Feet to $8533\frac{1}{3}$ Feet, which the Ball would be carried through in one Second by the accelerating Force of the Powder.

7. But since the accelerating Force of Gravity gives the Ball a Weight of 24 Pounds, the greater accelerating Force of the Powder will give it the Force of Weight $=12800$ Pounds. For the Weights of Bodies are as the accelerating Forces, and these are the Spaces passed thro' in the same Time; therefore 16 F. : 24 *lb.* : : $8533\frac{1}{3}$ F. : 12800 *lb.* as required.

8. But if the Cannon be free to recoil, Part of this Force will be spent in giving it a Velocity of 2,4 Feet in a Second, and to find what Part of the whole Weight 12800 Pounds; that is, we are to consider that as the whole Force produces a Velocity of 640 Feet in a Second one Way, so when the Piece recoils, this Velocity is divided into two Parts in contrary Directions, *viz.* into 640—2,4 and 2,4; the whole Force therefore of 12800 Pound must be divided into Parts of the same Proportion; thus, as 640—2,4 : 2,4 : : 637,6 : 2,4 : : 797 : 3 : : 12800—x : x : : 797+3 (=800) : 3 : : 12800 : x=48 Pounds. Wherefore of the 12800 Pounds only 48 are spent for giving a Recoil to the Cannon, the other 12772 Pounds spend themselves in Pressure on the Gun and Ball, giving it a Velocity of 637,6 Feet *per* Second.

while hard and elastic Bodies *rebound with equal Force*, and under *equal Angles*. Hence, the Reason of *Rowing with Oars*, and the *Swimming of Fishes* in Water; also the *Flying of Birds* in Air; the *Recoiling of Guns*; the retro-vertiginous Motion of the *Æolipile*; the *Hydrostatic Paradox*; and many other Things hereafter occasionally mentioned, are accountable for on this and no other Principle.

WE proceed next to consider the *Nature of Motion belonging to Bodies which descend freely by the Force of Gravity* in *Vacuo*, or an unresisting *Medium*: And this Kind of Motion we shall find affected with the following Properties, *viz.* (1.) That it is equably accelerated. (2.) That its Velocity is always proportional to the Time of the Fall. (3.) That the Spaces passed through are as the Squares of the Times or Velocities. And therefore, (4.) That the Velocity, and consequently the *Momentum*, which is proportional thereto, is as the Square Root of the Space descended through. (5.) That the Space passed through the first Second is very nearly $16\frac{13}{100}$ Feet. (6.) That a Body will pass over twice the Space in an horizontal Direction, with the last acquired Velocity of the

the descending Body, in the same Time (XXVI).

HENCE

(XXVI) 1. It has been shewn (*Annot.* XXI.) that the Motion of a Body falling freely by its Gravity is *equably accelerated*; and that *its Velocity is at all Distances proportional to the Time of the Fall*, is evident from hence, that in every equal Particle of Time, the Body receives an equal Impulse from Gravity, which generates an equal Increment of Velocity; which, therefore, as it increases with, must also be proportional to the Times.

2. *That the Spaces passed thro' are as the Squares of the Times or Velocities* is hence evident, that if the Time of the Fall thro' a certain Space be represented by A 1, and the Velocity acquired in the End of that Time be 1 *a*; then drawing A *a*, the Triangle A 1 *a* will represent the Space pass'd thro' in that Time. For if we conceive the Line A 1 to be divided into an infinite Number of equal Parts, and thro' each of these Divisions, Lines drawn parallel to 1 *a*, these will represent the Velocities in the several Particles of Time. Now the Space described in each Moment will be as the Velocity (because for a Moment the Velocity may be considered as uniform) consequently the Sum of all the Spaces described in the Moments in the Time A1, will be as the Sum of all the Velocities, or Lines parallel to 1*a*, which represent them: But the Sum of all these Lines make up the Area of the Triangle A1*a*, therefore the whole Space pass'd thro' in the Time A1 will be as the Area of the Triangle A1*a*. Plate III. Fig. 5.

3. Therefore the Triangle A2*b* represents the Space pass'd thro' in the Time A2, and the Triangle A3*c* the Space passed thro' in the Time A3, and so on. But the Triangles A1*a*, A2*b*, &c. are similar, and therefore are to each other as the Squares of their Sides A1, A2, or 1*a*, 2*b*, &c. That is, *The Spaces are as the Squares of the Times or Velocities.*

4. Consequently the *Velocities and Times of the Fall are as the Square Roots of the Spaces pass'd thro'*; and since the

HENCE it follows, that if one Leg AB of a right-angled Triangle represent the *Time*

the *Momentum* of a Body is always $M = QV$, and in this Case Q being a given Quantity, we have $M : V$, or the *Momentum of the Body at the Velocity, or as the Square Root of the Height thro' which the Body falls.*

5. It has been found by very accurate Experiments made by letting Bodies of various Sorts fall from the Height of the Dome of St. *Paul's*, to the Pavement, that Gravity accelerates Bodies in the beginning of their Fall, at the Rate of 16,13 Feet in the first Second of Time. This may also be otherways shewn *a priori*, because we demonstrate that the Time (T) of the Vibration of a Pendulum (in the Arch of a Cycloid) is to the Time (t) of a Body's falling thro' Half the Length of the Pendulum ($\frac{1}{2}$L) as the Circumference of a Circle (P) to the Diameter (D). (*Annotat.* XXVIII. 9.)

6. Again, it is found that the Length of a Pendulum vibrating Seconds is 39,2 Inches. Now since $T : t :: P : D :: 3{,}14159 : 1$, we have $T^2 : t^2 :: \overline{3{,}14159}^2 : 1^2 :: x : \frac{1}{2}L$; whence we have $x = \frac{1}{2}L \times 3{,}14159^2 = 16$ Feet $=$ the Space descended in the Time of one Vibration, or 1 Second.

Plate III. Fig. 5. 7. Since the Space pass'd thro' by an uniform Motion in the Time AB with the Velocity BC is $AB \times BC = ABCD = 2ACB$; it is evident that *a Body moving uniformly with the last acquired Velocity* BC *of a descending Body, will move thro' twice the Space in the Time* AB *of the Fall.*

8. It is farther evident, that since the Spaces descended thro' in each Second are as the odd Numbers, 1, 3, 5, 9, 11, 13, 15, &c. and these Numbers constantly approach nearer and nearer to an Equality, so the accelerated Motion of the Descent does by Degrees approach nearer and nearer to an uniform Motion; thus the Spaces 101, 103, 105, differ but little; and the Spaces 1001, 1003, 1005, differ less; but when we come

to

Time of the Fall, and the other Leg BC the *Velocity* acquired at the End of the Fall; then will the Area ABC of the Triangle repreſent the Space paſſed through. And hence the Spaces deſcended through at the End of every Second will be as the Square Numbers 1. 4. 9. 16. 25. 36. *&c.* and therefore the Spaces paſſed through in each Second ſeparately will be as the odd Numbers 1. 3. 5. 7. 9. 11. 13. 15. *&c.* as in the *Figure.*

THE next Sort of Motion is that of Bodies deſcending on *inclined Planes*, and *curved Surfaces*, which we find diſtinguiſhed with the following Properties. (1.) The Motion on the inclined Planes is equably accelerated, as ariſing from Gravity. (2.) The Force of Gravity compelling a Body, as A, to deſcend on an inclined Plane BD, is to the abſolute Force of Gravity as the Height of the Plane BC to its Length BD. (3.) The Spaces deſcended are

Plate V. Fig. 1.

to the Spaces 1000000001, 1000000003, 1000000005, theſe are ſo very near equal, that the Motion may now be eſteemed uniform.

9. If the Deſcent be in a *reſiſting Medium*, the Motion will actually become uniform at a certain Diſtance, and the ſooner as the Medium is denſer; thus a Body falling in Air will be longer in acquiring an uniform Motion

are as the Squares of the Times. (4.) The Times in which different Planes, BD, BH, of the same Altitude BC, are passed over, are as their Lengths respectively. (5.) The Velocities acquired by descending through such Planes at the lowest Points, DH, are all equal. (6.) Therefore if a Body

Motion than in Water, and in Quicksilver it will soon obtain it; because as the Density increases, so does the Resistance, and consequently the Increments of Velocity are annihilated in the same Ratio, and the Motion reduced to Uniformity.

10. Bodies of the same Matter and Figures will sooner come to an uniform Motion as the Magnitudes are lesser; for it is shewn (in Geometry) that the Quantities of Matter decrease in Bodies with the Cubes of the Diameters, but the Surfaces decrease only with the Squares of the Diameters. And since the Resistance is proportional to the Surfaces of Bodies, moving in the same Medium with the same Velocity, it will follow that smaller Bodies will be more resisted than larger ones, and so come sooner to an uniform Motion. Hence it is, that a Body reduced to Powder descends very slowly and with nearly an uniform Motion in Water, tho' in the Solid much heavier than Water. Also a Bullet shot from a Gun will go much farther and with a greater Velocity than a Charge of small Shot of the same Weight, or a Ball of Cork of the same Size, which *in Vacuo* would go much farther than the Bullet, as admitting a greater Velocity with a less Quantity of Matter, from the same *Momentum* of the Powder.

11. If Bodies (equal and homogeneous) move in the same Medium with different Velocities, those which move fastest will soonest acquire an uniform Motion, since (as will be shewn) the Resistance in such a Case increases with the Squares of the Velocity. If the Velocity with which a Body is projected downwards be equal

Plate V.

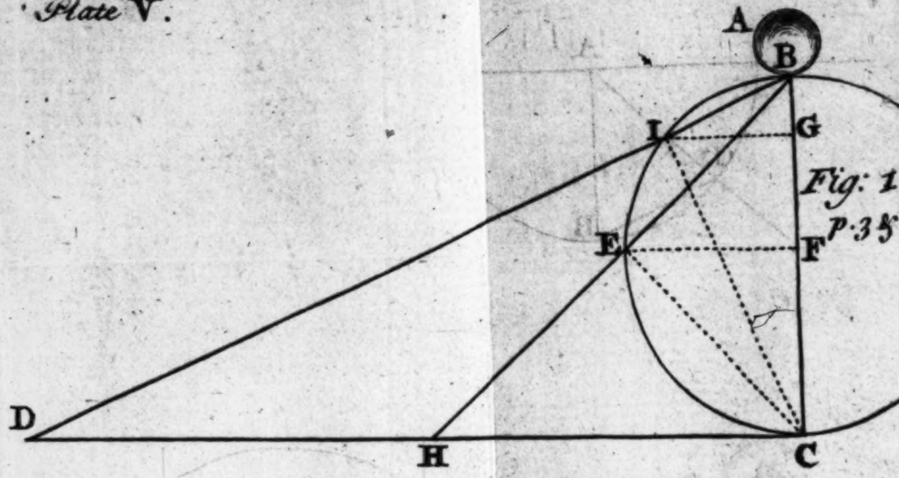


Fig: 1 p.35

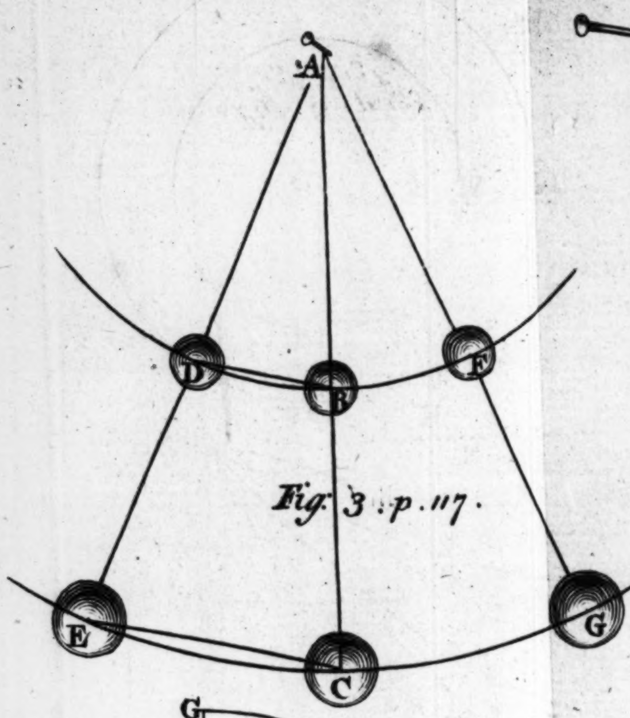


Fig: 3 . p. 117.

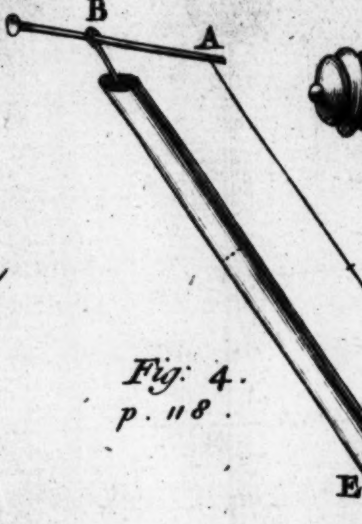


Fig: 4. p. 118.

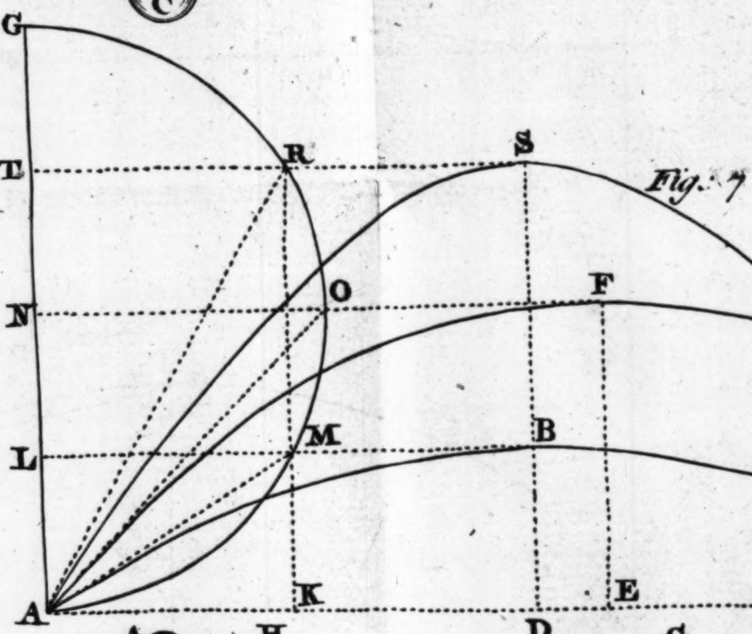


Fig. 7

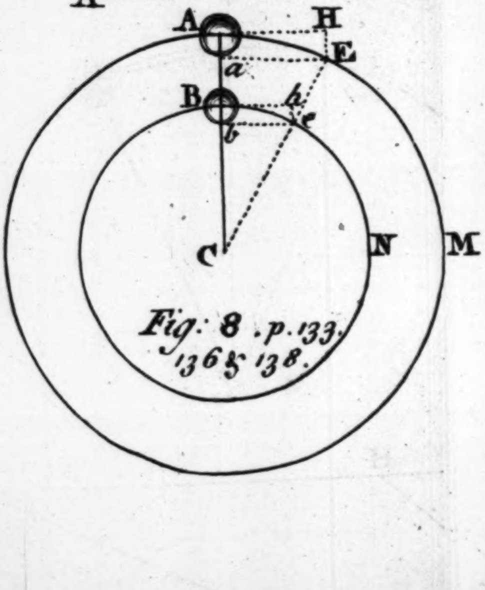


Fig: 8 . p. 133, 136 & 138.

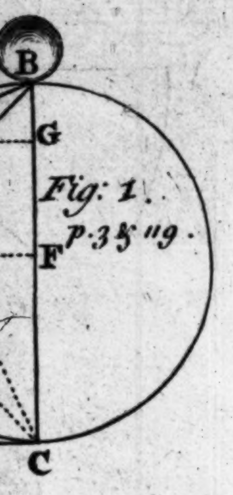


Fig: 1. p. 3 & 119.

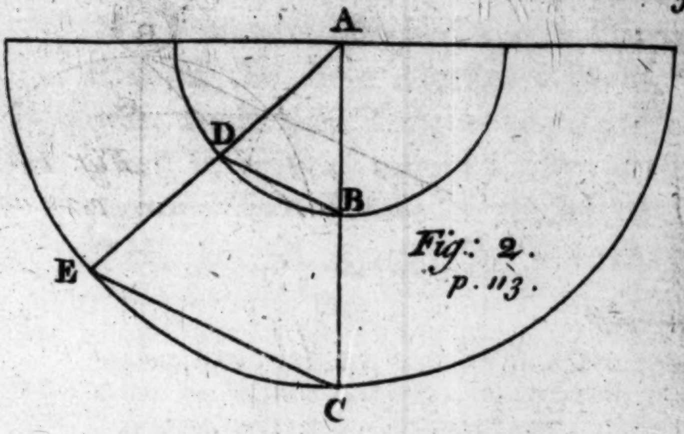
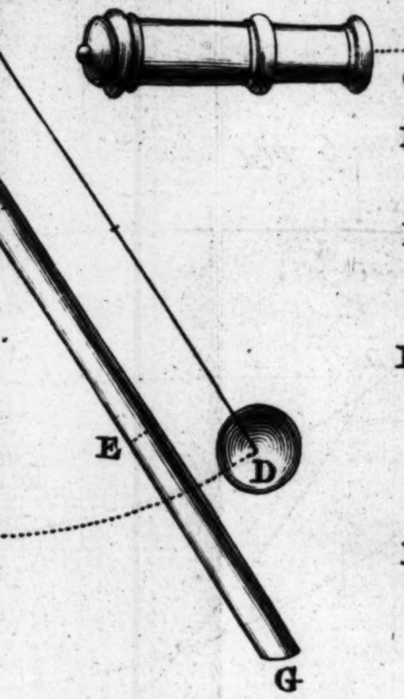


Fig: 2. p. 113.



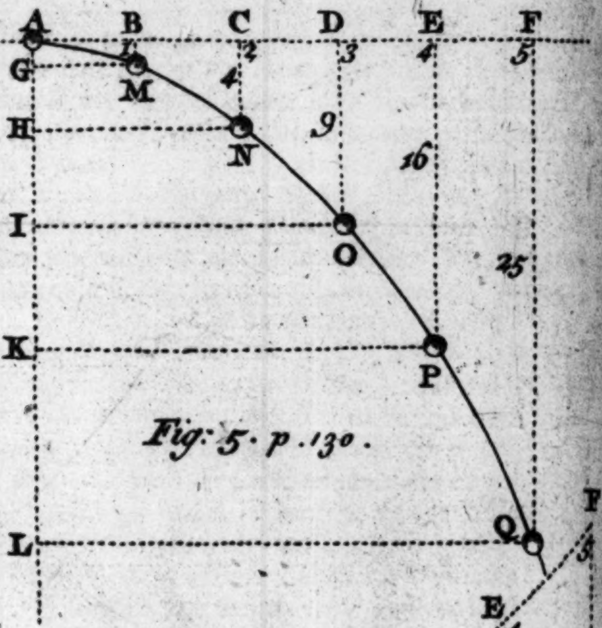


Fig: 5. p. 130.

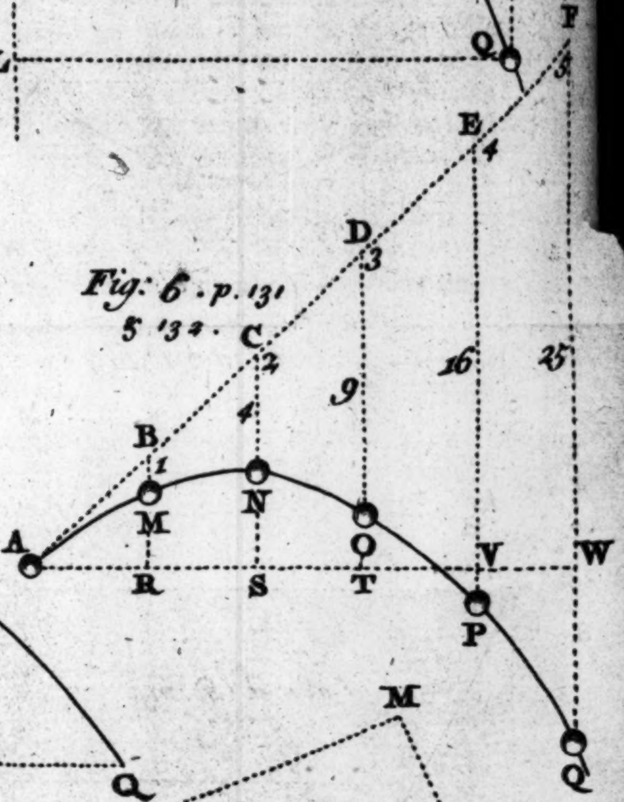
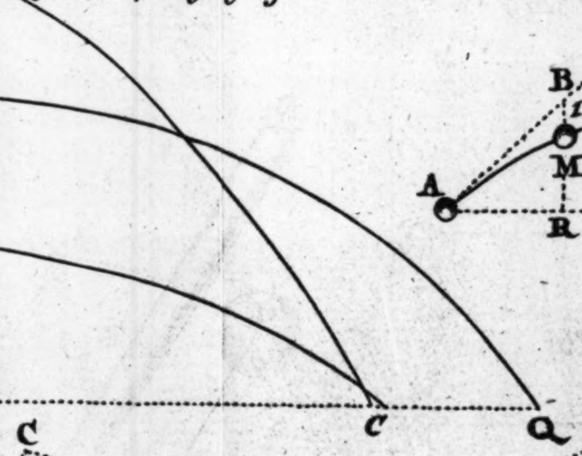


Fig: 6. p. 131 & 132.

Fig: 7: p. 131 & 132.



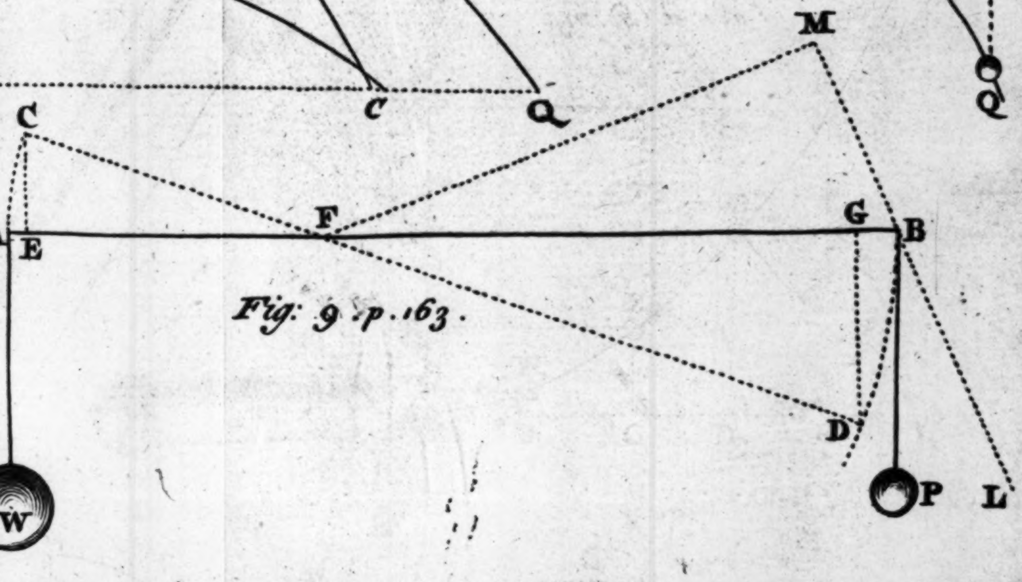
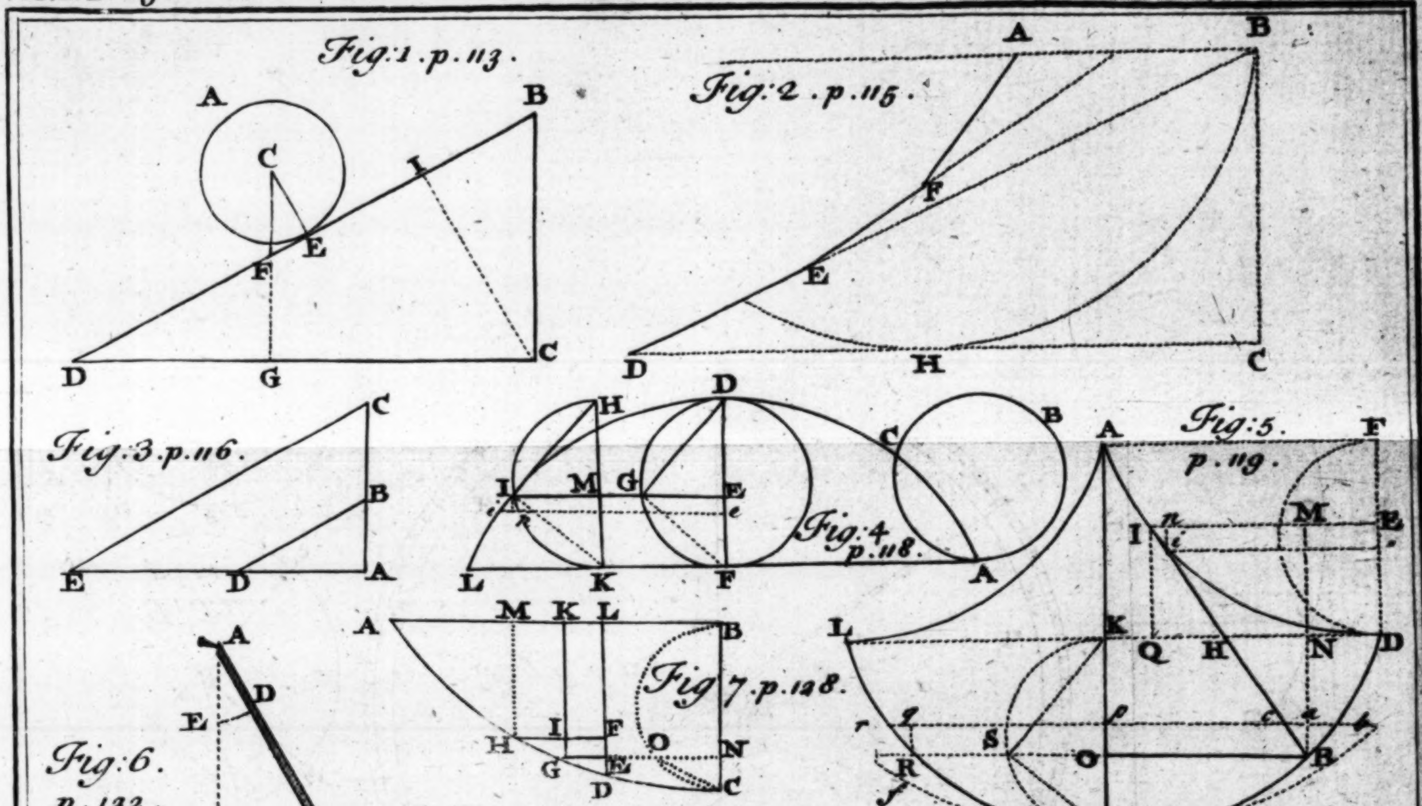


Fig: 9. p. 163.

E. Bowen sc.

Vol.I. P. 113. Plate VI.



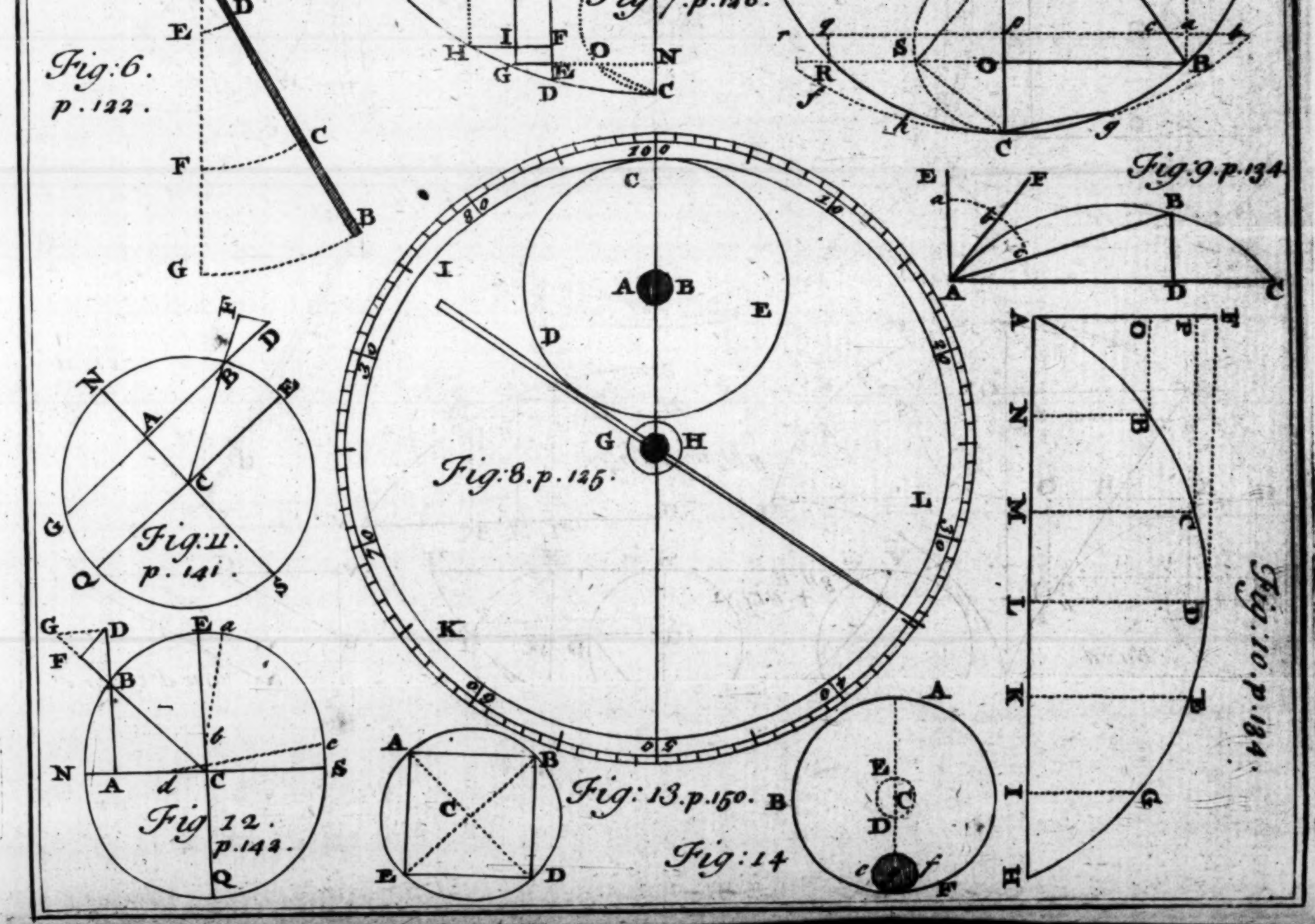


Fig:6. p.122.

Fig:8. p.125.

Fig:9. p.134.

Fig:10. p.134.

Fig:11. p.141.

Fig 12. p.142.

Fig:13. p.150.

Fig:14

Body descends from the same Height thro' several contiguous Planes, ever so different in Number or Situation, it will always acquire the same Velocity in the lowest Point. (7.) The Times or Velocities of Bodies descending through Planes similarly situated, or alike inclined to the Horizon, as DB, EC, will be as the Square Roots of their Lengths DB, EC (XXVII). Fig. 2.

FROM

equal to the uniform Motion of a free Descent, the Motion will be every where uniform, because the Resistance of the Medium is equal to Gravity. But if a Body (as a Ball from a Cannon) be projected with a Velocity greater than that, the Motion will be retarded, inasmuch as the Resistance is now greater than Gravity, and so will diminish the uniform Motion, arising from the Equilibrium just mentioned.

12. Hence light Bodies (as an Arrow) thrown directly upwards, spend less Time in rising than falling, because the Motion upwards is altogether retarded, whereas that of the Descent is in some Part uniform. Also the greater the Velocity with which a Body (as a Bullet) enters a dense Medium (as Water) the less the Effect at a given Distance therein; and with the greater Force they are reflected or thrown out again, when projected obliquely, as we see in a Stone thus thrown into the Water, &c.

(XXVII) Let BD be an inclined Plane, A a Body descending thereon; from the Centre C draw CE perpendicular to the Plane, and CG perpendicular to the Horizon or Base CD, and meeting the Plane in F. Now since the Body gravitates in the Direction CF, let CF represent the Weight of the Body or absolute Force of Gravity; this may be resolv'd into two Forces, acting in the Direction CE and EF, of which the first is perpendicular to the Plane, and is that by which it Plate VI. Fig. 1.

FROM these Properties of Bodies descending on Inclined Planes, we deduce the following Corollaries, *viz.* (1.) That the Times in which a Body descends through the

presses or acts upon the Plane; but since the Plane equally acts upon the Body in a contrary Direction, that Force CE is wholly destroy'd. The Remainder therefore, EF, is that which carries the Body down the Plane, as acting in a Direction parallel thereto; and this is called the relative Gravity.

2. Now because of the similar Triangles CFE, DFG, DBC, we have CF : FE :: DF : FG :: BD : BC; that is, *the absolute Gravity* CF *is to the Residual or Relative Gravity* FE, *as the Length of the Plane* BD *to the Height thereof* BC.

3. Also since the Triangles BDC and BCI are similar (CI being drawn perpendicular to BD); and since the Spaces described in the same Time in the Perpendicular and on the Plane will be as the Powers of Gravity, or accelerating Forces in those Directions, that is, as BD to BC, or as BC to BI (for BD : BC :: BC : BI). Therefore BI will be the Distance descended on the Plane in the same Time as a Body would descend freely in the Perpendicular from B to C.

4. Since the Motion on the Plane is accelerated, the Time of describing BI will be to the Time of describing BD, as $\sqrt{BI} : \sqrt{BD} :: t : T$; and so $t^2 : T^2 :: BI : BD$; but, (because BI : BC :: BC : BD) we have $BI : BD :: \overline{BI}^2 : \overline{BC}^2$; therefore $t^2 : T^2 :: \overline{BI}^2 : \overline{BC}^2$; and so $t : T :: BI : BC :: BC : BD$; *consequently the Time of describing* BI *or* BC *will be to the Time of descending through the Plane* BD, *as the Height of the Plane* BC *to its Length* BD.

5. Hence the Time $(\mathcal{T})$ of descending thro' any other Plane BH of the same Height (*Fig.* 6.) is to the Time (t) of the Descent thro' the Perpendicular BC, as BH to BC; that is, $\mathcal{T} : t :: BH : BC$; but it is also $\mathcal{T} : t :: BD : BC$; therefore *ex æquo*, we have $T : \mathcal{T} ::$ BD :

the Diameter BC, or any Chord BE of a Circle, are equal. Hence, (2.) All the Chords of a Circle are described in equal Times. (3.) The Velocities acquired in descending

BD : BH; *or the Times of descending thro' several Planes* BD, BH, *of the same Height, are as the Length of the Planes.*

6. The Velocity acquired in falling to C, is to the Velocity at I as BC to BI, (for the Velocities generated in the same Time, must be as the Powers which produce them). Also the Velocity at D is to that at I as $\sqrt{\mathrm{BD}}$ to the $\sqrt{\mathrm{BI}}$; or $\mathrm{V} : V :: \sqrt{\mathrm{BD}} : \sqrt{\mathrm{BI}}$; and so $\mathrm{V}^2 : V^2 :: \mathrm{BD} : \mathrm{BI} :: \overline{\mathrm{BC}}^2 : \overline{\mathrm{BI}}^2$; therefore $\mathrm{V} : V :: \mathrm{BC} : \mathrm{BI}$; since then the Velocity at C and D have both the same Ratio to the Velocity at I, they must be equal to each other.

7. In the same Manner it is shewn that the Velocity at H is equal to the Velocity at C; and consequently the Velocities acquired in descending thro' Inclined Planes of the same Height are always equal.

8. Hence a Body descending thro' several Inclined Planes AF, FE, ED, contiguous to each other, will have the same Velocity at the Point D, as it would have acquired in falling freely through the same perpendicular Height BC, for the Velocity at F is the same it would have had by descending thro' GF, which has the same Altitude; and the Velocity at E is the same it would have gained by descending thro' BE; consequently the Velocity of D is the same it would acquire by descending thro' BD; and therefore the same it would have by falling freely thro' the Perpendicular BC. Plate VI. Fig. 2.

9. If now we suppose the Number of those contiguous Planes infinite, and their Lengths infinitely small, they will then constitute a *Curve Line*; whence it follows, That a Body descending thro' the Arch of any Curve BH will acquire the same Velocity in the lowest Point H, as it would have at C, by falling thro' the same perpendicular Height BC.

descending through any Arch of a Circle EC, in the lowest Point C, is equal to that which would be acquired in falling through the same perpendicular Height FC. (4.) The Velocities acquired in descending through the Chords IC, EC, of a Circle, are,

Plate VI. Fig. 3.

10. If there are two Planes, BD and CE, similarly situated, or alike inclined to the Horizon AE; the Time of the Descent on DB, will be to the Time of the Descent on CE, as Square Roots of their Lengths; for the Time on BD is to the Time thro' BA, as DB to BA (by *Art.* 4.) and the Time on CE is to the Time thro' CA, as CE to CA; but the Times thro' BA and CA are as the Square Roots of BA and CA; and since by similar Triangles we have $\sqrt{BA} : \sqrt{CA} :: \sqrt{BD} : \sqrt{CE}$, it follows, *That the Times of Descent on two similar Planes* DB *and* CE *are as the Square Roots of their Lengths*, or as $\sqrt{BD}$ to $\sqrt{CE}$.

Plate V. Fig. 1.

11. From what has been demonstrated in the preceding Articles, it follows, (1.) That the Times of descending from B to I, and to E, are equal, being each equal to the Time of Descent thro' the Perpendicular BC. (2.) For the same Reason the Descent thro' IC and EC, and consequently thro' all the Chords of a Circle, are equal. (3.) The Velocities at C acquired by the said Descents are as the Lengths of the Chords IC and EC. For since BC : EC :: EC : CF, we have $BC = \frac{EC^2}{FC}$; and for the same Reason $BC = \frac{IC^2}{GC}$, whence $\frac{EC^2}{FC} = \frac{IC^2}{GC}$; and therefore $IC : EC :: \sqrt{GC} : \sqrt{FC}$; that is, as the Velocities in the Descents thro' IC and EC. (4.) Lastly, since the Times of Descent through the Planes DB and EC are as $\sqrt{DB}$ to $\sqrt{EC}$, they will also be as $\sqrt{BA}$ to $\sqrt{CA}$; for DB : EC :: BA : CA.

are, at the loweſt Point C, as the Lengths of thoſe Chords. (5.) The Times of Deſcent through Chords of ſimilar Arches DB, EC, are as the Square Roots of the Semidiameters AB, AC, of the reſpective Circles.

FROM theſe Properties, and their Corollaries, the Doctrine of PENDULUMS is derived. A PENDULUM is any Body B, ſuſpended upon, and moveable about a Point A, as a Centre. The Nature of a Pendulum conſiſts in the following Particulars. (1.) The Times of the Vibrations of a Pendulum in very ſmall Arches are all equal. (2.) The Velocity of the Bob in the loweſt Point will be nearly as the Length of the Chord of the Arch which it deſcribes in the Deſcent. (3.) The Times of Vibration in different Pendulums, AB, AC, are as the Square Roots of their Lengths. (4.) Hence the Lengths of Pendulums AB, AC, are as the Squares of the Times of their Vibrations. (5.) The Time of one Vibration is to the Time of Deſcent through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. (6.) Whence the Length of a Pendulum vibrating Seconds will be found 39 Inches nearly; and of an *Half-* Plate V. Fig. 3.

Fig. 4. *Second* Pendulum 9,8 Inches. (7.) An uniform homogeneous Body BG, as a *Rod*, *Staff*, &c. which is *one third Part* longer than a Pendulum AD, will vibrate in the same Time with it (XXVIII). (8.) This *Centre*

PlateVI. Fig. 4. (XXVIII) 1. As the Doctrine of Pendulums and Time-keeping Instruments depends in a great Measure upon the *Cycloid*, I think it necessary here to shew the Nature and Use of that Curve, with regard thereto. If a Circle ABC, insisting on a right Line AL, begin to revolve in the Manner of a Wheel, from A towards L, the Point A will by its twofold Motion describe the Curve ACDIL, while the Circle makes one Revolution from A to L.

2. This Curve is called the CYCLOID, and from the Definition 'tis evident (1.) That the *Base* of the Cycloid AL is equal to the Periphery of the *generating Circle* ABC. (2.) The Axis of the Cycloid FD is equal to the Diameter of the said Circle. (3.) That the Part of the Base KL is equal to the Arch of the Circle IK. (4.) Therefore KF (=ME=IG) is equal to the remaining Arch IH, or GD. (5.) That the Chord of the Circle KI is perpendicular to the Cycloid in the Point I; and (6.) Therefore the Chord HI (being at right Angles with IK) is a Tangent to the Curve in the Point I. (7.) The said Tangent HI is parallel to the Chord DG.

3. Parallel to EI draw *ei* infinitely near, and I*n* perpendicular thereto; then will the Triangles DGE, DGF, I*ni*, be similar, and so we have DE : DG : : DG : DF : : I*n* : I*i*; that is, (putting DF $= a$, and DE $= x$, DI$=z$,) $x : \sqrt{ax} : : \sqrt{ax} : a : : \dot{x} : \dot{z} = \text{I}i = \frac{a\dot{x}}{\sqrt{ax}} =$ Fluxion of the Arch DI, whose Fluent is $2\sqrt{ax} =$ 2DG $=$ the Arch DI; and consequently, the Semicycloid DIL $=$ 2DF, the Diameter of the generating Circle.

4. Let

4. Let AID be a Semicycloid inverted; and suppose a flexible String fasten'd at one End in A, and stretch'd along the said Curve AID, so that the other End of the String should be made to coincide with the Point D; whence the Length of the String will be equal to that of the Curve: If now the End of the String be taken from D, and with a tight Hand be drawn from the Cycloid, it will in its Evolution, describe the Curve DBC, the Nature of which we are now to investigate. Fig. 5.

5. Let the String, in evolving, be in any Situation AIHB, then is IB (called the *Radius of Evolution*) equal to the Part of the Cycloid ID. Put FD = AK = a, DE $= x$, CO $= y$, CB $= z$, then is $ni = \dot{x}$, $Ba = Op = \dot{y}$, $Bb = \dot{z}$; and (by *Art.* 3.) DI = (IB =) $2\sqrt{ax}$, whose Fluxion is $\dot{x}\sqrt{\frac{a}{x}} = \overline{Ii}$; therefore $\dot{x} \times \sqrt{\frac{a}{x}}^2 - \dot{x}^2 =$ $(\overline{Ii}^2 - \overline{in}^2 =) \frac{\dot{x}^2 a - x\dot{x}^2}{x} = \overline{In}^2$, whence $In = \dot{x}\sqrt{\frac{a-x}{x}}$; but $In : Ii :: ca : cB :: aB : Bb = \dot{z} = \frac{\sqrt{a} \times \dot{y}}{\sqrt{a-x}}$; and $Ii : ni :: IB : MB = 2x$; therefore $BM - BN = MN = IQ = ED = x$. Consequently $BH = HI$; and also $CK = KA$, because all the Radii of Evolution (of which AC is the last) are bisected by the base LD; therefore $CK - OK = a - x = OC = y$; whence $\sqrt{y} = \sqrt{a-x}$, and so $\dot{z} = \frac{\sqrt{a} \times \dot{y}}{\sqrt{y}}$; therefore $z = 2\sqrt{ay}$; consequently the Curve CBD, having the same Property with AID, is every Way equal and similar thereto.

6. Whence it appears that if AC be the Length of a Pendulum so disposed, as to vibrate between the two Semicycloids AD and AL, the Bob will describe in its Motion the Cycloid DCL. The Properties of which Motion will now easily appear: For with respect to the

Velocity acquired by descending thro' any Arch RC or BC, it is always as $\sqrt{OC}$ which is as the Chord $SC = \frac{1}{2}RC$; consequently *the Velocity at* C *is every where as the Space passed through, or as the Arch of the Cycloid described in the Descent.*

7. But since it is always $S = TV$, in all Kind of Motions; if in any Case S be as V, 'tis evident T must be a given Quantity, or always the same. That is, When the Spaces ($S, s,$) are as the Velocities ($V, v,$) the Times ($T, t,$) will be always equal; and therefore *all the Arches of a Cycloid, great or small, are described in equal Times.*

8. If we put $CK = a$, $KO = x$, then will $2SC = RC = 2\sqrt{aa - ax}$; if the Descent be from L to R, the Velocity at R will be $\sqrt{x} = \sqrt{OK}$. Now CO : CS (: : CS : CK) : : $Rq : Rr = \frac{a\dot{x}}{\sqrt{aa - ax}} = \dot{z}$, the Fluxion of the Arch LR, which divided by the Velocity $\sqrt{x}$ gives the Fluxion of the Time $= \frac{a\dot{x}}{\sqrt{aax - axx}}$

$= \frac{a\dot{x}}{\sqrt{a} \times \sqrt{ax - xx}}$; but $\frac{a\dot{x}}{2\sqrt{ax - xx}}$ is $=$ the Fluxion of the circular Arch KS; therefore the Fluent of the Time of Descent thro' LR is the Arch $\frac{2KS}{\sqrt{a}}$; wherefore when S coincides with C, LR will become LC; and so the Time of Descent thro' the Semicycloid will be $\frac{2KSC}{\sqrt{a}}$.

9. Therefore the Time of Vibration thro' the whole Cycloid LCD is $\frac{4KSC}{\sqrt{a}}$; and the Time of Descent thro' the Perpendicular $KC = a$, being as $2\sqrt{a}$, we have $\frac{4KSC}{\sqrt{a}} : 2\sqrt{a} : : 2KSC : a$. *The Time of a*

Vibration

Vibration in the Cycloid is to the Time of Descent thro' half its Length, as the Circumference of a Circle to its Diameter, or as 3,14159 to 1.

10. If A C be the Length of a Pendulum vibrating Seconds, it may be easily found, since it is known by most accurate Experiments, that a Body descends freely by Gravity thro' $193\frac{1}{5}$ Inches in the first Second of Time; therefore by making this Analogy, as $3,14159^2 : 1^2 :: 193\frac{1}{2} : 19,6 = AK$, and so $AC = 39,2$ Inches, *the Length of a second Pendulum, as required.* And since the Lengths are as the Squares of the Times of Vibrations, we have 4 : 1 : : 39,2 : 9,8 Inches, the Length of an half-second Pendulum.

11. Since the Length of a Pendulum A C, for vibrating Seconds, is so considerable, the Bob, without the Cycloidal Cheeks A L and A D, will indeed describe the Arch of a Circle, as f C e, when it vibrates, but then this circular Arch will, for some Distance on each Side the lowest Point C, coincide with the Cycloid very nearly; as for Example, to g and h; and hence it follows that a common Pendulum, vibrating through very small Arches g h will perform all its Vibrations in equal Times. And hence the above-mentioned Cycloidal Checks came into Disuse.

12. Hence the Time of Descent through the Chord Cg is to the Time of Descent through the Arch of the Circle or Cycloid belonging to it as $\sqrt{4}\,a$ (supposing $AC = 2a$) to $\frac{\frac{1}{2}P}{\sqrt{a}}$, or as $2a$ to $\frac{1}{2}P$, that is, as a to $\frac{1}{4}P = \frac{3,14159}{4} = 0,7854$, or as 1 to 0,7854, *which is the Proportion of a Square to its inscribed Circle.*

13. In all that has been hitherto said, the Power of Gravity has been supposed constantly the same. But if the said Power varies, the Lengths of Pendulums must vary in the same Proportion, in order that they may vibrate in equal Times; for we have shewn that the Ratio of the Times of Vibration and Descent through half the Lengths is given, and consequently the Times of Vibration and Descent through the whole Length is given: But the Times of Vibration are supposed equal,

Centre of Oscillation E in the Rod, is also the *Centre of Percussion*, or that Point in which the Force of the Stroke is the greatest possible (XXIX).

therefore the Times of Descent thro' the Lengths of the Pendulums are equal. But Bodies descending thro' unequal Spaces in equal Times are impell'd by Powers that are as the Spaces described, that is, *the Powers of Gravity are as the Lengths of the Pendulums*. See *Annot.* XXVII. *Art.* 2 *and* 3.

Pl. VI. Fig. 6. (XXIX) 1. If A B be an uniform Rod vibrating about the Point A, the Velocities of every Point D, C, B, will be as the Arches D E, C F, B G described in the same Time; but those Arches are as A D, A C, A B; consequently *the Velocity of every Point is proportional to its Distance from the Center of Motion* A.

2. Therefore if x represent the Length of the Line, it will represent also the Sum of all the Velocities, of which $\dot{x}$ will be the Fluxion; therefore $x\dot{x}$ will be the Fluxion of the *Momenta*, and so the Fluent $\frac{x^2}{2}$ will be the Sum of all the *Momenta*, which divided by the Quantity of Matter x, will give $\frac{x}{2}$ for an Expression of all the Velocities on each Side the common Center of Gravity of all the Points, which therefore is in the middle Point of the Line A B.

3. In regard of striking an Object, the *Momentum* of each Particle will be increased in Proportion to its Velocity and Distance from the Center A, and this is call'd the *Force* of the Particle; therefore if the Fluxion of the *Momenta* $x\dot{x}$ be multiplied by the Velocity x, the Product $xx\dot{x}$ will be the Fluxion of the Forces of all the Particles: The Fluent of which $\frac{x^3}{3}$ will be the Sum of all the Forces, which divided by the Sum of all the *Momenta* $\frac{x^2}{2}$ will give $\frac{2}{3}x =$ Distance of the Center of Forces from the Point A. In this Center C, therefore, the

FROM these Properties of the Pendulum we may difcern its Ufe as an *univerfal* CHRONOMETER, or Regulator of Time, as it is ufed in Clocks and fuch-like Machines. By this Inftrument alfo we can meafure the Diftance of a Ship, by meafuring the Interval of Time between the Fire and the Sound of the Gun; alfo the Diftance of a Cloud, by numbering the Seconds or Half-Seconds between the Lightning and Thunder. Thus, fuppofe between the Lightning and Thunder we number 10 Seconds; then, becaufe Sound paffes through 1143 Feet in one Second, we have the Diftance of the Cloud equal to 11420 Feet. Again; the Height of any Room, or other Object, may be meafured by a Pendulum vibrating from the Top thereof. Thus, fuppofe a Pendulum from the Height of a Room vibrates once in three Seconds; then fay, As 1 is to the Square

the Force of all the Particles will be united, and confequently this Point, and no other, will ftrike with the Force of the whole Rod, which is therefore called *the Center of* PERCUSSION.

4. This Point is alfo the *Center of Ofcillation or Vibration*; for fince it ftrikes with the Force of all the Particles in the Line A B, it will move in the fame Manner as if all the Particles were in that Point collected together, that is, it will ofcillate or vibrate in the fame Time with a fingle Pendulum, whofe Length is equal to that Diftance, *viz.* $\frac{2}{3}$ of A B.

Square of 3, *viz.* 9, ſo is 39,2 to 352,8 Feet, the Height required. Laſtly, by the Pendulum we diſcover the different Force of Gravity on diverſe Parts of the Earth's Surface, and thence the *true Figure of the Earth* (XXX).

THE

(XXX) 1. Since it is found by Experiment, that a Pendulum under the Equator vibrating Seconds is ſhorter by $\frac{1}{10}$ of an Inch than ſuch an one in our Latitude; it follows, that the Gravity under the Equator is to the Gravity with us, as 391 to 392; as being proportional to the Lengths of the Pendulums (*Annotat.* XXIII. *Art.* 13.) Whence it is evident the Surface of the Earth is farther diſtant from the Center under the Equator than in our Latitude, and therefore is not ſpherical. What the true Figure is, and whence it proceeds, will be ſhewn hereafter.

2. To the Uſes of a Pendulum above-mentioned may be added another moſt conſiderable, viz. *That a Pendulum is an univerſal Standard of Meaſure*, becauſe the Length of any Pendulum is known by the Time of its Vibration; and therefore may be compared with the Length of a Second Pendulum as a Standard. Thus ſuppoſe a *Japoneſe* was enquiring the Length of the *Engliſh* Foot, and was told it made 100 Vibrations in the Time a Second Pendulum made 55, he would immediately know the Time of a ſingle Vibration in each muſt be inverſely as thoſe Numbers, *viz.* as 55 to 100; and then, that the Lengths of thoſe Pendulums were as the Squares of the Times, *viz.* as $\overline{100}^2 : \overline{55}^2 :: 39{,}2 : 12 =$ the Length of one Foot.

3. Hence, if the Length of a Second Pendulum were made to conſiſt of 1000 equal Parts, the Lengths of the Meaſure made uſe of by all Nations might be compared therewith, and expreſs'd in thoſe Parts; thus the *Engliſh* Foot would be found equal to 306,9 nearly; for as 39,2 : 12 :: 1000 : 306,9 *ferè*. And thus for any other. If this Method of aſcertaining the Meaſures

THE greateſt Inconvenience attending this moſt uſeful Inſtrument is, that it is conſtantly liable to an Alteration of its Length from the Effects of *Heat* and *Cold*, which very ſenſibly expand and contract all Metalline Bodies, as will be very evident by the *Pyrometer* (XXXI).

WHEN

ſures were uſed by all Nations, it would entirely preclude all Doubts and Obſcurity in this Affair to Poſterity.

(XXXI.) 1. The PYROMETER which I have contrived to ſhew the Extenſion of heated Metals, *&c.* is perhaps the moſt ſimple, exact, and eaſy, that the Nature of ſuch a Machine will admit; and the Manner of computing its Effect is as follows: Plate VI. Fig. 8.

2. AB is the Diameter of the Axis or Spindle on which the heated metalline Bar is laid, and is moved by its Extenſion; on the other End is fixed a ſmall Wheel CDE, which is connected by a ſilken Thread to a ſmall Pinion Wheel GH, which on its Extremity carries a fine Index over the Diviſions of a large graduated Circle IKL.

3. Now admitting the metalline Rod extends $\frac{1}{10000}$ Part of an Inch, it is plain the Circumference of the Axis AB, on which it lies, muſt be moved through the ſame Space; and ſuppoſing the Wheel DE to be in Diameter 10 Times as large as the ſaid Axle AB, the Motion at D will be 10 Times as great, or the Point D will move through $\frac{1}{1000}$ Part of an Inch. But juſt ſo much as D moves, will the Periphery of the Pinion GH move, they both moving with the ſame Thread; if therefore the Circle IKL be in Diameter 10 Times as large as that of the Pinion GH, the Motion of the Index (which it carries) on its Circumference will be 10 Times as great, *viz.* $\frac{1}{100}$ Part of an Inch, which will be viſible to the Eye: Conſequently every Inch on the graduated Circle will ſhew the Extenſion of ($100 \times \frac{1}{10000} =$) $\frac{1}{100}$ Part of an Inch in the Metal.

WHEN Pendulums were first applied to Clocks, they were made very short; and the

Metal. And so if the whole Circumference of the Circle I K L be 10 Inches, one Revolution of the Index will shew the Expansion of $\frac{1}{10}$ of an Inch in the Metal.

4. Hence it is easy to observe, that if the Wheel C D E, and Circle I K L, be enlarged in any Proportion to the Axes A B and G H greater than what I have assigned, the Power or Effect of the Machine will be increased in the same Degree; and hence this Instrument may be constructed to shew very great Expansions of the Metals with the utmost Degree of Accuracy.

5. The Reader may be curious to know in what Proportion the several Sorts of Metals are expansible, which I shall here shew in a Table made from the Experiments of Professor *Muschenbroek*, with 1, 2, 3, 4, 5, Flames of Spirits of Wine.

	Iron.	*Steel.*	*Copper.*	*Brass.*	*Tin.*	*Lead.*
Flame 1	80	85	89	110	153	155
2	117	123	115	220	*	274
3	142	168	193	275	*	*
4	211	270	270	361	*	*
5	230	310	310	377	*	*

6. Here we observe the Expansion of *Iron* to be the least of all Metals, and that of *Tin* and *Lead* the greatest, and nearly equal with one Flame. Hence the Rods of Pendulums ought to be made of Iron, and also all Measures of Length, as Yards, &c. and whatever else we would not have altered by Heat and Cold.

7. The Method of remedying the Inconvenience arising from the Extension and Contraction of the Metalline Rod of Pendulums, is by applying the Bob with a Screw, by which Means the Pendulum is at any Time made longer or shorter, as the Bob is screw'd downwards or upwards, and so the Time of its Vibration is continually the same.

8. If a Glass or Metalline Tube, uniform throughout, filled with Quicksilver, and the Length of 58,8 Inches,

the Arches of the Circle defcribed being large, the Times of Vibration through different Arches could not, in that Cafe, be equal; to effect which, the Pendulum was contrived to vibrate in the Arch of a *Cycloid*, the Property of which Curve is, *that a Body will defcend from any Part thereof to the loweft Point in the fame Time*, and fooner than by any other Way (XXXII.)

THE

Inches, were applied to a Clock, it would *vibrate Seconds* (for 39,2 = $\frac{2}{3}$ of 58,8) and fuch a Pendulum admits of a twofold Expanfion and Contraction, *viz.* one of the Metal and the other of the Mercury, and thefe will be at the fame Time contrary, and therefore will correct each other.

9. For by what we have fhewn, the Metal will extend in Length with Heat, and fo the Pendulum will vibrate flower on that Account. The Mercury alfo will expand with Heat, and fince by this Expanfion it muft extend the Length of the Column upward, and confequently raife the *Center of Ofcillation*, fo that by this Means its Diftance from the Point of Sufpenfion will be fhortened, and therefore the Pendulum on this Account will vibrate quicker: Wherefore if the Circumftances of the Tube and Mercury are fkilfully adjufted, the Time of the Clock might by this Means, for a long Courfe of Time, continue the fame, without any fenfible Gain or Lofs.

10. This is the Invention of Mr. *Graham* in the Year 1721, who made a Clock of this Sort, and compared it with one of the beft of the common Sort for three Years together, and found the Errors of the former but about $\frac{1}{8}$ Part of the latter; of which the Reader may fee a farther Account in *Phil. Tranf.* N° 392.

(XXXII) 1. How far the Cycloid is concern'd in the Pendulum, has been fhewn; I fhall now fhew that this

is

THE Motion of PROJECTILES comes next to be considered. A PROJECTILE is any

PlateVI. Fig. 7. is the *Curve of quickest Descent*; for let A H C be the Curve of quickest Descent, then will a Body descending from A pass from H to D sooner in an Arch of this Curve than in any other, between H and D; let the Point G be taken such, that, drawing H F, G E parallel to the Axis A B, they may cut off I G = E D, in the Perpendicular K G and L D. Since the Point D is given, L D is constant, as also I G = D E, because the Point H is invariable, and therefore also K G is constant: but the variable Quantities are H I, H G, and G E, G D.

2. Now put K G = b, L D = c, and I G = E D = d; also H I = v, and G E = z; then will the Velocity of the Body at G be as $\sqrt{KG}$, and at D as $\sqrt{LD}$. And since the Times are as the Spaces pass'd over directly, and the Velocities inversely, the Time of passing thro' H G will be as $\frac{HG}{\sqrt{KG}} = \sqrt{\frac{d^2+v^2}{b}}$; and the Time of describing G D, as $\frac{GD}{\sqrt{LD}} = \sqrt{\frac{d^2+z^2}{c}}$.

3. But the Sum of those Times is the least possible, *viz.* $\sqrt{\frac{d^2+v^2}{b}} + \sqrt{\frac{d^2+z^2}{c}}$, a *Minimum*. Consequently its Fluxion $\frac{v\dot{v}}{\frac{1}{2}\sqrt{d^2+v^2}} + \frac{z\dot{z}}{c^{\frac{1}{2}}\sqrt{d^2+z^2}} = 0$, or $\frac{v\dot{v}}{b^{\frac{1}{2}}\sqrt{d^2+v^2}} = \frac{-\dot{z}z}{c^{\frac{1}{2}}\sqrt{d^2+z^2}}$; but since H F = $v + z$ = a constant Quantity, we have $\dot{v} + \dot{z} = 0$, and so $\dot{v} = -\dot{z}$; and therefore $\frac{v}{b^{\frac{1}{2}}\sqrt{d^2+v^2}} = \frac{z}{c^{\frac{1}{2}}\sqrt{d^2+z^2}}$; therefore $v c^{\frac{1}{2}}\sqrt{d^2+z^2} = z b^{\frac{1}{2}}\sqrt{d^2+v^2}$, whence

any Body A, thrown or projected in an *upright*, *oblique*, or *horizontal* Direction; as a *Stone* from the Hand, an *Arrow* from the Bow, or a *Ball* or *Bomb* from a Cannon or Piece of Ordnance. The Force with which the Body is projected is called the IMPETUS, and the Distance to which it is thrown is called the HORIZONTAL RANDOM or AMPLITUDE of the Projection.

EVERY Projectile is acted upon by two Forces or Powers, *viz.* the *Impetus* of the *projectile*

$\sqrt{d^2+v^2} : \sqrt{d^2+z^2} \mid v c^{\frac{1}{2}} : z b^{\frac{1}{2}}$; that is, GD : GH :: HI $\times \overline{LD}^{\frac{1}{2}}$: GE $\times \overline{KG}^{\frac{1}{2}}$.

4. But GD, HG are the Fluxions of the Curve; HI, GE, the Fluxions of the *Abscissæ* AM, AK; and KG, LD are the Ordinates to the Points G and D. *Therefore the Fluxions of the Curve of swiftest Descent are every where as the Fluxions of the* Absciffæ *directly, and the Square Roots of the Ordinates inversely.*

5. But this is the Property of the Cycloid; for putting AK $= x$, KG $=$ BN $= y$, and BC $= a$; the Triangles ONC and GED are similar by the Nature of the Curve; and therefore ON : OC :: GE : GD; that is, $\overline{ay-yy}^{\frac{1}{2}} : \overline{aa-yy}^{\frac{1}{2}} :: \dot{x} : \dfrac{\dot{x}\times\overline{a^2-y^2}^{\frac{1}{2}}}{\overline{ay-yy}^{\frac{1}{2}}} =$

$\dfrac{\dot{x}\times a^{\frac{1}{2}}\times\overline{a-y}^{\frac{1}{2}}}{y^{\frac{1}{2}}\times\overline{a-y}^{\frac{1}{2}}} = \dfrac{\dot{x}\times a^{\frac{1}{2}}}{y^{\frac{1}{2}}} =$ GD; but $a^{\frac{1}{2}}$ is a given Quantity, therefore GD is every where a $\dfrac{\dot{x}}{y^{\frac{1}{2}}}$, or directly as the Fluxion of the Abfcifs AK, and inverfely as the Square Root of the Ordinate RG, and confequently *the Cycloid is the Curve of swiftest Descent.* See *Annotat.* XXVIII. 3.

projectile Force, and that of *Gravity*. By the first, the Body passes over *equal Spaces*, AB, BC, CD, *&c. in equal Times*; and by the second, it descends through Spaces AG, AH, AI, *&c.* which are as the *Squares of the Times*; and therefore by these two Forces compounded, the Body will describe, not a *Right Line*, but a *Curve* AQ; and of that Sort which we call a *Parabola*; and this will be the Case in all Directions but that in the Perpendicular, when the Path of the Projectile will be (to Appearance) a Right Line. The greater the Angle of Elevation KAM of the Cannon is, the greater will be the Height DB to which the projected Body will rise. Also, the greater will be the Distance or Amplitude of the Projection, till the said Angle becomes equal to 45 Degrees KAO; upon which Elevation the Random AC will be the greatest possible, and equal to twice the Altitude AG of the perpendicular Projection. On any Elevation AM or A*M*, equally above or below 45 Degrees, as on 40 and 50, 30 and 60, 20 and 70 Degrees, the Random AC will be the same; which Case an Engineer frequently finds of very great Use.

Plate V. Fig. 7.

If the Object be situated above the Horizon, then, in order to strike it with the least *Impetus*, let a Piece of Looking-glass be

be fixed to the Cannon perpendicular to its Axis; and holding a Plumb-Line over the Glaſs directly under the Eye, the Cannon is to be elevated till the Object appears exactly under the Plummet, and there fixed; if then it be diſcharged, it will ſtrike the Object as required.

FROM what has been ſaid, we may eaſily underſtand how a Body projected upright from the Earth's Surface does really deſcribe a Parabola, though to Appearance it aſcends and deſcends in a Right Line. For it is urged by two Forces, *viz.* the Projectile upwards, and the Force ariſing from the Motion of the Earth about its Axis from *Weſt* to *Eaſt*; in which Caſe it muſt neceſſarily deſcribe a Parabola (XXXIII).

I SHALL

(XXXIII) 1. In the *Parabola*, it is demonſtrated that the ſeveral *Abſciſſæ* A G, A H, A I, &c. are as the Squares of the Ordinates G M, H N, I O, &c. and ſince this is the Property of the Curve which the Projectile deſcribes, it is evident the Projectile deſcribes a *Parabola*. Plate V. Fig. 5,6.

2. Let A M repreſent the Force with which a Ball is projected from the Cannon in the Direction A M; this may be reſolved into two others A L and L M, of which the firſt is perpendicular, and the latter parallel to the Horizon. The perpendicular Force or Velocity A L is that by which the Ball riſes, and the Heights to which it will riſe with the Velocities A L and A M are as the Squares of the Velocities, *viz.* as $\overline{AL}^2$ to $\overline{AM}^2$; but (becauſe $AL : AM :: AM : AG$) $\overline{AL}^2 : \overline{AM}^2 :: AL : AG$; therefore ſince A G repreſents the Fig. 7.

 Height

I SHALL in the last Place consider the Nature of CIRCULAR MOTION and CENTRAL

Height to which the Ball will rise with the Velocity A M, A L will be the Height to which it will rise with the Velocity A L, or in the Projection on the Elevation A M; thus B D = A L.

Fig. 6, 7. 3. The Velocity of the Projection L M in the horizontal Direction is every where uniform, or carries the Ball through equal Spaces A R, R S, S T, T V, V W, in equal Times. For by the single *Impetus* of the Powder, the Ball describes equal Spaces A B, B C, C D, *&c.* in equal Times, in the Direction A F, as has been shewn; therefore by the Similarity of Triangles A R, R S, S T, *&c.* will be as A B, B C, C D, *&c.* that is, *the horizontal Spaces passed over in the Projection, will, in equal Times, be equal.*

Fig. 7. 4. Now since the perpendicular Velocity is to the horizontal Velocity as A L to L M, and A L is the Space pass'd through by the former in half the Time of the Projection, L M would be the Space pass'd through in the horizontal Direction in the same Time, if the Velocity L M were of the same Sort with A L, viz. *uniformly retarded and accelerated*; but since L M is not retarded, but uniform, it will carry the Body through twice the Space L M in the same Time (*Annot.* XXVI. 7.) that is, through A D = L B = 2 L M.

5. And since the Time of the Ascent from A to B is equal to the Time of Descent, the Ball will pass over D C = A D in the Time of the Descent from B to C; therefore the horizontal Space A C, which the Ball passes over during the whole Time of the Projection, is equal to 4 Times L M, wherever the Point M be taken in the Circumference A O G.

6. Hence when A O is the Direction of the Cannon, (O being the middle Point of the Semicircle) the Line N O, which is the horizontal Velocity, will be the greatest possible (from the Nature of the Circle) and consequently 4 N O = A Q, *the greatest possible horizontal Random*; which therefore is made on an Angle

O A K

TRAL FORCES. If a Body A be suspended at the End of a String A C, moveable about Plate V. Fig. 8.

O A K = 45 Degrees. For O A K = A O N = O A N, because N O = A N; therefore since A N O = 90°, A O N = O A N = O A K = 45°.

7. Hence A N = F E (= N O = $\frac{1}{4}$ A Q) will be the Height of the Projection on the Angle O A K = 45°. *Whence* A Q *will be the Parameter, or* Latus Rectum, *of the Parabola* A F Q.

8. If M O = O R, then will L M = T R; and consequently the Horizontal Random on the Elevation A M, and A R, will be equal, *viz.* A C; but the Height of the Projection on A R will be A T = S D. *Whence the Height* S D *is to the Height* D B D *(as* R K *to* M K) *as the Tangent of the Angle of Elevation* R A K *to the Tangent of the Angle* M A K.

9. From what has been demonstrated it is easy to understand, that an Object situated on the Horizon may be hit by a Ball discharged on an Elevation of 45° with a less Charge of Powder than on any other Angle of Elevation.

10. The Time of the Projection A B C is equal to the Time of the perpendicular Ascent and Descent through A L or D B, (because the horizontal Velocity does no Way affect that in the Perpendicular.) But the Time of Ascent and Descent in A L is to that in A G, as $\sqrt{AL}$ to $\sqrt{AG}$; that is, (since A L : A G :: $\overline{AM}^2$: $\overline{AG}^2$) as A M to A G; that is, (because A G : A M :: A M : A L = M K) as M K to A M, or *as the Sine of the Angle of Elevation* M A K *to the Radius.*

11. Therefore the Times of two different Projections A B C, A S C, will be as the Chords A M and A R, or as the Sines of the Angles of Elevation to the Radius A G.

12. Suppose it required to know from what Height a Body ought to fall to acquire such a Velocity, that, being reflected in that Moment in the Direction A F, it should describe the given Parabola A O Q. In order to this we are to consider that (the Equation being $xp=yy$)

about a Point or Pin C as a Centre, and in that Position it receive an Impulse or Blow

the Velocity of a Ball shot from the Cannon commences in the Direction of the Axis A L (or Abscis x) in the Point A; but that in the Direction A F (or of the Ordinate y) is from first to last the same; consequently there must be a certain Point, to which the Ball having descended, acquires a Velocity equal to that in the Direction A F, which is uniform; and this must be when $\dot{x} = \dot{y}$. To determine which, putting the Equation $xp = yy$ in Fluxions, we have $\dot{x}p = 2y\dot{y}$; but since in this Case we have $\dot{x} = \dot{y}$, therefore $p = 2y$, and $\frac{p}{2} = y = \sqrt{xp}$, (*per* Equation;) therefore $\frac{1}{4}p^2 = xp$, that is, $\frac{1}{4}p = x$.

13. Consequently, if an heavy Body fall from the Height of $\frac{1}{4}$ of the Parameter of the given Point A, and in that Point it be reflected (with the Velocity there acquired) in the Direction A F, it will describe the given Parabola A O Q.

Plate VI. Fig. 9. 14. If it be required to strike the Point B, whose Distance A D and Height B D are given, with the least *Impetus* of all that will hit it, then in order to find the Elevation of the Cannon, draw A B, and E A perpendicular to A C; on the Point A describe the Arch abc, which bisect in b, then a Line A F drawn through that Point will be the Elevation of the Cannon as required; and the least *Impetus* will be equal to $\frac{AB + BD}{2}$. See the Demonstration of this by the Inventor, Dr. *Halley*, in the *Phil. Transactions*, and by Dr. *Keill*, *Introd. Pag.* 280, 281. From a bare View of the Diagram, the Reason of the Use of the *Looking-Glass* and *Plummet* is evident.

Fig. 10. 15. If a Ball be projected perpendicularly upright in the Direction A F with a Velocity that will carry it to the Height of one Mile, or 5280 Feet; it will be 18 Seconds (nearly) in the Ascent, and as much in the Descent, (for, as 16,2 Feet : 5280 Feet :: $1^2 : x^2$; whence

Blow in an horizontal Direction, it will be thereby compell'd to describe a Circle about the central Pin: While the circular Motion continues, the Body will have a continual Endeavour to recede or fly off from the Centre, which is call'd its *Centrifugal Force*, and arises from the *Horizontal Impetus*; with this Force it acts upon the fix'd Centre Pin, and that, by its Renitency or Immobility, re-acts with an equal Force on the Body by means of the String, and solicits it towards the Centre of Motion,

whence $x = 18''$, *ferè* the Time of Ascent or Descent.) Now in the Parallel of *London*, the Motion of the Earth's Surface, and of all Bodies upon it, is at the Rate of 10,8 Miles *per* Minute, or 950 Feet *per* Second, from West to East: Therefore in 36 Seconds, the Time of the Ascent and Descent, the Ball will be carried over 34214 Feet, or more than six Miles, in the horizontal Direction Eastwards from the Point A to H. Now since the Ball is carried at the same Time with a retarded Force upwards, and an uniform Force forwards, it will describe the Parabola ADH. And because the horizontal Motion in the *Spectator* and in the *Project* is the same, they will be carried through equal Spaces AN, OB; AM, PC; AL, FD, *&c.* in the same Time; and consequently the projected Ball will always appear perpendicularly over the Spectator in every Point of the Curve; which is the Occasion of the Deception abovementioned.

16. Hence the Objection usually urged against the Motion of the Earth, from the apparent Ascent and Descent of Projects in a Right Line, is the Result of Ignorance; as indeed are all other Arguments of this Kind.

tion; whence it is call'd the *Centripetal Force*; and when we ſpeak of either, or both of them indefinitely, they are call'd the *Central Forces* of the revolving Body.

IT is of the laſt Importance to underſtand the Nature of this Kind of Motion, ſince by it all the Machinery of the Planetary Syſtem is perform'd, as will be eaſy to underſtand, if, for the revolving Body, we ſubſtitute a Planet; for the Centre, we place the Sun; for the *Centripetal Force*, or String, its Power of Attraction; and for the *Projectile Force*, the Almighty Power of God in the firſt Creation of Things.

Plate V. Fig. 8. THE Theory of this Species of Motion is compriſed in the following Propoſitions, *viz.* (1.) The projectile Force A H is infinitely greater than the centrifugal Force H E. (2.) The central Force is proportional to the Quantity of Matter in the revolving Body A, all other Things being equal. (3.) If two equal Bodies A, B, deſcribe unequal Circles A M, A N, in equal Times, the central Forces will be as the Diſtances A C, B C, from the Centre C. (4.) If equal Bodies deſcribe unequal Circles with equal Celerities, the central Forces will be inverſely as the Diſtances. (5.) If equal Bodies deſcribe equal Circles, the

the central Forces will be as the Squares of the Celerities. (6.) If equal Bodies describing unequal Circles have their central Forces equal, their Periodical Times will be as the Square Roots of the Distances. (7.) If equal Bodies describe unequal Circles with equal Celerities, the Periodical Times will be as the Distances directly. (8.) Therefore the Squares of the Periodical Times are proportional to the Cubes of the Distances, when neither the Periodical Times nor the Celerities are given. And in that Case, (9.) The central Forces are as Squares of the Distances inversely.

THESE are the Theorems of circular Motions, the two last of which are found by Astronomers to be strictly observed by every Body of the Planetary and Cometary System. For Example: The Periodical Time of *Venus* is 225 Days, and that of the *Earth* 365; the Squares of which Numbers are 50625 and 133225: Again, the Distance of *Venus* from the *Sun* is to that of the *Earth* as 72 to 100; the Cubes of which Numbers are 373248 and 1000000; but 50625 : 133225 :: 373248 : 1000000; that is, the Squares of the Periodical Times are as the Cubes of their Distances very nearly. From hence also it will easily ap-

pear, that Bodies under the Equator have the greatest centrifugal Force, which there acts in direct Opposition to Gravity, and diminishes towards the Poles, with the Squares of the Distances from the Earth's Axis. *Hence also it is evident, that if ever the Earth was in a fluid State, and at the same Time moving about its Axis, it must necessarily put on the Figure, not of a perfect Sphere or Globe, but of an oblate Spheroid, flatted towards both Poles; as is manifestly shewn by Experiment* (XXXIV).

Plate V. Fig. 8. (XXXIV) 1. Let $AD = a$, the Diameter of the Circle AMD; AH the Space which a Body A moves through in a constant Particle of Time by an uniform *Projectile Force*; HE the Distance it is drawn from the Tangent AH by the *Centripetal Force* (which is just sufficient to retain the Body moving in a circular Orbit) in the same Time.

2. Then since AH is indefinitely small, the Arch AE, and its Chord, will nearly coincide and be equal to AH, and will therefore also represent the Projectile Force. But $AD : AE :: AE : Aa = HE = \frac{AE^2}{AD}$ = the Central Force. Now AE is infinitely small in respect of AD, therefore HE or Aa is infinitely small with respect to AE.

3. If the Projectile Force or Velocity $AE = V$, and $AD = a$, the general Expression of the Central Force will be $\frac{V^2}{a}$; which therefore in different Circles will be as *the Squares of the Velocities directly, and inversely as the Diameters or Semi-diameters of the Circles.*

4. But in the same Circle it will *be directly as the Squares of the Velocity*; because in that Case a is given. But

But if the Velocity be given, *it will be as the Distance from the Centre inversely*, (for then $\frac{V^2}{a}$ becomes $\frac{1}{a}$.)

5. Hence if the Central Forces in two Circles be equal, the *Diameters of those Circles, or Distances from the Centre of Force, will be as the Squares of the Velocities directly.*

6. As AE : 1 : : 3,1416 a : P = the *Periodical Time* of describing the whole Circle (for in equable Motions the Spaces are as the Times.) Therefore $P = \frac{3,1416\,a}{V}$,

whence $V^2 = \frac{\overline{3,1416}^2 \times a^2}{P^2}$, which substituted in the Expression for the Central Force, we have $\frac{\overline{3,1416}^2 \times a}{P^2}$

$= \frac{a}{P^2}$ (because $\overline{3,1416}^2$ is given) for the Central Force in this Case; which therefore *is always as the Diameter or Radius of the Circle directly, and the Square of the Periodical Time inversely.*

7. Hence if the Central Force be as any Power (n) of the Distance (a) from the Center C, we have $\frac{a}{P^2}$ $= a^n$; whence $P = \frac{1-n}{a\,2}$; and therefore when $n=0$, we have $P = a^{\frac{1}{2}}$, or *the Periodical Time will be as the Square Root of the Distance.*

8. If $n = 1$, that is, if the Force be directly as the Distance; then will $P = a\frac{0}{2} = a^0 = 1$; that is, *the Periodical Time will be given, or the same in every Circle.*

9. If $n = 2$, that is, if the Force be directly as the Square of the Distance, then $P = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$; or *the Periodical Time will be as the Square Root of the Distance inversely.*

10. If $n = -1$, that is, if the Force be inversely as

as the Distance, then $P = a^{\frac{2}{2}} = a$, or *the Periodical Time will be directly as the Distance from the Centre.*

11. If $n = -2$, or the Force be inversely as the Square of the Distance, then $P = a^{\frac{3}{2}}$, or (squaring both Sides) $P^2 = a^3$, that is, *The Square of the Periodical Time will be as the Cube of the Distance from the Centre.*

12. When the Central Force is given, $\frac{a}{P^2} = 1$, and $P = a^{\frac{1}{2}}$, that is, *The Periodical Time will be as the Square Root of the Distances.*

13. Since the Central Force is as $\frac{V^2}{a}$, and as $\frac{a}{P^2}$, we have $PV = a$; *whence the Distance from the Centre is always in the Compound Ratio of the Periodical Time and Velocity.*

14. Hence if the Distance be given, or $a = 1$, then the *Periodical Time is inversely as the Velocity, and the Velocity inversely as the Periodical Time.*

15. If the Velocity be given, or $V = 1$, the *Periodical Time will be directly as the Distance from the Centre.*

16. If the Periodical Time be given, the *Velocity will be directly as the Distance likewise.*

17. If we put $p = 3{,}1416$; then $pa : P :: AE = \frac{AE \times P}{pa}$ = Time of describing AE. Again, as $1^2 : 16{,}2 :: \frac{\overline{AE}^2 \times P^2}{p^2 \times a^2} : \frac{16{,}2\,\overline{AE}^2 \times P^2}{p^2\, a^2}$ = the Distance descended by an heavy Body (in the Time of describing AE) by the Force of Gravity. Lastly, this Force of Gravity $\left(\frac{16{,}2\,\overline{AE}^2 \times P^2}{p^2\, a^2}\right)$ is to the Central Force $\left(\frac{\overline{AE}^2}{a}\right)$ as 1 to $\frac{a p^2}{16{,}2 P^2}$, that is, as $16{,}2 P^2$ to $a p^2$, or as P^2 to $0{,}615\, a$.

18. If now we suppose a Body revolving about the Centre of the Earth at the Distance of its Surface with a Centrifugal Force equal to that of Gravity; then $P^2 = 0{,}615\, a$, and so $P^2 = \sqrt{0{,}615\, a}$; and putting $a =$

42000000

42000000 Feet in the Diameter of the Earth, we have $P = 5083$ Seconds, or $84' \; 43''$; which is the Time of Revolution to acquire a Centrifugal Force equal to that of Gravity at the Earth's Surface. Conſequently were the Earth to revolve in $84' \; 43''$ inſtead of 24 Hours, the Bodies on its Surface would loſe all their Weight, and be as liable to fall off as to abide thereon.

19. From what has been demonſtrated we may diſcover the Law of Gravitation at the Moon; for ſince the Diſtance of the Moon from the Earth's Centre is to the Diſtance of the Earth's Surface as 60 to 1; and ſince the periodical Time at the Earth's Surface is $5083''$, therefore the Periodical Time at the Moon may be found (by *Art.* 11.) thus; $1^3 : 60^3 :: \overline{5083}^2 : P^2$; whence $P = 5083 \times 60^{\frac{3}{2}} = 2362000'' = 27,3$ Days, the Periodical Time of the Moon, if moved by a Centripetal Power, as *Gravity decreaſing with the Squares of the Diſtances inverſely:* And therefore ſince the *Periodical Time, thus found, is equal very nearly to the real Periodical Time of the Moon*, it ſhews that Law takes Place between the Earth and Moon. This might alſo be ſhewn, from the Time of the Moon's Revolution, but *one Proof* is enough. I ſhall alſo hereafter ſhew, that this is the general Law of the whole Planetary Syſtem.

20. Since we had $\frac{a}{P^2} =$ the Central Force, when a is given, the Force will be as $\frac{1}{P^2}$; that is, in equal Circles the Centrifugal Forces will be as the Squares of the Velocities inverſely. And therefore ſince in the Time of the Earth's Revolution, *viz.* 24 Hours, there are $68400''$; if we ſay, as $\overline{5083}^2 : \overline{68400}^2 :: 1 : 289$ nearly; that is, *the Centrifugal Force under the Equator, ariſing from the Earth's Rotation, is to the Power of Gravity on its Surface as* 1 *to* 289 *nearly.*

21. Since the Time of Revolution of a Body under the Equator EQ, and in any parallel of Latitude BG, is equal, the Centrifugal Forces are as the Diſtances EC, BA, from the Centre of Motion (by *Art.* 16.) or, as Radius CB to the Co-ſine AB of the Latitude. Plate VI. Fig. 11.

22. But

22. But in any Latitude B, the Centrifugal Force is not (as under the Equator) opposite to the whole Gravity, but only a Part thereof, which is to the Whole as the Co-sine of Latitude to the Radius. For continue out AB (the Direction of the Centrifugal Force) to D; and CB (in which Gravity acts) to F; from D let fall a Perpendicular to F, then will BD represent the whole Centrifugal Force, and BF that Part of it which acts directly against Gravity; but BF : BD :: AB : BC = EC. Therefore on both these Accounts, the Centrifugal Force decreases from the Equator towards the Poles N and S in the Proportion of the Squares of the Co-sines of the Latitude.

Plate VI. Fig. 12. 23. We proceed now to demonstrate *the true Figure of the Earth*, which we shall find to be *Spheroidical* and not *Spherical*, if ever its Parts were in a Fluid or yielding State, from the Consequence of a Centrifugal Force. In order to this, let N S be the Axis of Rotation, BC a Column of fluid Particles gravitating towards the Centre C, which (because we suppose the Parts of the Fluid every where quiescent) will be of the same Weight with every other Column of Particles CN, or CE. Let $CE = a$, $CN = b$; $CB = x$, $AB = s$ Sine of the Angle BCN; and supposing the gravitating Power to be every where as the Power n of the Distance from the Centre; that is, suppose the Power of Gravity at E (g) to be to that at B as CE^n to CB^n, then will the Gravity at B be $= \frac{g \times CB^n}{CE^n} = \frac{g \times x^n}{a^n}$.

24. Also the Centrifugal Force at E (f) is to that at B as CE to AB (as we shew'd *Art.* 16.) But $AB : CB :: s : 1 =$ Radius; whence $AB = s \times CB = sx$; consequently we have the Centrifugal Force at B equal to $\frac{fsx}{a}$.

25. But since this Force acts in the Direction BD, that Part which opposes Gravity is FB (as was shewn above) whence since $BD : FB :: 1 : s :: \frac{fsx}{a} : \frac{fssx}{a}$ = to Centrifugal Force at B, which opposes Gravity.

26. Hence the Power of Gravity on a Particle at B will

will be as $\frac{g\times x^n}{a^n} = \frac{fs^2x}{a}$ impelling it towards the Center C. Such a Particle will be represented by $\dot{x}$, and therefore its Weight will be $\frac{g\times x^n\dot{x}}{a^n} - \frac{fssx\dot{x}}{a}$; and as this is the Fluxion of the whole Weight of the Column C B, the Fluent thereof $\frac{g\times x^{n+1}}{\overline{n+1}\,a^n} - \frac{fssx^2}{2a}$ will be the Weight of the said Column of Particles C B.

27. If now we consider the Column of Particles C E, which is of equal Weight, since in this Case the Angle N C E is a Right one, we shall have $x = a$, and $s = 1$; wherefore the Weight of the Column C E will be $\frac{g\times a^{n+1}}{\overline{n+1}\,a^n} - \frac{faa}{2a}$; therefore the Weight of a Column every where is equal to $\frac{2g-nf-f}{2\,\overline{n+1}}\times a$ $\left(= \frac{g\times x^{n+1}}{\overline{n+1}\,a^n} - \frac{fssx^2}{2a}\right)$ therefore we have (by proper Reduction) $2gx^{n+1} - \overline{n+1}\times fssa^{n-1}x^2 = \overline{2g-nf-f}\times a^{n+1}$ for a general Equation.

28. Whence, putting $s = o$, we have $2gx^{n+1} = \overline{2g-nf-f}\times a^{n+1}$; and therefore $a : x :: 2g^{\frac{1}{n+1}} : \overline{2g-nf-f}^{\frac{1}{n+1}}$ :: C E : C N, *so is the Diameter of the Equator to the Axis of the Earth.*

29. Hence, upon the Hypothesis of an uniform Gravity, we have $n = o$; in that Case $2g : 2g - f$: C E : C N. But the Gravity (g) is to the Centrifugal Force (f) under the Equator, as 289 to 1 (*per Art.* 20.) Wherefore in this Case C E : C N :: 578 : 577.

30. If the Earth were to revolve in 84′ 43″, the Centrifugal Force would be equal to Gravity (*Art.* 18.) or $g = f$; and then C E : C N :: 2 : 1. If the Earth were to revolve quicker than that, the Particles would fly off, and the Earth be reduced to a single Atom.

31. If the Gravity be supposed proportional to the Distance from the Center; then $n = 1$, and we have

C E:

CE : CN :: $g^{\frac{1}{2}}$: $\overline{g-f}^{\frac{1}{2}}$ and if in this Hypothesis the Earth were to revolve in 84′ 43″, or $g=f$, then would C E be infinitely greater than C N; that is, C N would be nothing, or the Spheroid would then become a circular Plane.

32. If Gravity be supposed inversely as the Square of the Distance, we have $n=-2$; and then we shall have $2g^{\frac{1}{-n+1}} : \overline{2g+nf-f}^{\frac{1}{-n+1}}$ :: CE : CN :: $2g^{\frac{1}{-1}} : \overline{2g+f}^{\frac{1}{-1}} :: 2g+f : 2g :: 579 : 578$. And since of all the above Hypotheses, this last is found to be the only true one (see *Art.* 19.) it follows *that the Earth in its Chaotic or Fluid State, revolving about its Axis, must necessarily put on a Spheroidical Figure*, having the Equatorial Diameter E Q, to the Axis N S, as 579 : 578.

33. And this would be the Case, were all the Circumstances of the Hypothesis the same in Nature as we have presumed in the Theory; but since they are found to be otherwise, the *Mathematical Theory* (which gives only the true Sort of Figure, but not the true Figure itself) must be adapted to Nature, and the Figure which the Earth really has investigated from other Principles. For Bodies on the Earth's Surface do gravitate in such Directions as pass not thro' the Centre C (as has been supposed) any where but in the Equator or under the Poles, *viz.* at E and N.

34. In order to find the true Proportion between E Q and N S, Sir *Isaac Newton* makes a Supposition they are as 101 to 100, and then by a Method (too prolix to be here explained) he finds the Gravity at E and N to be as 500 to 501. Now supposing the Matter of the Earth uniform and at Rest, the Weight of the Column of Particles E C will be to that of the Column N C in the Ratio of their Magnitudes 101 to 100, and of their specific Gravities 500 to 501 conjointly, (See *Annot.* LVIII. 10.) that is, as 505 to 501.

35. Now it is plain if the Weight of each Particle in the Column C E were divided into 505 equal Parts, a Centrifugal Force, that should take off four of those equal

equal Parts, would leave the Weight of the Column EC equal to that of the Column NC, whence an Equilibrium must ensue between them.

36. Whence our Author makes this Analogy: If a Centrifugal Force which is $\frac{4}{505}$ of Gravity causes an Excess of $\frac{1}{100}$ Part in the Altitude EC, a Centrifugal Force which is but $\frac{1}{289}$ Part of Gravity can cause an Excess only of $\frac{1}{229}$ Part of E C. Therefore the true Proportion between the Diameter of the Equator E Q, and the Earth's Axis N S, is that of 230 to 229. Whence, since the former is about 8000 Miles, it will exceed the latter by about $34\frac{3}{10}$ *English* Measure.

37. Whence, since in an Equilibrium of Fluids communicating with each other, the Altitudes are as the specific Gravities inversely, it follows that the Gravity at the Equator E is to that at the Pole N as 229 to 230; or a Body which under the Poles weighs 230 *lb.* will under the Equator weigh but 229 *lb.*

38. The Gravity increases from the Equator E towards the Pole N *with the Squares of the Sines of the Latitudes.* For continue out C F to G, and draw G D perpendicular to A D; then if B D be the whole Centrifugal Force at the Equator, and B G the whole Diminution of Gravity occasioned there by it; and since in the Latitude B the Centrifugal Force which acts directly against Gravity is reduced to F B, the Line F B will also represent the Diminution of Gravity in the Latitude B, and therefore the Difference G F will be the Increase of Gravity at B. And so (because B G : G G : : G D : G F) we have B G : G F : : ($\overline{GB}^2$: $\overline{GD}^2$: :) $\overline{BC}^2$: $\overline{AC}^2$: : *Radius Square* : *Square of the Sine of the Latitude* : : *Decrease at the Equator* : *the Increase at* B.

39. If we put the Diminution of Gravity at the Equator = 10000, then will the Gravities at the Equator, *London*, and the Pole, be as the Numbers 2290000, 2296124, 2300000; therefore the Lengths of Pendulums vibrating Seconds in each of those Places will be in the same Proportion (by *Annot.* XXVIII. 13.) But 2296124 : 2290000 : : 39,2 : 39,1 = Inches of a Second Pendulum under the Equator, the same nearly as found by Observation.

 40. Since

40. Since in the elliptic Meridian ENQS, the Curvature at E, Q, is much greater than that at N, S; the Radius of Curvature (ſuppoſe E*b*) in the former will be ſhorter than that in the latter (ſuppoſe S*d*;) and therefore if at the Equator the Angle E*ba*, or the Arch E*a*, be equal to *one Degree*, the Angle of one Degree S*dc* at the Pole S will contain a greater Arch *c*S, as is evident from the Diagram. Whence it appears that in this ſpheroidical Figure of the Earth, the Degrees increaſe from the Equator to the Pole; ſo that if in the Equator a Degree conſiſts of 60 Miles, in the ſeveral Latitudes the Miles will be as below:

Lat.	0	10°.	20°.	30°.	40°.	50°.	60°.
Miles in a Deg.	60.	59,5.	59,57.	59,67.	59,8.	59,93.	60,06.
Lat.	70°.	80°.	90°.				
Miles in a Deg.	60,16.	60,235.	60,26.				

SCHOLIUM.

What has been ſaid hitherto, has been upon Suppoſition that the Earth is of an uniform Denſity throughout, but it is reaſonable to ſuppoſe (from the Earth's having been originally in a fluid State) that the heavieſt Matter ſubſided firſt, according to the Laws of Gravity; and therefore that the Earth is more denſe and compact the nearer we go to the Centre, whence it muſt follow, that as there is a greater Number of Particles, there will be, on the Whole, a greater Quantity of Centrifugal Force in the Column E C, and conſequently a greater Exceſs in its Altitude above that of NC, than was before ſtated; that is, E C will exceed NC more than in the Proportion of 230 to 229, which is alſo found to be ſtill more conſonant to Experiments. See farther on this Head my *New Principles* of GEOGRAPHY and NAVIGATION.

LECTURE III.

Of the CENTRES *of* MAGNITUDE, MOTION, *and* GRAVITY. *The* METHOD *of finding the* CENTRE *of* GRAVITY *in all* BODIES. *The* PHÆNOMENON *of the* DOUBLE CONE *and* CYLINDER. *Of the* COMMON CENTRE *of* GRAVITY *between two or more* BODIES; *considered between the* EARTH *and* MOON, *the* SUN *and* PLANETS. *The* FUNDAMENTAL PRINCIPLE *of* MECHANICS *demonstrated. The Impossibility of a* PERPETUAL MOTION. *The* NATURE *and* KINDS *of* LEVERS, BALANCES, PULLIES, *the* AXIS *in* PERITROCHIO, INCLINED PLANE, WEDGE, *and the* SCREW. *Of* FRICTION *and* FRICTION-WHEELS. *Of* COMPOUND MACHINES. *The* THEORY *of* CLOCK-WORK. *A new* ORRERY, COMETARIUM, *&c. Of the* MOVEMENT *of* WIND-MILLS, WATER-MILLS, SHIPS, *&c. Of the* MAXIMA *and* MINIMA *in* MECHANICS. *The* THEORY *of* WHEEL-CARRIAGES *at large.*

HAVING considered the Nature of every Kind of Motion, with the Properties of each, we should

have come immediately to treat of the MECHANICAL POWERS OR MACHINES, but that ſomething ſtill remains to be premiſed thereto, relating to the *Centres of Magnitude*, of *Motion*, and of *Gravity* in Bodies.

THE CENTRE *of* MAGNITUDE is that Point which is equally diſtant from all the external Parts of the Body: And in Bodies that are uniform and homogeneal, it is the ſame with

THE CENTRE *of* MOTION, which is that Point which remains at Reſt, while all the other Parts of the Body move about it: And this is again the ſame in uniform Bodies, of the ſame Matter throughout, as

THE CENTRE *of* GRAVITY, which is that Point about which all the Parts of a Body do in any Situation exactly balance each other.

THIS Centre of Gravity in Bodies is of the greateſt Conſequence to be well underſtood, as being the ſole *Principle of all Mechanical Motions.* The particular Properties hereof are as follow. (1.) If a Body be ſuſpended by this Point, as the Centre of Motion, it will remain at Reſt in any Poſition indifferently. (2.) If a Body be ſuſpended in any other Point, it can reſt only in two Poſitions, *viz.* when the ſaid Centre of Gravity is exactly above or below

low the Point of Suspension. (3.) When the Centre of Gravity is supported, the whole Body is kept from falling. (4.) Because this Point has a constant Endeavour to descend to the Centre of the Earth; therefore, (5.) When this Point is at Liberty to descend, the whole Body must also descend or fall, either by sliding, rolling, or tumbling down. (6.) The Centre of Gravity in regular, uniform, and homogeneal Bodies, as *Squares*, *Circles*, *Spheres*, &c. is the middle Point in a Line connecting any two opposite Points or Angles. (7.) In a Triangle it is in a Line drawn from any Angle bisecting the opposite Side, one Third of the Length distant from that Side or Base. (8.) It is also *one Third* of the Side distant from the Base of an hollow Cone. (9.) But in a solid Cone it is *one Fourth* of the Side distant from the Base. (10.) In the Human Body, the Centre of Gravity is situated in that Part which is call'd the *Pelvis*, or in the Middle between the Hips.

HENCE the Solution of several very curious *Phænomena* will be evident with the least Attention; as why some Bodies stand more firmly on their Bases than others; why some stand more firmly in an inclined Position; why some Bodies fall in one

manner, ſome in another; why ſome may ſeem to riſe, while the Centre of Gravity really deſcends, as the *rolling* CONE, and CYLINDER. Hence the Form of that particular Bucket which deſcends empty with the Mouth downwards, but is drawn up full with the Mouth upwards. Hence alſo it appears that a Waggon loaded with heavy Matter, as Iron, Stone, &c. will go ſafely on the Side of a Hill or riſing Ground, where a Load of Hay or Corn would be overturn'd. Again, we hence ſee the Reaſon why no Man, ſtanding ſtill, can move or ſtir, without firſt moving the Centre of Gravity out of its Place; alſo, why we ſtand firmly, while the Centre of Gravity falls between, or on the Baſe Line of the Feet; and how neceſſarily we fall, when the Centre of Gravity falls on one Side or other of the ſame: With many other Particulars, which naturally reſult from this Principle (XXXV).

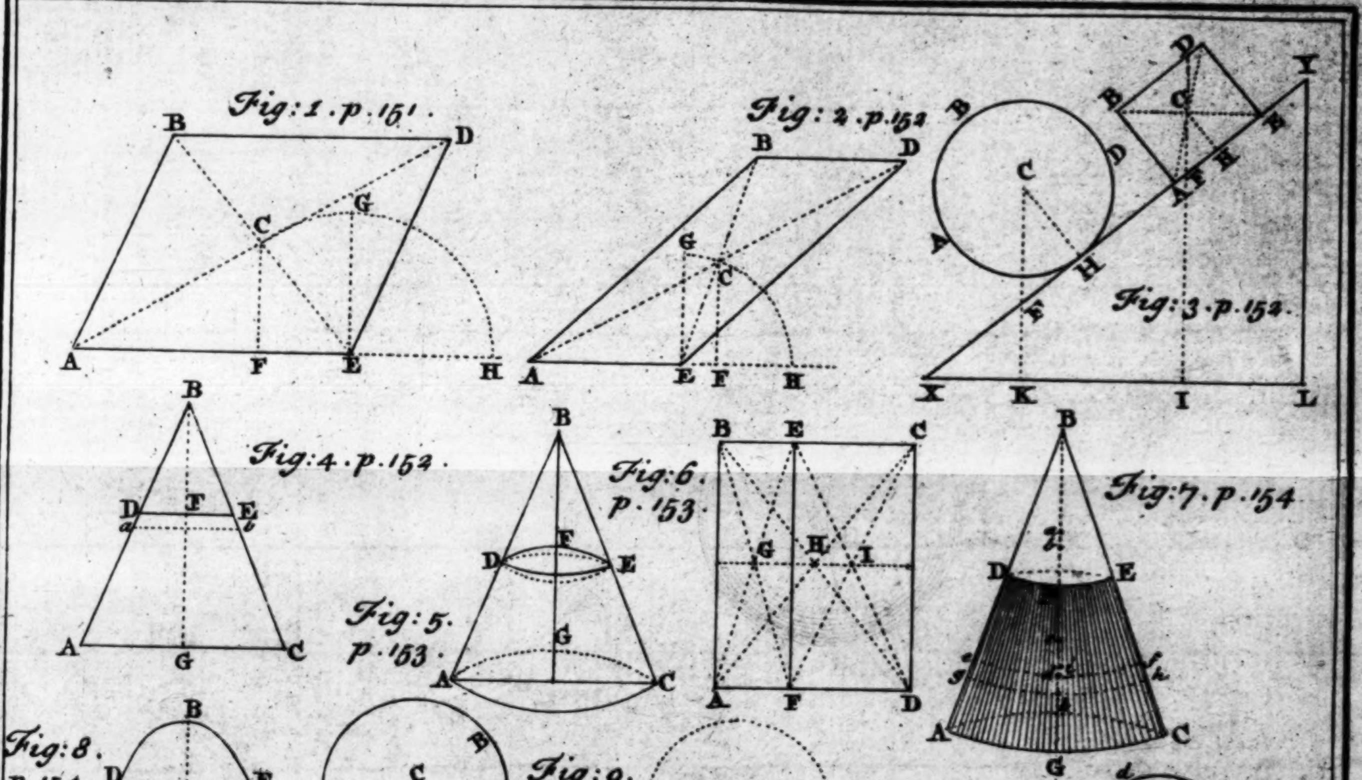
(XXXV) 1. That the Centre of Magnitude, of Motion, and of Gravity is the ſame Point C in regular and homogeneous Bodies, as the circular Area, or Square A B D E, is evident; becauſe any Right Line as A D drawn through the ſaid Point and terminated by the Peripheries of thoſe Bodies, has an equal Number of Particles on each Side; therefore the ſaid Point is Centre of Magnitude and Gravity, and conſequently of uniform Motion in every ſuch Line, and therefore of the whole Area, which is made up of them.

Plate VI. Fig. 13.

2. But

Vol. I. P. 151.

Plate VII.



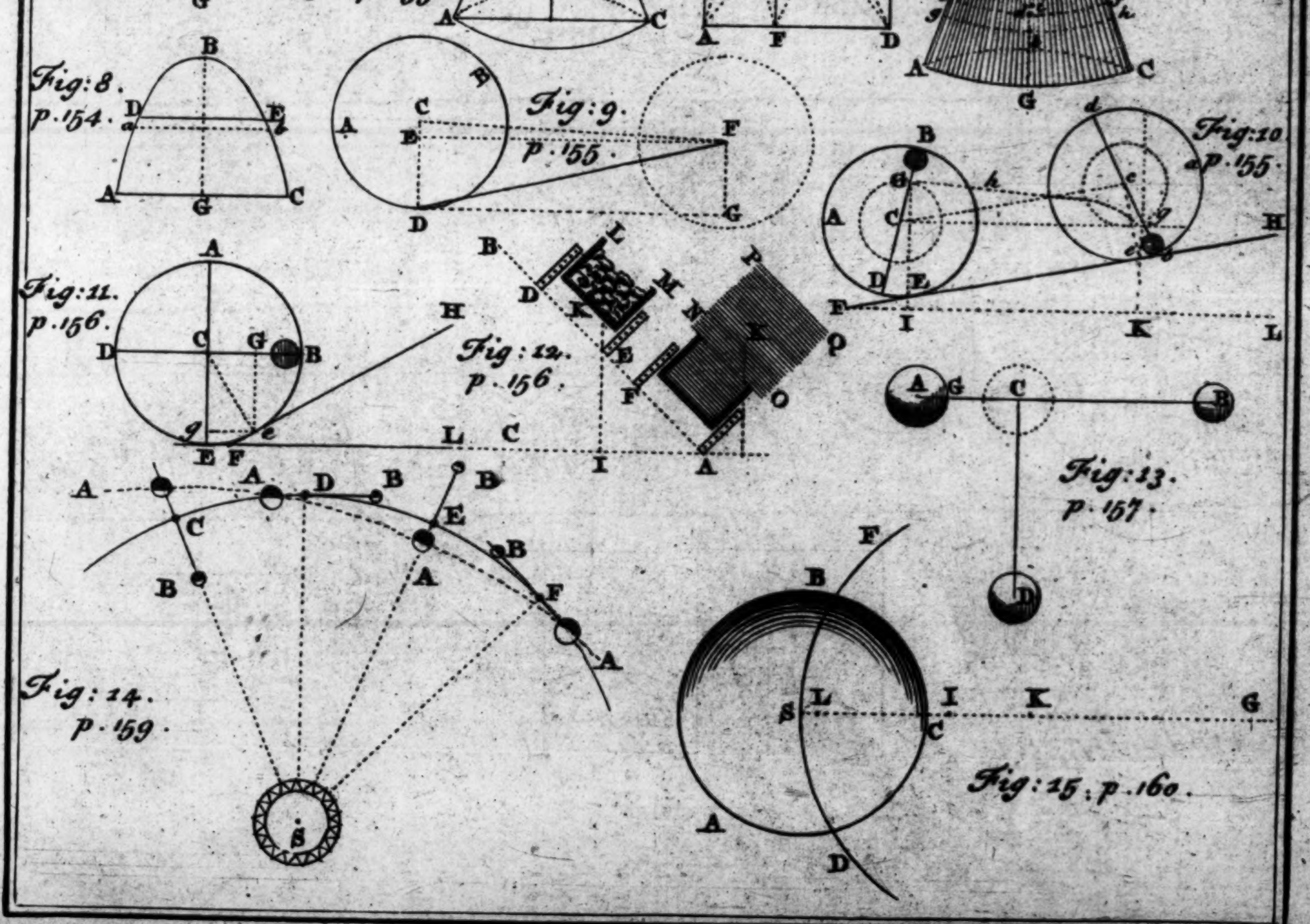


Fig:8. p.154.
Fig:9. p.155.
Fig:10 p.155.
Fig:11. p.156.
Fig:12. p.156.
Fig:13. p.157.
Fig:14. p.159.
Fig:15 ; p.160.

If two or more Bodies in Motion be connected together, as *Chain-Shot*, &c. or any how

2. But if a Body be not homogeneous, but consists of different Matter in different Parts, the Centre of Magnitude C will not be the Centre of Gravity; thus suppose ABF a circular Piece of Wood with a round Piece of Lead *e f* in one Side of it; this Lead will remove the Centre of Gravity from C to some other Point D in the Diameter passing through the Centers C and *g*; and this Centre of Gravity D will of Course be also the Centre of Motion and Rest, for on no other Point can a Body rest but that on which it is equilibrated, since Rest is only the Result of an Equilibrium of the Parts.

3. If the Centre of Gravity be supported, the Body will be supported likewise; for since the Power of every Particle to descend is there referred, and all unite in one; if that one Point be upheld, all the rest must, as having a mutual Dependence thereon by a necessary Connection of Parts.

4. If a Body be suspended on any other Point C, than the Centre of Gravity D, it will descend in every Position but two, *viz.* when the Centre of Gravity is in E above, or in D below the Point of Suspension C, because in two Cases it gravitates on a fixed Point, which suspends the Body, and therefore upholds it; but in all other Cases it has Liberty to descend, and therefore will carry the Body down, till it comes to rest in D.

Pl. VII. Fig. 1.

5. If the Centre of Gravity C in the Body ABDE gravitate on the Base AE in the Perpendicular CF, that Body will stand firm, and so much the more so, as the Point F falls nearer the middle Point between A and F; for since, were the Body to move, the Point or Angle E would be the Centre of Motion, it follows, that in that Case the Centre of Gravity C must rise in the Arch of a Circle from C to G, which it cannot do without some extraneous Force; and therefore, when left to itself, the Body must stand firm.

6. But in the Body of the next Figure, because the Line CF, in which it gravitates, falls off from the

how depend on each other, they will have a *Common Centre of Gravity*, which will be a Point

Pl. VII. Fig. 2. Baſe AE, the Centre of Gravity C will be free to deſcend, as not being ſupported, and conſequently the Body muſt fall.

Fig. 3. 7. If a Body ABDE, laid on an inclined Plane XY, gravitates in the Direction CI, within the Baſe AE, that Body will move down the Plane (by *Annot.* XXVII. 1.) but it cannot move over the angular Point A (by 5th of this) it muſt therefore deſcend by *ſliding down* the Plane. If the Baſe AE, had been ſo ſmall that the Point F had fallen off it, it would have turned on the Point A in its Motion, and ſo have *tumbled down* the Plane. But if the Baſes AE be ſuppoſed infinitely ſmall, or a Point, the falling from one Point to another, will be ſo momentaneous, that the Body (which then becomes a Circle or Sphere) will deſcend by *rolling down* the Plane.

Fig. 4. 8. To find the Centre of Gravity of a *Line*, has been ſhewn (*Annot.* XXIX. 2.) To find that of a Superficies, as of a *Triangle*, *Parabola*, &c. this is the Method. Let AG biſect the Baſe AC of the Triangle ABC, it will alſo biſect every other Line DE drawn parallel to the Baſe; conſequently the Centre of Gravity of the Triangle will be found ſomewhere in the Line BG. The Area of the Triangle may be conſidered, as conſiſting of an infinite Number of indefinitely ſmall Parallelograms DE*ba*D, each of which is to be conſidered as a ſmall Weight, and alſo as the Fluxion of the Area of the Triangle, and ſo may be expreſſed by $2y\dot{x}$, (putting BF $= x$, and FE $= y$;) if this fluxionary Weight be multiplied by its Velocity x, we ſhall have $2yx\dot{x}$ for its *Momentum*.

9. Now put BG $= a$, and AC $= b$; then BG (a) : AC (b) :: BF (x) : DE $= \frac{bx}{a} = 2y$; therefore the Fluxion of the Weights $2y\dot{x} = \frac{bx\dot{x}}{a}$; and the Fluxion of

Point in the Line joining the Centres, so situated, that its *Distance from the said Centres*

of the *Momenta* $2yx\dot{x} = \frac{bxx\dot{x}}{a}$; whence the Fluent of the latter, *viz.* $\frac{bx^3}{3a}$, divided by the Fluent of the former, *viz.* $\frac{bx^2}{2a}$, will give $\frac{2}{3}x$ for the Distance of the Point from B in the Line B F, which has a Velocity equal to the mean Velocity of all the Particles in the Triangle D B E, and is therefore its Centre of Gravity. *Consequently the Centre of Gravity of any Triangle* A B C, *is distant from the Vertex* B, $\frac{2}{3}$ B G *a right Line drawn from the Angle* B *bisecting the Base* A C.

10. And since the Section of a superficial or hollow Cone is a Triangle, and Circles have the same Ratio as their Diameters, it follows *that the Circle whose Plane passes through the Centre of Gravity of the Cone is* $\frac{2}{3}$ *of the Length of the Side distant from the Vertex of the said Cone.*

11. But in a *solid Cone*, which consists of an infinite Number of circular Areas, which therefore may be considered as so many Weights, the Centre of Gravity may be found as before, by putting B E $= x$, B G $= a$; the circular Area D F E $= y$, and A G C $= b$; and from the Nature of the Cone, $a^2 : x^2 :: b : y = \frac{bx^2}{a^2}$, but $\dot{x}y = \frac{bx^2\dot{x}}{a^2}$ = Fluxion of the Weights; and $yx\dot{x} = \frac{bx^3\dot{x}}{a^2}$ = Fluxion of the *Momenta*, whence the Fluent of the latter, *viz.* $\frac{bx^4}{4a^2}$ divided by the Fluent of the former $\frac{bx^3}{3a^2}$, will give $\frac{3}{4}x$ of the Centre of Gravity of the Part D B E F; *consequently the Centre of Gravity of the Cone* A B C G *is distant from the Vertex* B, $\frac{3}{4}$ *of the Side* B G, *in a Circle parallel to the Base.* Pl. VII. Fig. 5.

12. If A B C D be a regular Superficies, and A B E F another Fig. 6.

Centres will be reciprocally as the Quantity of Matter in each Body.

WHENCE,

another similar thereto, their Difference will be F E C D; and the Centres of Gravity of each will be in the Points H, G, I, as is evident from what has been shewn. Now (from *Euclid's* Elements) we have F E C D to A B E F as the Triangle F B D to the Triangle A C F; but F B D : A C F : : F D : A F : : G H : H I; wherefore if any two regular and similar Superficies or Solids together with their Centres of Gravity G, H, are given, the Centre of Gravity I of their Difference will be given also by the above Analogy. This in many Mechanical Affairs will prove of very considerable Use.

Fig. 7. 13. Thus for Example, suppose ABCG and BDEF were two similar Cones, and their Centres of Gravity when hollow or superficial were *c* and *a*; but when solid, *d* and *b*; if then we suppose the Cone D B E F cut off from the Cone A B C G, there will remain the Difference A D E C G, whose Centres of Gravity, when hollow and when solid, may be found as above; which let be in the Circles *e i f* and *g k h*, at *i* and *k*. If now the truncated Part were made into a Bucket by fixing in a thin Bottom at A C, and suspending it moveable in a Handle by Pins fixed between the said Circles *e i f* and *g k h*, on each Side, it is evident such a Bucket when empty must hang with its Mouth downwards; and when full of Water, with the Mouth upwards; thus may such a Bucket be applicable to divers Uses, as being capable of filling, and emptying, and moving itself in different Forms, as the Bottom is fixed in the wide or narrow End.

Pl. VII. Fig. 8. 14. The Centre of Gravity of a Parabola A B C, is found as in the Triangle and Cone; thus let B F $= x$, D E $= y$; then will $y\dot{x}$ be the Fluxionary Weight, and $yx\dot{x}$ the Fluxion of the *Momenta*; but from the Nature of the Curve we have $y = x^{\frac{1}{2}}$, whence $y\dot{x} = x^{\frac{1}{2}}\dot{x}$, and $yx\dot{x} = x^{\frac{1}{2}}\dot{x}x$, whose Fluent $\frac{2}{5}x^{\frac{5}{2}}$ divided by $\frac{2}{3}x^{\frac{3}{2}}$ (the Fluent of $x^{\frac{1}{2}}\dot{x}$) will give $\frac{3}{5}x = {}^{3}$ B F for the

WHENCE, since the Earth and Moon are to each other as about 40 to 1, and the

the Distance of the Centre of Gravity from the Vertex B in the Part D B E; and so $\frac{3}{5}$ of B G is the Point in the Axis of the whole Parabola A B C, from the Vertex B.

15. The Phænomenon of the double Cone is easily accounted for, from what we have said of the Centre of Gravity. For let A B D be the common Base of the two Cones, its Centre C will be the Centre of Gravity of the Whole; therefore if D F be the Leg of a Ruler elevated to an Angle F D G, whose Sine F G is less than the Semidiameter of the Cone C D, it is plain the Centre of Gravity C at the Position of the Cone in D is more distant from the Centre of the Earth than in its Position between the Legs of the Ruler at F; and therefore it will descend (as on an inclined Plane C F E) from C to F, where it will stop, as being supported on the Ends of the Ruler. Fig. 9.

16. Let A B E D represent a Section of a Cylinder of Wood bias'd on one Side with a cylindric Piece of Lead as B, this will bring the Centre of Gravity out of the Centre of Magnitude C to some Point G between C and B; let F H be an inclined Plane, whose Base is F L. It is evident the Cylinder laid upon the Plane will no where rest but there, where a perpendicular to the Horizon F L passes through the Centre of Grav ty G, and that Point of the Plane E in which the C linder touches it; and this in all Angles of Inclination of the Plane less than that whose Sine is equal to C G (the Radius being C D) will be in two Situations A B E D and *a b e d*, because when the Cylinder moves, the Centre of Gravity describing a Circle round the Centre of Magnitude C, this Circle will meet the Perpendicular in two Points G and *g*, in each of which the Centre of Gravity being supported the Cylinder will rest. Therefore the Cylinder moves from E to *e* by the Descent of the Centre of Gravity from G to *g*, in the Arch of the Cycloid G *h g*. Fig. 10.

17. If

the Distance of the Moon 60 Semidiameters of the Earth, the Distance of the common

Pl. VII. Fig. 11. 17. If the Cylinder ABED insisting on the horizontal Line EL in the Point E, has the Centre of Gravity G in the horizontal Diameter DB, it will gravitate in the Perpendicular G*e*; if therefore a Plane FH touch the Cylinder in the Point *e*, it is plain the Cylinder cannot either ascend or descend on such a Plane; because G in any Situation between *e* and H, or *e* and F, it will gravitate to the Left or Right from the Point in which the Cylinder touches the Plane, and so will in either Case bring it back to the Point *e*.

18. In this Case the Angle of the Plane's Inclination HFL is equal to the Angle EC*e* or C*e*G, because EL and FH are Tangents to the Cylinder in the Points E and *e*, and consequently the Angles EC*e* + EF*e* = two Right Angles = EF*e* + HFL; therefore EC*e* = HFL. Hence it follows, that a Cylinder cannot ascend on a Plane whose Inclination is greater than the Angle EC*e*, but on a Plane of less Inclination it may; and there is a certain Inclination of the Plane on which the Ascent will be a *Maximum*, but it is not worth while to insist farther on so useless a Subject.

Fig. 12. 19. If DLME and FPQO represent two Waggons loaded, the former with heavy Matter, as *Iron*, *Stone*, &c. which lies low, and the latter with a light Substance that rises high, both on the Side of an Hill AB; it is evident the Centre of Gravity of the Load LM, acting in the Direction RI, which falls between the Wheels DE will keep the Waggon firm on the Side of the Hill, (by *Art*. 5.) But in the other Load NPQO, where the Centre of Gravity K rises so high above the Bottom of the Waggon, it must act in a Direction KH, which falling without the Base Line FG of the Wheels, must necessarily cause the Waggon to overturn.

20. For the same Reason a Man must fall when the Centre of Gravity acts in a Direction falling without the Base-line of the Feet; and it is by the artful adjusting

common Centre of Gravity of the Earth and Moon will be found about 1854 Miles from the Earth's Surface; and it is this common Centre of Gravity that describes the *Annual Orbit* about the Sun, and not the Earth itself, as is commonly said and thought.

IN like manner there is a common Centre of Gravity of the Sun and all the Planets which circulate about him; and were they all placed in a Right Line on one Side the Sun, then would the common Centre of Gravity of the whole System be distant from the Sun's Surface *eight Tenths* of his Semidiameter: And it is about this common Centre of Gravity, and not about the Sun, that not only all the Planets, but even the Sun itself, do constantly move (XXXVI).

THE

ing of this Point over the Rope that People can walk or fly thereon; and that Tumblers and Equilibrists perform such Wonders. In fine, it is this single Principle that regulates every Kind of Motion, both Animal and Mechanical, and to which we are naturally prone to have a much stricter Regard than we commonly think of; as would be evident were we nicely to consider the Gait, Posture, and Configuration of the Body under the various Modes of Acting, *viz.* of *Walking*, *Sitting*, *Riding*, *Stooping*, *Carrying*, *Lifting*, *Drawing*, *Pushing*, *Reaching*, *Striking*, *Throwing*, &c. and the particular Manner in which Artists fix a Wind-Mill, Crane, or any such like Machine.

(XXXVI) 1. If two Bodies A and B be connected together, by a Wire or Chain A B, they will each of them Pl. VII. Fig. 13.

THE common Centre of Gravity of any Number of Bodies being supported, none of them affect the other by their gravitating on the Line A B; and also every Particle of that Line will be affected with those Gravities, or be carried downwards, but with an unequal Force, except *one*, on which the Force of each Body is equal; and since we shewed the Force of any Body A or B arises from its Quantity of Matter multiplied into its Velocity, it is evident, that one Particle must be so situated at C, that $B : A$ $(:: v : V) :: AC : BC$. Because then we have $\overline{B \times V} = \overline{A \times v} = \overline{B \times AC} = \overline{A \times BC}$. Or the Product of each Body into its Velocity is the same at the Point C, but no other.

2. The Point C is therefore called the *common Centre of Gravity* of both the Bodies. If the Distance between the Centres of the Bodies be given, and the Magnitude of each, the Distance of the Point C from either may be thus found; as $B + A : B :: AC + BC$ $(= AB) : AC$. Let A represent the Earth, and B the Moon; then will $A : B :: 40 : 1$, nearly; and since $AB = 60$ Semidiameters of the Earth; we have $40 = 1 : 1 :: 60 : \frac{60}{41} = AC$; and since a Semidiameter contains about 4000 Miles, we have $\frac{60 \times 4000}{41}$ $= 5854 = AC$, from whence taking one Semidiameter $AG = 4000$, there will remain $GC = 1584$ Miles, for the Distance of the Point C from the Earth's Surface.

3. Since the Earth and Moon act on each other by Attraction, it is evident they must both revolve about the common Centre of Gravity C; whence this Point C, and not the Centre of the Earth A, is that which the Moon regards in her periodical Revolutions, and were there no other Bodies in the Heavens but the Earth and Moon, this common Centre of Gravity C would be at rest, or a fixed Point.

4. But since the large Body of the Sun commands (by the same Power of Attraction) the Earth and Moon to

of those Bodies can fall; which is the Reason of many very surprising Appearances in

to revolve about itself, it will follow that the Point C is that which must describe the Circle (or *Orbis Magnus*) about the Sun; because no other Point between A and B can keep always at the same Distance from the Sun, on Account of the mutual Revolutions of those Bodies about that Point at the same Time they are carried about the Sun.

5. But to illustrate this Matter further, let S be the Sun, and CDEF a Part of the annual Orbit; A and B the Earth and Moon in her Conjunction at C, in her Quadrature at D, in her Opposition at E, and in her last Quarter at F; during all these Motions from C to F, the Centres of the Earth and Moon will describe Curves of the *Cycloid* Kind, every where concave to the Sun; and though this may appear a Paradox, yet it is capable of a Geometrical Demonstration, and may be also shewn by Experiment. In the common Astronomical Tables, the Centre of the Earth is supposed to describe the *Orbis Magnus*; and the Place and Distance of the Sun or Earth is computed accordingly; but these will be different from the true ones, which must be computed from the Circle which the Earth does monthly describe about the common Centre of Gravity; and this Difference is call'd the MENSTRUAL PARALLAX. Pl. VII. Fig. 14.

6. The Point C is that in which the Force of each Body A, and B, is exerted; and therefore every Action of that Point is the same as it would be if both the Bodies were there united in one; thus if A and B were two Chain-Shots, the Point C in the Chain, and no other, would strike an Obstacle with the greatest Force, or that of a Body equal to the Sum of A and B.

7. If therefore a Wire be inserted in the Point C, and at the other End any Body D were affixed, the common Centre of Gravity of all the three Bodies A, B, and D, may be found as before; for let the Body $C = A + B$; then will $C + D : D :: CD : CE$; or $A + B + D : D :: CD : CE$; and thus E will be the Fig. 13.

 the

in Nature, *as that common Experiment of suspending a Bucket of Water at the End of a Stick*

the Point sought. And in the same Manner the common Centre of Gravity in a System of any Number of Bodies may be found.

8. Hence the common Centre of Gravity of the Sun and Planets may be easily found by comparing their Quantities of Matter and Distances; thus let A B C represent the Sun's Body, and S its Centre. Now since the Bulks and Distances of the four first Planets, *Mercury*, *Venus*, the *Earth* and *Mars*, are very inconsiderable in regard of the Sun, they would not (if placed in the right Line C G) remove the common Centre of Gravity between the Sun and themselves far from the Centre of the Sun, suppose to L. But when we come to *Jupiter*, his Bulk and Distance give him a considerable *Momentum*, which will (if placed in the same Line CG) remove the Centre of Gravity from L to I, a Point without the Surface of the Sun's Body. Lastly, if we consider *Saturn* placed in the Line C G, with all the rest; though his Quantity of Matter falls short of, yet his Distance far exceeds that of *Jupiter*, and therefore his *Momentum* will be considerable enough to bring the common Centre of Gravity from I to K, at such a Distance C K from the Sun's Surface as is equal to $\frac{8}{10}$ of the Sun's Semidiameter S C; or S C : C K : : 10 : 8. Now it is this Point K which is the fixed and immoveable Centre of the System, about which the Sun, as well as all the Planets, continually move. But the Sun being always very near it, and its Distance therefrom varying with the different Positions of the Planets, the Motion of the Sun about this common Centre K will be very irregular and unequal; while that of the Planets, on account of their great Distance, may be esteem'd nearly uniform and circular.

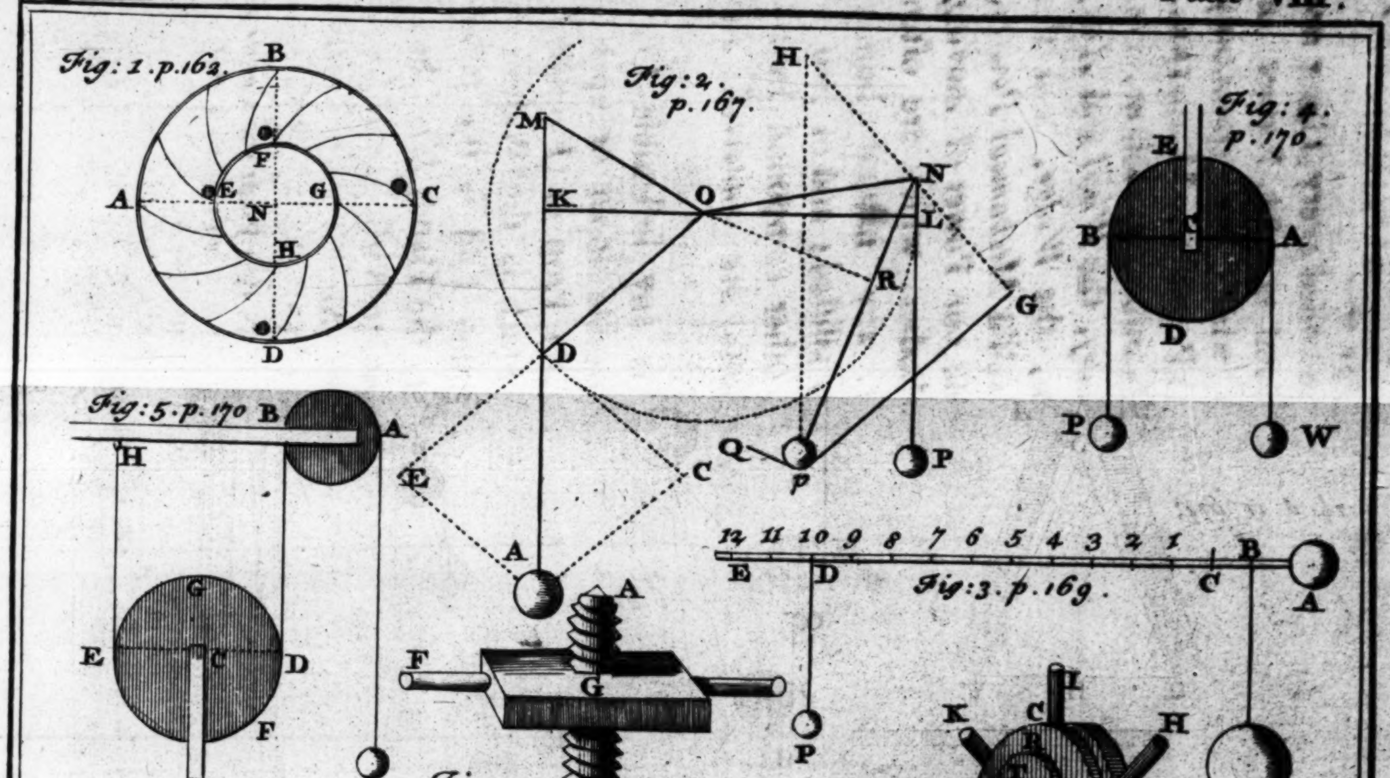
Fig. 15.

9. If the two Bodies A and B move to or from each other in the same right Line A B, with Velocities proportional to their Quantities of Matter inversely, the common Centre of Gravity C will remain at Rest, because their Distances from it will by that means be always in the same Ratio, *viz.* of their Masses inversely, which is

Fig. 13.

Vol.I. P.161.

Plate VIII.



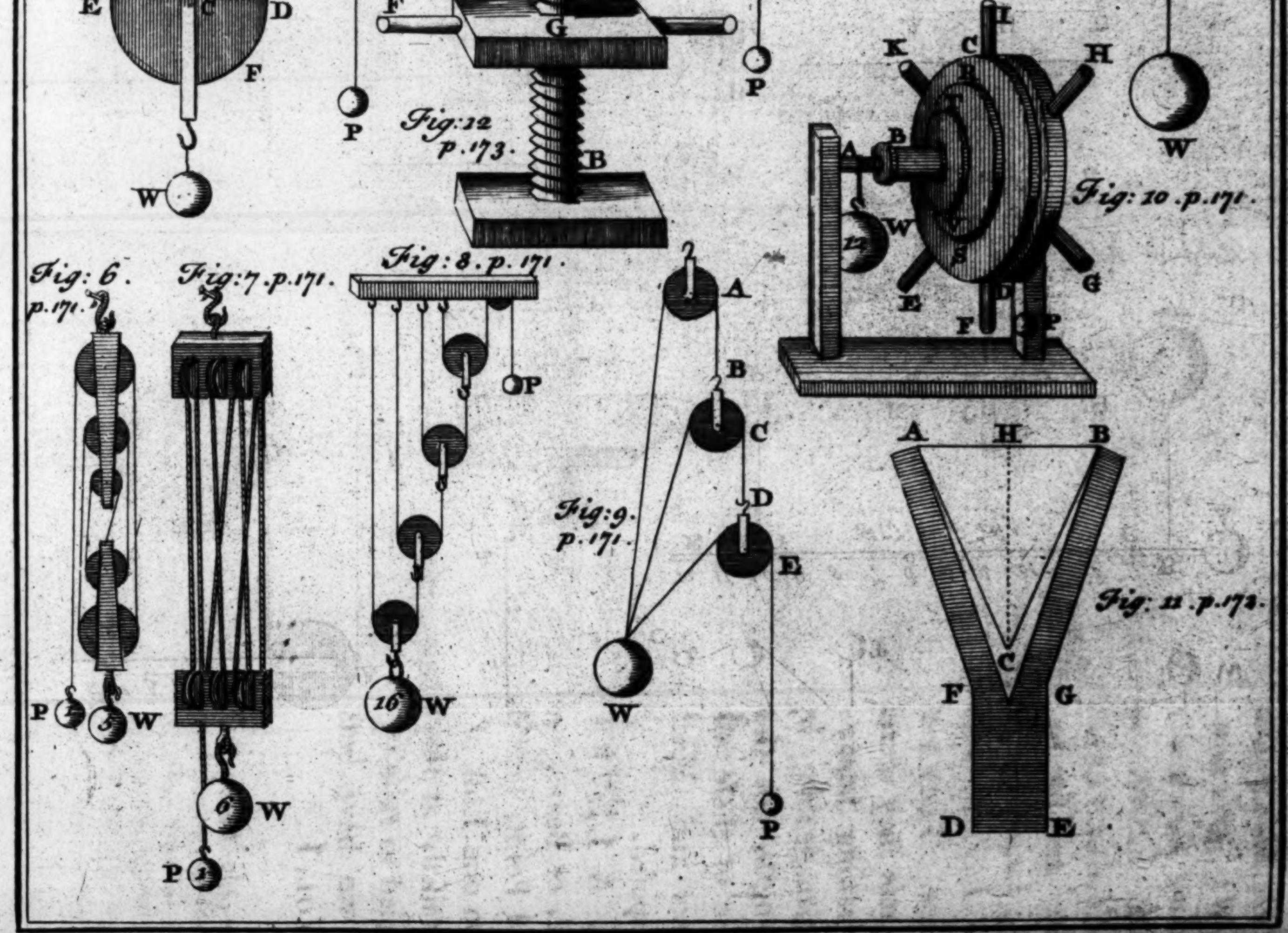


Fig: 6. p. 171.
Fig: 7. p. 171.
Fig: 8. p. 171.
Fig: 9. p. 171.
Fig: 10. p. 171.
Fig: 11. p. 172.
Fig: 12 p. 173.

a Stick off the Table without falling; the Toy of the Man a sawing, &c.

We have now premised every Thing necessary for understanding the Nature of those Instruments which are commonly call'd MECHANICAL POWERS or MACHINES: They are six in Number, as follow, *viz.* The *Lever*, the *Pulley*, the *Wheel* and *Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw*. They are call'd *Mechanical Powers*, because they increase our Power of moving or raising heavy Bodies, which are often unmanageable by any natural human Strength, not thus assisted; and of two or more of these all other compound Instruments and Machines are contrived and composed.

I. A LEVER is any inflexible Line, Rod, or Beam, moveable about or upon a fix'd Point, (call'd the Prop or *Fulcrum*); upon one End of which is the Weight to be raised, at the other End is the Power applied to raise it, as the Hand, *&c.* Since (as we have before proved) the *Momen-*

is the only Thing that makes that Point the Centre of Gravity between them. But if both Bodies were to move in two different Planes, either in the same or different Directions, the Centre of Gravity C would describe a Right Line, and with a Velocity of the same Sort as that with which the Bodies themselves do move.

tums of the Weight, and Power, are as the Quantities of Matter in each multiplied by their respective Celerities; and the Celerities are as the Distances from the Centre of Motion, and also as the Spaces pass'd through in a perpendicular Direction in the same Time; it must follow, *that there will be an Equilibrium between the Weight and Power, when they are to each other reciprocally as the Distances from the Centre*, or *as the Celerities of the Motions*, or *as the perpendicular Ascent or Descent in the same Time*; and this universally in all Mechanical Powers whatsoever, which is therefore the *fundamental Principle* of all *Mechanics* (XXXVII).

To

(XXXVII) 1. The Nature of this Proposition being not understood by Smatterers in Mechanics, gave them Occasion to imagine the Possibility of a *Perpetual Motion* from one Part of it, which they did not see was utterly impossible from another Part of it.

Pl. VIII. Fig. 1. 2. That Part which seem'd to promise the Possibility thereof, was this, viz. *That the* Momenta *of equal Bodies were as their Distances from the Centre of Motion.* Hence, say the Perpetual-Motion-Men, if a Wheel were constructed of the Form of that in the Figure ABCD, with circular Cells going from the inner Part EFGH to the outer, containing equal Balls C, D, E, F, then upon turning the Wheel, they must move towards the Centre N on one Part, as the Ball E, and from it on the opposite Part, as the Ball C; and by this Means the Ball C will have a greater *Momentum* than the Ball F, and so will determine the Wheel to move round; and since this must be the Case of all the Balls E and C that come into the Situation E, C, the Wheel must

To illustrate this, let AB be the Lever supposed without Weight, and F its *Fulcrum* or Prop; let W be a Weight suspended from the End A, and P the Power applied to the other End B. Then let the Lever be moved into the Situation CD, 'tis evident the Velocities of the Points A and B will be as the Arches AC and BD described in the same Time: Also the perpendicular Distances, through which the Weight

Plate V. Fig. 9.

must necessarily move continually, because it will continually bring two Balls into that Situation.

3. 'Tis true, were there but two Balls E and C, the Ball C would by this Contrivance move the Wheel one Quarter round, *viz.* while it descended from C to D; and by this Means would raise the Ball E to F; and there they will abide in the Situation DF; but, say the Gentlemen of this Persuasion, two other Balls succeeding to the Places E and C, will still keep the Wheel moving.—Yes, so they would, if the Balls at D and E could be taken away the Moment they come into that Position, not else; for the Balls C and E, in order to move the Wheel, must move the Balls D and F, which have equal *Momenta*, (as being at the same Distance each from the Centre, as are the other two respectively) which is absurd by the *general Proposition.*

4. The Absurdity of a Perpetual Motion will still farther appear, if we consider, that the *Momenta* of Bodies are always proportion'd to the perpendicular Descent or Ascent to or from the Centre of the Earth. Since, therefore, in the Wheel, the Bodies are all equal by Supposition, and the perpendicular Spaces through which they *descend and ascend* below and above the Horizontal Line or Diameter AC, are equal; it follows, that an *Equilibrium* must necessarily ensue. Thus so far is this Wheel from producing a Perpetual Motion, that it admits of none at all.

Weight W and Power P move in the same Time, are CE and GD, which are as the Arches AC and BD; and these are as the Radii CF and DF, which are equal to AF and BF. Therefore in order to produce an *Equilibrium*, it must be W × AC = P × BD, or W × CE = P × DG, or W × AF = P × BF: Consequently, P : W :: AC : BD :: CE : DG :: AF : BF. Note, *that in estimating the Effects of Machines, we regard only the Distances of the Power or Weight which are perpendicular to the Lines of Direction in which those Powers act, as* FB, *or* FM, *which are perpendicular to the Directions* PB *and* LM.

THE *Lever* is of five Kinds. (1.) The common Sort, where the Prop is between the Weight and the Power, but nearest the former. (2.) When the Prop is at one End, the Power applied at the other, and the Weight between both. (3.) When the Prop is at one End, the Weight at the other, and the Power applied between both. (4.) The bended Lever, which differs only in Form from the first Sort. (5.) When the Prop is placed at an equal Distance between the Weight and the Power, and this is commonly call'd

THE BALANCE, whose Use is, with a Pair of Scales, to bring one Body to an equal

equal Weight with another that is a Standard. The *Proportional Balance* is without Scales, and is used for discovering or assigning any Proportion of Weight in Bodies. The *False Balance* makes Bodies of unequal Quantities of Matter appear to have equal Weight. Lastly, the *Statera*, or *Roman Balance*, commonly called the *Steelyard*, is a most useful Kind of universal Balance, the Structure and Use whereof will be easy to understand from the above Principles.

II. THE PULLEY is an Instrument well known; if single, it is reduced to the Lever of the fifth Sort, or Balance, and so affords no Advantage in raising Weights. If two or more be combined together in the common Way, *Then the Power is to the Weight as Unity to the Number of Ropes which goes to the Pulleys of the lower Box.* But there are different Ways of applying Pulleys, whose *Advantage or Power* will be obvious from a View of the Structure of the several Sorts of *Tackles*, and the Experiments with them.

III. THE WHEEL and AXLE (commonly call'd the *Axis in Peritrochio*) is the third Mechanical Power. We easily see by its Make it is reducible to a Lever of the first Sort; and that therefore the *Power is*

to the Weight, as the Diameter of the Axle to the Diameter of the Wheel, in an *Equilibrium* in this Machine.

IV. THE INCLINED PLANE is the fourth Mechanical Power, and from a due Consideration of it, it will appear, that, for an *Equilibrium, the Power must be to the Weight, as the Height of the Plane to the Length.*

V. THE WEDGE is only a double Inclined Plane, intended to separate the Parts of Wood, *&c.* which strongly cohere together; whence, in the common Form of it, *the Power will be to the Resistance to be overcome, as half the Thickness of the Wedge to the Length thereof.*

VI. THE SCREW is the last mention'd Mechanical Power, whose Use is both for *Pressure* and *raising of Weights*, but chiefly the former. *The Power is to the Weight, as the Velocity of the Weight to the Velocity of the Power,* that is, *as the Distance between two Threads of the Screw to the Circumference of a Circle described by the Power* (XXXVIII).

WE

(XXXVIII) 1. Notwithstanding the Demonstration of the Fundamental Principle of Mechanics, as deliver'd in the Lectures, is most natural and perspicuous, and easily results from what has been said of the *Momentum* of

WE have here considered the Action or Effect of each of these Machines, as they would

of Bodies and their common Centre of Gravity: Yet as Sir *Isaac Newton* has demonstrated the same Thing in a different and most extensive Manner, which also is the Consequence of a different Principle, *viz.* the *Composition and Resolution of Forces and Ratios*, it will be proper to exhibit and explain that also, which is as follows.

2. Suppose two Weights A, P, appended by the Strings M A, N P, at the Ends of unequal Radii O M, O N, issuing from the Centre O of any Wheel, in a State of *Equilibrium*; those Weights will be to each other reciprocally as O L to O K, that is, A : P :: O L : O K, or as the nearest Distances of their Lines of Direction from the Centre reciprocally. Pl. VIII. Fig. 2.

3. For on the Centre O with the Radius O L describe an Arch cutting the Thread M A in D, and draw O D, which continue out to E, to which draw A E perpendicular, and compleat the Parallelogram A E D C. Now since K L passes through the Centre O, and is perpendicular to each String in the Points K and L by Supposition; it matters not whether the Body A and P be suspended from the Points M and N, or K and L, or D and L, since the Weight of the Bodies is the same in either of those Points respectively.

4. Therefore let A D expound the whole Force of the Weight of the Body A, and let it be resolved into the two Forces D E and A E, of which the former drawing directly from the Centre avails nothing in moving round the Wheel; but the other Part A E or C D, acting perpendicularly upon the End of the Radius O D, has the same Force or produces the same Effect as if it had acted perpendicularly at the End of the equal Radius O L.

5. Therefore the Weight of P will be express'd by D C (or A E) because of the *Equilibrium*; and hence the Weight of A is to that of P as A D to A E; and because of the similar Triangles A E D and D O K,

would answer to the Strictness of the Mathematical Theory, were there no such thing

we have A D : A E : : D O (= L O) : O K; therefore A : P : : O L : O K.

6. If the Weight p, (equal to P) suspended by the String N p, does at the same Time in part rest upon the inclined Plane p G; let p Q be drawn perpendicular to p N, p H perpendicular to the Horizon, and H G thro' the Point N perpendicular to p G: Then will the Tension of the String P N be to that of the String p N, as p H to p N. For suppose the Body p wholly supported by the two Planes p G, p Q, it will press them with its whole Force, which may be express'd by p H, and which is resolvable into the two Forces H N and p N, of which the first presses the Plane p G, and the last the Plane p Q; if therefore the Plane p Q be removed, the same Force will stretch the String p N; but since p = P, the Force with which P stretches the String P N is as p H; therefore the Tension of the String P N is to that of the String p N as p H to p N.

7. Let the Force of p to move the Wheel (as supported on the Plane) be called x. Then P : x : : p H : p N; and so $P = \frac{x \times pH}{pN} = \frac{A \times OK}{OL}$ (by 5th), whence we have this Equation, $x \times pH \times OL = A \times OK \times pN$; therefore A : x : : p H × O L : p N × O K. But here it is to be observed that as the String P N is changed into the Position p N, its nearest Distance O L will become O R, so that the Analogy between A and x corrected will be A : x : : p H × O R : p N × O K.

8. From what is demonstrated it follows, (1.) That the Line K O L is a LEVER, whose Power is always express'd by the Ratio $\frac{OL}{OK}$. (2.) That the Lines M O L, M O N, D O L, are also Levers of equal Force, because the nearest Distance of their Lines of Direction from the Centre O is the same in all. (3.) Whenever the Lever K L is moved, the Velocity of the Points K and

thing as Friction or rubbing of Parts upon each other, by which means *one Third Part*

K and L will be proportional to the Distances from the Centre KO and OL. (4.) Therefore the Velocities also, and perpendicular Spaces pass'd through by the Bodies A and P, will be proportional to the said Distances from the Centre.

9. Since P : A : : KO : LO, 'tis evident the Power P may be in any Proportion less than the Weight A, if its Distance from the Centre of Motion OL be in the same Proportion greater than the Distance of the Weight KO, and yet its Force or *Momentum* shall be equal to that of the Weight A. Whence the Nature of a Lever for increasing a Power sufficiently appears.

10. If the Distances from the Centre OK = OL; then will the Lever become a *common Balance*, because then, in order to an *Equilibrium*, the two Bodies A and P must have equal Weight.

11. If the Arms of the Balance KO and OL differ but a very little in Length, there will be the same Difference in the Weights A and P. Whence if OK : OL : : 31 : 32 : : P : A; and therefore if A is an *Averdupois* Pound, P will want ½ an Ounce of it and yet be *in Equilibrio*. Hence the Nature of a *False Balance* is evident; and in order to detect it we need only interchange the Weights, for then, in order to an *Equilibrium*, we must have KO : OL : : (31 : 32 : :) 32 : 33 $\frac{1}{31}$; but of this there wants 2 $\frac{1}{31}$ half Ounces, which will render the Fraud very notorious.

12. The Arms of a Balance being nicely equated in Weight and Length, and divided into an equal Number of equal Parts, becomes the *Proportional Balance* to be used without Scales. For any two Bodies hanging on the Arms of this Balance *in Equilibrio*, will have the Proportion of their Weights express'd by the Numbers at the Divisions (whence they hang) alternately.

13. If AC the shorter Arm of a Balance be made equal in Weight to CE the longer Arm, by the Addition of the Ball A, so that the whole Beam AE may Pl. VIII, Fig. 3.

Part of the Effect of the Machine is, at a Medium, destroy'd, as is evident by an Experiment

may be nicely equipoised on the Centre of Motion C, it then becomes the *Roman Statera* or STEEL-YARD in common Use. For if B be taken as a fixed Point from whence to hang any Weight W, and P be any constant Weight, which by moving backwards and forwards on the longer Arm C E, comes to a Point D of the *Equilibrium*; then will P : W : : C B : C D; but because P and C B are given Quantities, or always the same, therefore W will always be as C D. Consequently if the Arm C E be divided into Parts each equal to C B, and numbered; the Number at which P hangs will always shew how many Times P is contained in W, and thence its Weight will be known.

14. If A D B E be a Pulley, upon which hang the Weights P, W; then since the nearest Distances of the Strings A W and B P, from the Centre of Motion C, are A C and B C, the Pulley will be reduced to the Lever or Balance A B with respect to its Power; and from thence it appears that since A C = B C, we shall always have P = W for an *Equilibrium*; and therefore no Advantage in raising a Weight, *&c.* can be had from a *single Pulley*.

Fig. 4.

15. In a Combination of two Pullies A B and D F E G, the Power is doubled; for the Pulley D F E G is reducible to the Lever E D, which must be considered as fixed in the Point E to the immoveable String H E; and the Power acting at D is equal to P, and the Weight W is sustained from the Centre C of the Pulley, but P : W : : C E : D E; therefore since D E = 2 C E, it is W = 2 P, or P = $\frac{1}{2}$ W.

Pl. VIII. Fig. 5.

16. The Force of the PULLIES may also be easily shewn by comparing the Velocities of the Power and Weight; for it is evident, if the Weight W be raised one Inch, each String H E, B D, will be shortened one Inch, and consequently the String A P will be lengthen'd *two Inches*, and so P will pass through twice the Space that W does, in the same Time, whence its Velocity will

periment of the Inclined Plane. And farther, concerning Friction we are to observe, that it is not proportional to the *Quantity*

will be twice as great, and therefore it will be equipollent to a Body W of twice its Weight.

17. In the two other Forms of Pullies it is evident the *Power* P *is to the Weight* W *as Unity or* 1 *to the Number of Ropes going to the lower Pullies*, because if the Weight be raised *one Inch*, each Rope belonging to the lower Pullies will be shortened one Inch, all which will go into the Rope to which the Power P is applied, which therefore must descend through so many Inches in the same Time. Consequently the Tackle of Pullies in the Form of *Fig.* 6. will increase the Power five Times; and that of *Fig.* 7. will increase it six Times. Pl. VIII. Fig. 6,7.

18. In the Disposition of Pullies according to *Fig.* 8. it is plain, since each Pulley has a fixed Rope, it must be considered as a Lever of the second Sort, and so will double the Power of the foregoing Pulley, and so four Pullies will increase the Power sixteen Times. Fig. 8.

19. For the Force and Conveniency of a Tackle of Pullies all together, none is equal to that in the Form of *Fig.* 9. where the uppermost Pulley is fixed, and each has a Rope annexed to the Weight; its Power is therefore thus estimated. When the Weight W is raised one Inch, the Rope A B will be lengthened as much, and so the Pulley C will descend one Inch, by which means the Rope C D will be lengthened two Inches, and one by the Rising of the Weight W; wherefore the Pulley E will descend three Inches; and thus the Rope E P will be lengthened six Inches by that means (*viz.* three on each Side) also the Rising of the Weight will cause it to lengthen one Inch more; so that the Power P goes through seven Inches while the Weight W rises one; therefore P : W :: 1 : 7; and thus you proceed for any other Number. Fig. 9.

20. The WHEEL and AXLE is a Mechanical Power upon the same Principle; for the Weight W hanging from the Axis A will be to the Power P, which keeps it in *Equilibrio*, inversely as their Velocities; but the Velocities Fig. 10.

Velocities are as the Circumferences of the Wheel and Axle; which again are as their Diameters or Semidiameters, that is, as their Distances from the Centre of Motion. If the Spokes or Handles F, G, H, I, K, &c. be added, the Power of the Machine is still farther augmented in Proportion to their Lengths.

21. That the INCLINED PLANE is a Mechanical Power appears sufficiently in its diminishing the Weight of a Body laid upon it in regard to the Power which holds it in *Equilibrio*. Let A be a Body sustained on the inclined Plane B D; from the Centre C, draw C F perpendicular to the Horizon or Base D C; and C E perpendicular to the Plane, then C F will represent the whole Weight or Force of Gravity of the Body A, which is resolvable into the two Forces C E and E F; but the Force C E being perpendicular to the Plane acts wholly upon it, and is equally re-acted on or sustained by the Plane; the other Force E F, being parallel to the Plane, is that by which the Body descends, or is kept from descending by an equal Power acting in a contrary Direction. Therefore the whole Weight of the Body is to the Power which keeps it in *Equilibrio* on the Plane as C F to F E, or (because the Triangles C F E and B D C are similar) as B D to B C, that is, as the Length of the Plane to its Height, as was more particularly shewn in *Annot*. XXVIII. 1, 2.

Pl. VIII. Fig. 11. 22. The Power of the WEDGE A C B H is evident from its consisting of two equal Inclined Planes A H C and B H C; as it is chiefly of use to separate the adhering Parts of Wood; and since the Power of Cohesion in Wood is every where variable and uncertain, it is evident there can be no regular Calculation of the actual Effect of the Wedge; but if we suppose the Power of Cohesion in the Wood A D E B to be uniform, or to make every where an equal Resistance to the Wedge A B C, dividing its Parts A F and B G, then the Power of the Wedge would be to the Resistance of the Wood, as their Velocities inversely, that is, as the Spaces moved through in the same Time, that is, *as the Height of the Wedge* H C *to Half its Width* A H. But the Wedge being only a double inclined Plane, is not really a *different Mechanical Power*, tho' usually reckoned as such.

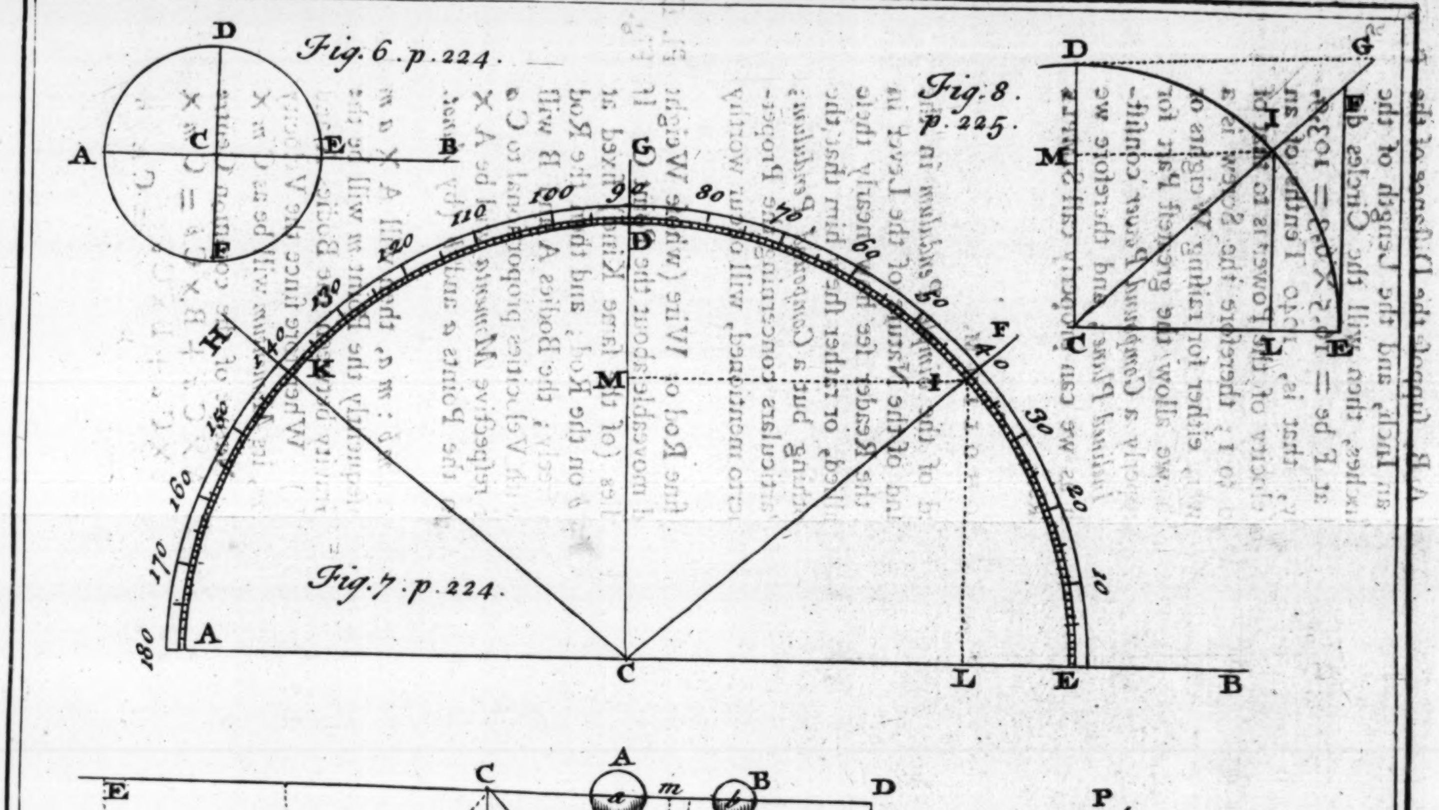
[illegible] the circumferences of the Wheels, [illegible] which again are as their Diameters, or [illegible], that is, as their Diftances from the Centre of Motion. If the Spokes or Handles F G, H I, K [illegible] &c. be added, the Power of the Machine [illegible] augmented in Proportion to their Length.

21. That the INCLINED PLANE [illegible] Power appears fufficiently in its diminifhing [illegible] of a Body laid upon it in regard to the Power which holds it in *Equilibrio*. Let A be a Body [illegible] the inclined Plane B D; from the Centre C, [illegible] perpendicular to the Horizon or Bafe D C, and C F perpendicular to the Plane, then C F will repre[illegible] whole Weight or Force of Gravity of the Body, which is refolvable into the two Forces C E and [illegible] but the Force C E being perpendicular to the [illegible] acts wholly upon it, and is equally [illegible] fuftained by the Plane; the other Force F E [illegible] parallel to the Plane, is that by which the Body [illegible] or is kept from defcending by an equal Power [illegible] a contrary Direction. Therefore the whole W[illegible] the Body is to the Power which keeps it [illegible] the Plane as C F to F E, or (becaufe the [illegible] C F E and B D C are fimilar) as B D to B C, [illegible] as the Length of the Plane to its Height, [illegible] particularly fhewn in *Annot.* XXVIII. [illegible]

22. The Power of the Wedge A C B [illegible] from its confifting of two equal Inclined Planes [illegible] and B H C; as it is chiefly of ufe [illegible] hering Parts of Wood; and fince the [illegible] hefion in Wood is every where variable [illegible] it is evident [illegible] can be no regular [illegible] actual Effect of the Wedge; but if we [illegible] Power of Cohefion in the Wood [illegible] form, or to make every where an equal [illegible] the Wedge A B C, dividing its [illegible] then the Power of the Wedge [illegible] ance of the Wood, as their [illegible] is, as the Spaces moved through [illegible] that is, as the *Height of the Wedge* [illegible] A H. But the Wedge being [illegible] Plane, is not really a *different Machine* [illegible] ufually reckoned as fuch.

Vol: I. P. 173.

Plate IX.



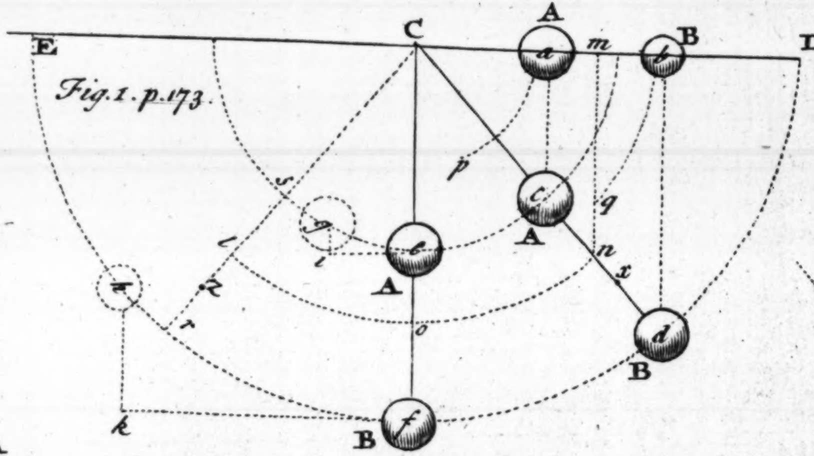


Fig. 1. p. 173.

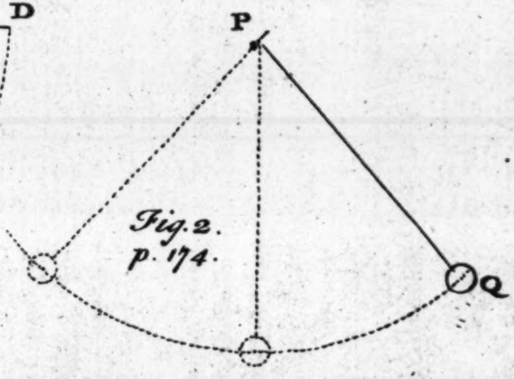


Fig. 2. p. 174.

Fig. 4. p. 177.

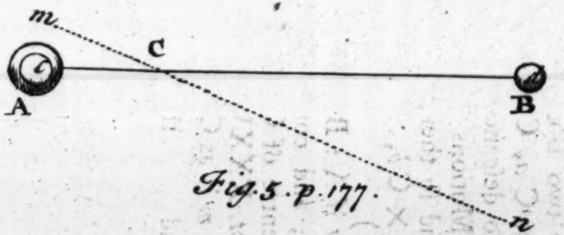


Fig. 5. p. 177.

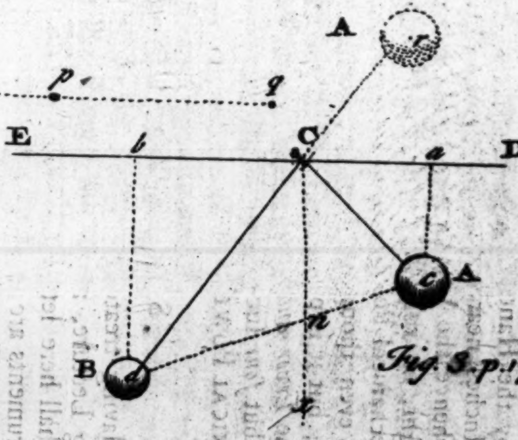


Fig. 3. p. 176.

23. In the SCREW A B, suppose the Distance of the Spiral Threads $\frac{1}{10}$ of an Inch, and the Length of the Lever F G = 16 $\frac{1}{2}$ Inches, then will the Circles described by the Hand at F be = 16,5 × 6,3 = 103,9, or 104 Inches nearly, that is, 1040 Tenths of an Inch; whence the Velocity of the Power is to that of the Weight, as 1040 to 1; therefore the Screw is a great Mechanical Power, either for raising Weights or Pressure, even though we allow the greatest Part for Friction; but it is properly a *Compound Power* consisting of the *Lever and Inclined Plane*; and therefore we can find but *four* such as we can properly call SIMPLE MECHANICAL POWERS. Pl. VIII. Fig. 12.

SCHOLIUM.

24. Having treated of the *Simple Pendulum* in the preceding Lecture, and of the Nature of the Lever in this; I shall here let the Reader see how nearly these two Instruments are allied, or rather shew him that the *Lever* is in reality nothing but a *Compound Pendulum*; from whence many Particulars concerning the Properties of each, not hitherto mentioned, will occur worthy of Observation.

25. Let C D be a fine Rod or Wire (whose Weight is inconsiderable) and moveable about the Point C. If A and B be two Bodies (of the same Kind) fixed at any Distance C a, C b on the Rod; and then the Rod permitted to descend freely; the Bodies A and B will begin their Motions with Velocities proportional to C a and C b, and so their respective *Momenta* will be A × C a and B × C b, in the Points a and b (by *Annot.* XXXVIII.) Pl. IX. Fig. 1.

26. Now if $A : B :: mb : ma$, then will $A \times am = B \times bm$; and consequently the Point m will be the common Centre of Gravity between the Bodies A and B (by *Annot.* XXXV.) Wherefore since the Velocity of the Point m is as C m, its *Momentum* will be as $Cm \times \overline{A+B}$; and from the Nature of the common Centre of Gravity, we have $A \times Ca + B \times Cb = Cm \times \overline{A+B}$; and therefore $\frac{A \times Ca + B \times Cb}{A+B} = Cm$; or

by

by putting $Ca = a$, $Cb = b$, and $Cm = d$; then $\frac{Aa + Bb}{A + B} = d$. Suppose the Moment the Rod CD began to descend, the Bodies A and B were to be disengaged from it in such a Manner as to descend upon it freely; then will they descend (not in the Curves *ap* and *bq*, but) in the Tangents *ac* and *bd*, describing the Spaces *ac*, *bd*, which will be as the Inceptive Velocities C *a*, C *b*; and of course when the Rod comes into the Situation C *d*, the Bodies will be at *c* and *d*; it being in this Case C *a* : C *b* : : *ac* : *bd* : : C *c* : C *d*. And *mn* will be the Line described by the common Centre of Gravity.

27. At *c* and *d*, let us suppose the Bodies again fix'd to the Rod, they will then constitute a *Compound Pendulum*, which will now vibrate forwards and backwards through the Angle *d* C *r*, the Body A describing the Arch *ces*, and B the Arch *dfr*, and the common Centre of Gravity *n* the Arch *nol*. As this Pendulum vibrates, the Velocities of the Bodies will be every where as their Distances, and greatest of all in the Perpendicular Situation C *ef*: Where the Velocities of the Points *e*, *o*, *f*, may still be represented by *a*, *d*, and *b*.

Plate IX. Fig. 2.

28. Now this *Compound Pendulum* is isochronous to (*i. e.* vibrates in the same Time with) some simple Pendulum of a determinate Length P Q. If then in the *Compound Pendulum* C *d* there be taken C *x* = P Q, the Point *x* is called the Centre of Oscillation in this *Compound Pendulum*; for if the Bodies A and B were there fixed, the Times of Vibration would be the same then as now: To determine this Point therefore, is to determine the Time of Vibration in any *Compound Pendulum* whatsoever.

29. In order to this; let us once more suppose the Moment the Bodies A and B come to the Points *e* and *f* in the Perpendicular, they were disengaged from the Rod, and were to ascend at the End of a String separately, with the Velocities they had acquired by their Descent in the Points *e* and *f*. Then the Altitudes *ig* and *kh*, to which they will ascend above the Horizontal Line *ei* and *fh*, will be as the Squares of the Velocities

locities at e and f; that is, as $a\,a$ and $b\,b$; as is evident from what has been elsewhere shewn (*Annotat.* XXVI. 2, 3.)

30. The Bodies A and B will in this Case arrive to the Points g and h before their Motion will be destroy'd. And it is plain that $e\,g$ is less than $e\,c$ or $e\,s$, because the Body A is retarded in coming from C to e by the Body B. Also $f\,h$ is greater than $d\,f$ or $f\,r$, because B has its Motion quickened by the Body A. And as when in passing from d to f, the perpendicular Spaces descended through are as the Velocities singly, and the *Momenta* as the Velocities and Masses of Matter conjointly; so on the other Side, in passing from f and e to h and g, the perpendicular Spaces ascended through are as the Squares of the initial Velocities, the *Momenta* will be as the Masses of Matter, and those perpendicular Spaces conjointly; that is, as A $a\,a$ and B $b\,b$.

31. Now in all Cases, the Sum of the *Momenta* divided by the Sum of the Masses gives a Quotient expressing the Velocity, or Ascent or Descent of the Centre of Gravity. (See *Art.* 26.) Therefore $\frac{A\,a\,a + B\,b\,b}{A + B}$ $= d =$ the Ascent of the Centre of Gravity; for the *Momentum* of this Centre of Gravity is on both Sides the Perpendicular Cf the same at equal Distances, and therefore it will always ascend or descend through the same perpendicular Heights.

32. Having thus obtained the Expression of the Descent of the Centre of Gravity, we have that for the Centre x of Oscillation of course, for we have As Cn : Cx : : d : x : : $\frac{A\,a\,a + B\,b\,b}{A + B}$: $\frac{A\,a\,a\,x + B\,b\,b\,x}{A\,d + B\,d}$ = Descent of the Centre of Oscillation. But the Descent is equal to the Ascent; for the Point x is actuated and moved in the same Manner as the Body Q, which in one Oscillation descends and ascends through equal perpendicular Spaces, and those Spaces are always as the Squares of the Velocities in the lowest Point; and therefore for the Point x, it will be as C$x^2 = x\,x$. Whence we have $\frac{A\,a\,a\,x + B\,b\,b\,x}{A\,d + B\,d} = x\,x$; and therefore

A $a\,a$

$\frac{Aaa + Bbb}{Ad + Bd} = x = Cx$, the Distance of the Centre of Oscillation from the Centre of Motion.

33. Because (in *Art.* 26.) we had $\frac{Aa + Bb}{AB} = d$; if we substitute this Equivalent for d in the above Equation, we have $\frac{Aaa + Bbb}{Aa + Bb} = x$; and therefore $Aaa + Bbb = Aax + Bbx$. Hence $Aaa - Aax = Bbb - Bbx$; and so $Aa : Bb :: b - x : x - a$; that is, $A \times Cc : B \times Cd :: xd : xc$. From whence it appears, *that the* Momenta *of the Bodies or their Power to move the Pendulum, are inversely proportional to their Distance from this Centre*; which is the true Definition of the Centre of Oscillation.

34. Therefore when $A : B :: b : d$, and so $Aa = Bb$; then also $b - x = x - a$, or $xd = xc$; or the Centre of Oscillation would bisect the Distance between the Bodies A and B.

35. If $a = o$, or the Body A be removed to the Centre C, then $Aaa = o$, and $Aa = o$; whence $x = \frac{Bbb}{Bb} = b$, that is, the Instrument then becomes a simple Pendulum C d.

Pl. IX. Fig. 3. 36. If the Bodies A and B are not placed both on one Rod, but on two Rods C c and C d, making an Angle c C d, being united in the Angle C. Then these Rods suspended on the Angle C will be the same *Compound Pendulum* as before, and the Centres of Gravity and Oscillation n and x, when the Bodies are at rest, will be both in the Perpendicular C x, as in the other Case, and express'd by the same Equations, *viz.* $Cn = d = \frac{Aa + Bb}{A + B}$; $Cx = x = \frac{Aaa + Bbb}{Aa + Bb}$. And it will oscillate in the same Time with the simple Pendulum P Q, when $Cx = PQ$.

37. If the Body A be supposed removed on the other Side the Line E D, its Distance $Cr = a$, is now negative, or it will be $-a$ in the Equations for the Centres; thus $d = \frac{-Aa + Bb}{A + B}$; and $x = \frac{Aaa + Bbb}{-Aa + Bb}$

because

because $-a \times -a = aa$. Hence in this Case 'tis very obvious that the *Compound Pendulum* is now become a *Lever*, whose *Fulcrum* is the Point C; and A may be consider'd as a Weight to be moved by the Power B. And hence because when Bb is greater than Aa, d will remain affirmative, therefore n the Centre of Gravity will be in the Part Cd, and so will cause B to preponderate.

38. On the contrary, if Aa be greater than Bb, then will d be negative, or the Centre of Gravity n will be in the Arm Cr, and cause the Weight A to descend. But when Aa = Bb, then $-\text{A}a + \text{B}b = o$, and $d = \frac{o}{\text{A}+\text{B}} = o$, that is, n or the Centre of Gravity will fall upon the Point C, and produce an *Equilibrium*; and A : B :: b : a :: Cd : Cr. As was before shewn to be the Property of the Lever. (See *Annot.* XXXVIII).

39. Thus also, with respect to the Centre of Oscillation, $x = \frac{\text{A}aa + \text{B}bb}{-\text{A}a + \text{B}b}$; 'tis plain, in case of an *Equilibrium* where Aa = Bb, we have $x = \frac{\text{A}aa + \text{B}bb}{o}$ = Infinite; for since the Lever in this Case has no Motion, it can only be isochronous to a simple Pendulum of an infinite Length, whose Times of Vibration are infinitely great, and its Motions, of Course, not sensible. Plate IX.

40. It is farther evident, that the Centres of Oscillation and Gravity are both on the same Side of the Centre of Motion C; and that when Bb is greater than Aa, the Centre of Oscillation will be somewhere on the Side of B in the Line Cd continued out; if Bb exceeds An but a little, it will be at a great Distance, as at q; if the Excess be greater, it will be nearer, as at p. If it be very great, the Centres will be very near, as at o; and when Bb is infinite with respect to Aa, then the said Centre will be in the Centre of the Body B. Fig. 4.

41. Let the Lever cCd be *in Equilibrio* with the Weights A and B; and let it be required to raise the Weight A from c to m, by the Power B; then if Bb Fig. 5.

be ever so little greater than A a it will descend, till the Lever comes to the Situation $m\,n$, as required. The whole Action of the Machine consists in raising the given Weight A to a given Height H, in a given Time; and this Effect of the Machine will (*cæteris paribus*) be greater as the Time is less. But the Time is the Time of half the Vibration of the Lever; and the Times of Vibrations are less as the Intensity of the Power B is increased, while its Distance continues the same, because the Distance of the Centre of Oscillation is diminished.

42. Now putting A = Body to be raised, H = Height, and T = Time, then $\frac{AH}{T} = E =$ the Effect, or Intensity of the Power B; whence $AH = ET =$ whole Action of the Machine; and therefore when ET is least of all, the Machine will be in its greatest Perfection; because as E increases, T will decrease, and though it be not in the same Proportion, yet it will cause that ET will become a *Minimum* at a certain Limit; as will be evident by the following Example.

43. Suppose A = 100 *lb.* and B = 10 *lb.*; and if $Cc : Cd :: 10 : 100$; then there will be an *Equilibrium*, in which Case T will be infinite, and therefore also TE, and consequently cannot be express'd in Numbers.

44. If B = 11 *lb.* then $\frac{Aaa + Bbb}{-Aa + Bb} = \frac{100 + 1100}{-100 + 110} = \frac{1200}{10} = 120 = x$, the Distance of the Centre of Oscillation, the Square Root of which 10,95 will be proportional to the Time (T) of Vibration, (see *Annotat.* XXIX.) This multiplied by E = 11, the Intensity of the Power, gives 120,45 for the Expression of the Action of the Lever in this particular Case.

45. If B = 15 *lb.* then, as before, we shall find $x = 32$, whose Square Root $5{,}66 \times 15$ $(= T \times E)$ $= 84{,}9$ the Lever, which is now less than before. Again, suppose B or E = 20 *lb.* then we shall have $x = 21$, and its Square Root 4,6; then $4{,}6 \times 20 = T \times E = 92$, which is again greater than the last;

the

Quantity of Surface, but *to the Weight* of the incumbent Part; as we ſhall alſo ſhew by Experiment (XXXIX).

As

the limit then is between 11 *lb.* and 20 *lb.* and is very near 15 *lb.* as will appear by the following Table; where the Numbers in the ſecond Column expreſs the whole Action of the Lever in Parts, of which the leaſt contains 100000, all which correſponds to the Intenſities of the Power increaſed from 10 to 20 *lb.*

Powers.	*Actions.*	*Powers.*	*Actions.*
10 ——	Infinite.	15,16 ——	100000
11 ——	142360	16 ——	100368
12 ——	114036	17 ——	101611
13 ——	104677	18 ——	103397
14 ——	101053	19 ——	105575
15 ——	100016	20 ——	108330

46. This Conſideration of the Power is but of little Uſe in the Lever; but as that is the moſt ſimple Machine, it was beſt adapted to explain and exemplify this Doctrine. In the Axis *in Peritrochio*, and Pulley, it is of greater Uſe, and ſhould not be neglected; but as it is there a Buſineſs of great Difficulty and Labour, I ſhall refer the Reader to *S'Graveſande*'s third Edition of his *Principia*, where he may ſee the Computations at large; the Reſult of which in general is, *That the Power which ſuſtains the Weight* in Equilibrio *ſhould be increaſed by one half in the Pulley and Axis* in Peritrochio, *that the Action of thoſe Machines may be a* Minimum.

(XXXIX) The Doctrine of FRICTION is contain'd under the following Particulars, *viz.*

1. When one Body inſiſts on another upon an horizontal Plane, it preſſes it with all its Weight, which being equally re-acted on (and therefore the whole Effect of its Gravity deſtroy'd) by the Plane, it will be abſolutely free to move in any lateral or horizontal Direction by any the leaſt Power applied thereto, provided both the touching Surfaces be perfectly ſmooth or even.

 2. But

As to *Compound Engines and Machines*, they are as *numerous*, as they are *various* in

2. But ſince we find no ſuch Thing as perfect Politure or Evenneſs in the Surfaces of Bodies (at leaſt ſuch as are produced by Art) but an evident Roughneſs or Unevenneſs of the Parts in the Surface, ariſing from the Poroſity and peculiar Texture of the Body, it is eaſy to underſtand that when two ſuch Surfaces come together the prominent Parts of the one will in ſome Meaſure fall into the concave Parts of the other, and therefore, when an horizontal Motion is attempted in one, the fix'd prominent Parts of the other will give more or leſs Reſiſtance to the moving Surface by holding and detaining its Parts, which is what we call *Friction*.

3. Now ſince any Body will require a Force proportional to its Weight to draw it over a given Obſtacle, it follows that the Friction ariſing to the moving Body will always be in proportion to its Weight only, and not the Quantity of the Surface, by which it bears upon the reſiſting Plane or Surface. Thus if a Piece of Wood four Inches wide, and one Inch thick be ground, and thereby made exactly fit to the Surface of another fix'd Piece of the ſame Wood, it will require the ſame Weight to draw it along on the ſame, whether it be laid on its broad or narrow Side.

4. For though on the broad Side there be four Times the Number of touching Particles (*cæteris paribus*) yet each Particle is preſs'd with but $\frac{1}{4}$ of the Weight that thoſe are on the narrow Side; and ſince four Times the Number multiplied by $\frac{1}{4}$ of the Weight, is equal to $\frac{1}{4}$ of the Number multiplied by four Times the Weight, 'tis plain the Effect, that is, the Reſiſtance, is equal in both Caſes, and ſo requires the ſame Force to overcome it.

5. The Reaſon why the Friction is proportional to the Weight of the moving Body, is, becauſe the Power applied to move the Body muſt *raiſe* it in ſome Meaſure *upon and over* the prominent Parts of the Surface on which it is drawn; and this Motion of the Body, as it is

Plate X.

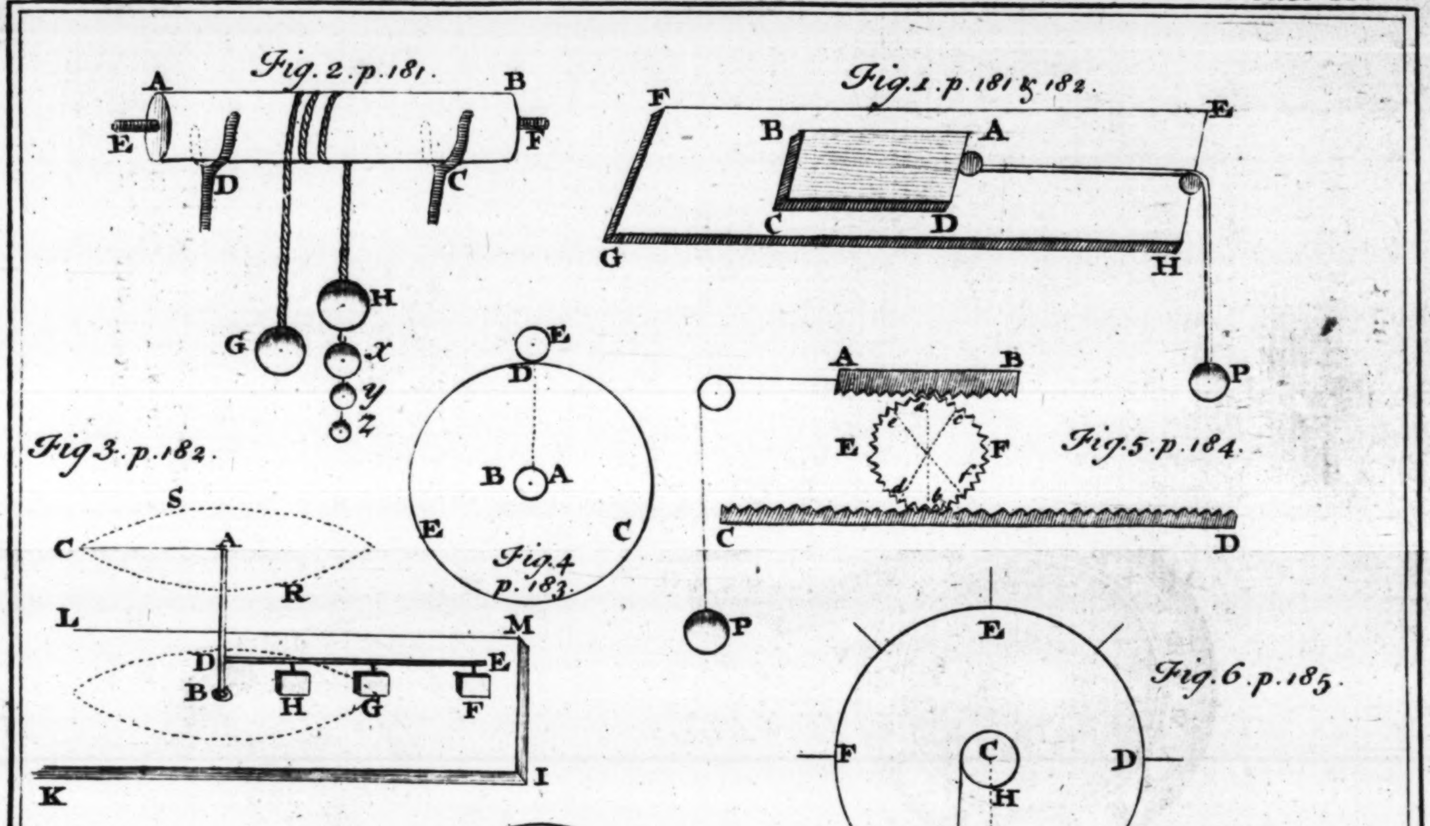
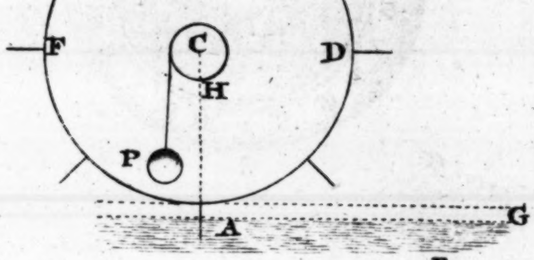


Fig. 2. p. 181.
Fig. 1. p. 181 & 182.
Fig 3. p. 182.
Fig. 4 p. 183.
Fig. 5. p. 184.
Fig. 6. p. 185.
A
B
C
D
E
F
G
H
x
y
z
P
S
R
L
M
I
K



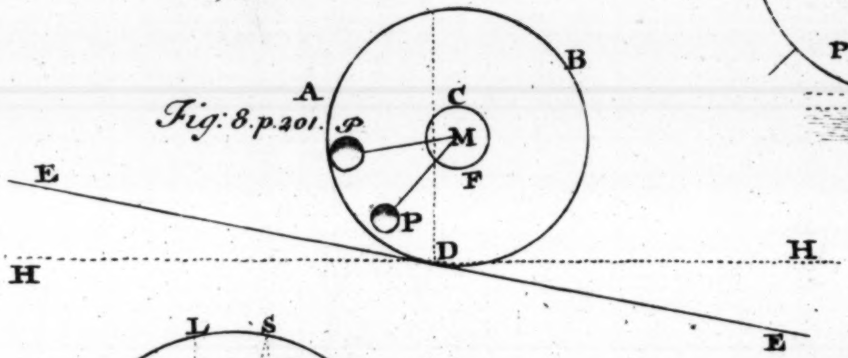


Fig: 8. p. 201.

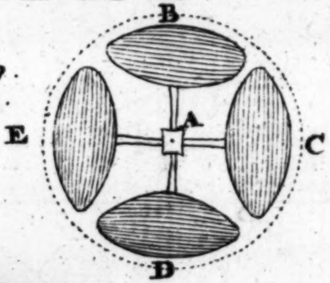


Fig. 9. p. 223.

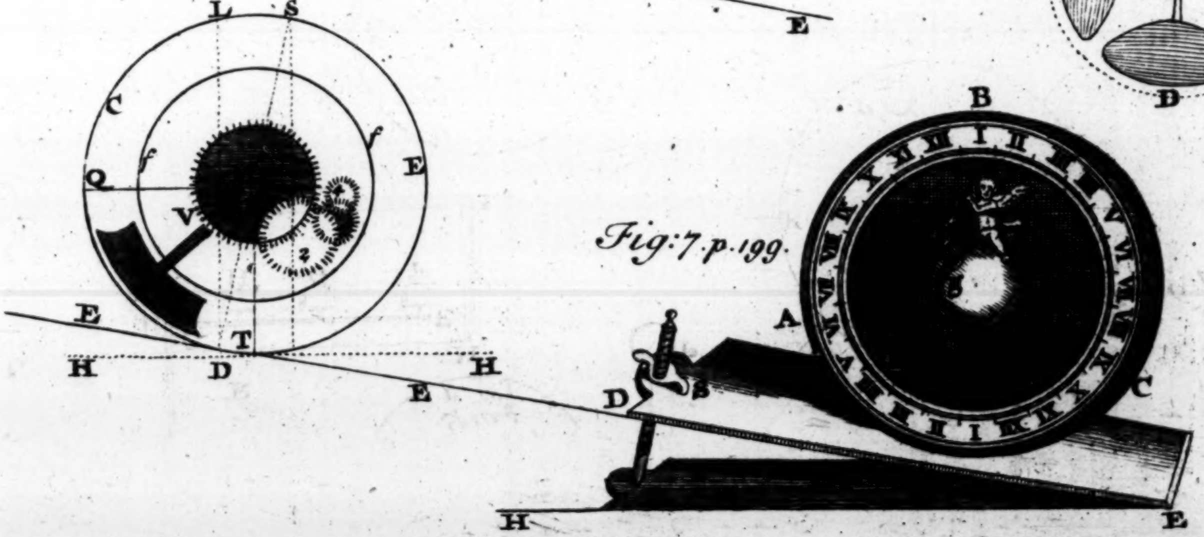


Fig: 7. p. 199.

in their Structure; concerning all which we may obſerve in general, that they con-
ſiſt

is not upright, ſo it will not require a Power equal to its whole Weight; but being in the Nature of the Motion on an *Inclined Plane*, (ſince the Body bears on the prominent Parts all the while) the Power which moves it will be proportional to but *a Part of its Weight only*; and this will vary with the various Degrees of Smoothneſs or Aſperity between the rubbing Surfaces, and the other concurring Circumſtances.

6. I find by Experiment, that a Body ABCD (of Wood, Braſs, *&c.*) laid on the Surface EFGH, will be drawn along by a Weight P, nearly equal to *one third* of its own Weight; if the Surfaces be hard and well poliſh'd, it will be leſs than a *third Part*; but if the Parts be ſoft or rugged, it will be much greater. Thus alſo the Cylinder of Wood AB, if very ſmooth, and laid on two well-poliſh'd Supporters C, D, (having been firſt oil'd or greas'd) and then charged with the Weight of two Pounds in the two equal Balls G, H, it will require an additional Weight x (equal to about a third Part of the two Pounds) to give Motion to, or overcome the Friction of the ſaid Cylinder. Pl. X. Fig. 1. Pl. X. Fig. 2.

7. Now this additional Weight, as it cauſes a greater Preſſure of the Cylinder, will likewiſe encreaſe the Friction, and therefore require the Addition of another Weight y, equal to the third Part of its own; for the ſame Reaſon the Weight y will require another z, a third Part leſs; and ſo on *ad infinitum*. Hence upon Suppoſition that the Friction is preciſely equal to a *third of the Weight*, the firſt Weight with all the additional ones, *viz.* 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, *&c.* will be a Series of Numbers in Geometrical Progreſſion decreaſing. Now the Sum of all thoſe Terms, except the firſt (*i. e.* the Sum of all the infinite Number of additional Weights $x + y + z$, *&c.*) is found (by a well known Theorem in Arithmetic) to be equal to *one Pound*. So that if the Weight of the Cylinder be inconſiderable, the Way to

sist of one, two, or more of the Simple Powers combined together; that in most of

overcome the Friction would be to double the Power G, or H at once.

8. But though we may at a Medium allow about a *third Part* of the Weight with which any simple Machine is charged from the Friction arising from thence, yet this is very precarious and seldom the Case; for if
Pl. X. Fig. 1. ABCD be a Piece of Brass of six Ounces, and EFGH be also a Plate of Brass, and both the Surfaces well ground and polish'd, the Weight P of near two Ounces will be requir'd to draw along the Body AC alone; but if AC be loaded with 6, 8, or 10 *lb.* then a sixth Part of the Weight will be sufficient to draw it along the Plane. If the Plane be covered with a linen or woollen Cloth, then a third, or half Part, and sometimes more, will be requisite to draw it along on the Plane.

9. Yet notwithstanding the Difficulty and Uncertainty attending the Estimation of the Quantity of Friction, 'tis still a most useful and necessary Enquiry how, and by what Means the Friction of any Machine may be abated or diminished. In order to this we must consider it mechanically, that is, we must consider Friction as a Force acting against a Power applied to overcome it. Thus suppose AB an upright Stem or Shaft turning
Fig. 3. freely in the Socket B fix'd in the Table or Plane IKLM; and AC, DE two Arms fix'd in the said Shaft, the latter of which DE has three Pins going into a Socket in the Middle of a heavy Weight at F, G, or H, in such a Manner, that when a Power applied at C moves the Lever AC, it causes the Lever DE to protrude or thrust along the Weight at F, G, or H, in a circular Manner upon the Table.

10. Now since we suppose the Weight all the while it is in Motion is freely and wholly supported by the Plane, it follows that all the Resistance it can give to the Power applied at C, is only what arises from its Friction on the Plane. What this Friction is, will be found

of them the *Axis in Peritrochio*, the *Lever*, and the *Screw*, are the conſtituent Parts; that

found by applying the Weight at G, ſo that BG be equal to AC; for then the Power applied to C, acting in a Tangent to the Circle CRS, that ſhall juſt move the Weight G, will be equal to its Friction (by *Annot.* XXXVIII. 10.) But if the Weight be applied at F, becauſe BF is greater than AC, the ſame Power at C, as before, will not move it, becauſe here its Force is increaſed by having a greater Velocity than the Power; as on the other Hand, if placed at H, a leſs Power at C ſhall move it, becauſe of its having there leſs Velocity than the Power. All which is plain from the Properties of the Lever, demonſtrated in *Annotat.* XXXVIII.

11. Hence we underſtand, that though the Weight of a Machine remains the ſame, yet the Friction may be diminiſhed by contriving that the Parts on which it moves and rubs ſhall have leſs Velocity than the Power which moves it. Thus if the Cylinder AB were to move on the two ſmall Pins or Gudgeons E, F, the Friction would be abated in the ſame Proportion as the Diameter of thoſe Gudgeons is leſs than the Diameter of the Cylinder.

12. The Friction on theſe Gudgeons is ſtill farther diminiſh'd by cauſing them to move on the Circumference of a Wheel; thus let F be the Gudgeon of the Cylinder revolving on the Wheel CDE; the Velocity of the Wheel's Circumference will be the ſame with that of the Gudgeon; but the Velocity of the Wheel's Axis AB (which is now to be conſider'd as the rubbing Part) is leſs than that of the Wheel in Proportion as its Diameter is leſs than that of the Wheel. For Example, if the Friction of the Cylinder moving on its Surface be $\frac{1}{3}$ Part of the Weight, and the Gudgeons be to the Cylinder as 1 to 10, they will reduce the Friction to $\frac{1}{30}$ Part; and if again the Axis of the Wheel be to the Wheel as 1 to 10, the Wheel will reduce it to $\frac{1}{300}$ Part; and if the Axis of this Wheel be laid on the

Pl. X. Fig. 4.

sist of one, two, or more of the Simple Powers combined together; that in most of

overcome the Friction would be to double the Power G, or H at once.

8. But though we may at a Medium allow about a *third Part* of the Weight with which any simple Machine is charged from the Friction arising from thence, yet this is very precarious and seldom the Case; for if
Pl. X. Fig. 1. ABCD be a Piece of Brass of six Ounces, and EFGH be also a Plate of Brass, and both the Surfaces well ground and polish'd, the Weight P of near two Ounces will be requir'd to draw along the Body AC alone; but if AC be loaded with 6, 8, or 10 *lb.* then a sixth Part of the Weight will be sufficient to draw it along the Plane. If the Plane be covered with a linen or woollen Cloth, then a third, or half Part, and sometimes more, will be requisite to draw it along on the Plane.

9. Yet notwithstanding the Difficulty and Uncertainty attending the Estimation of the Quantity of Friction, 'tis still a most useful and necessary Enquiry how, and by what Means the Friction of any Machine may be abated or diminished. In order to this we must consider it mechanically, that is, we must consider Friction as a Force acting against a Power applied to overcome it. Thus suppose AB an upright Stem or Shaft turn-
Fig. 3. ing freely in the Socket B fix'd in the Table or Plane IKLM; and AC, DE two Arms fix'd in the said Shaft, the latter of which DE has three Pins going into a Socket in the Middle of a heavy Weight at F, G, or H, in such a Manner, that when a Power applied at C moves the Lever AC, it causes the Lever DE to protrude or thrust along the Weight at F, G, or H, in a circular Manner upon the Table.

10. Now since we suppose the Weight all the while it is in Motion is freely and wholly supported by the Plane, it follows that all the Resistance it can give to the Power applied at C, is only what arises from its Friction on the Plane. What this Friction is, will be found

3

of them the *Axis in Peritrochio*, the *Lever*, and the *Screw*, are the conſtituent Parts; that

found by applying the Weight at G, ſo that BG be equal to AC; for then the Power applied to C, acting in a Tangent to the Circle CRS, that ſhall juſt move the Weight G, will be equal to its Friction (by *Annot.* XXXVIII. 10.) But if the Weight be applied at F, becauſe BF is greater than AC, the ſame Power at C, as before, will not move it, becauſe here its Force is increaſed by having a greater Velocity than the Power; as on the other Hand, if placed at H, a leſs Power at C ſhall move it, becauſe of its having there leſs Velocity than the Power. All which is plain from the Properties of the Lever, demonſtrated in *Annotat.* XXXVIII.

11. Hence we underſtand, that though the Weight of a Machine remains the ſame, yet the Friction may be diminiſhed by contriving that the Parts on which it moves and rubs ſhall have leſs Velocity than the Power which moves it. Thus if the Cylinder AB were to move on the two ſmall Pins or Gudgeons E, F, the Friction would be abated in the ſame Proportion as the Diameter of thoſe Gudgeons is leſs than the Diameter of the Cylinder.

12. The Friction on theſe Gudgeons is ſtill farther diminiſh'd by cauſing them to move on the Circumference of a Wheel; thus let F be the Gudgeon of the Cylinder revolving on the Wheel CDE; the Velocity of the Wheel's Circumference will be the ſame with that of the Gudgeon; but the Velocity of the Wheel's Pl. X.
Axis AB (which is now to be conſider'd as the rubbing Fig. 4.
Part) is leſs than that of the Wheel in Proportion as its Diameter is leſs than that of the Wheel. For Example, if the Friction of the Cylinder moving on its Surface be $\frac{1}{3}$ Part of the Weight, and the Gudgeons be to the Cylinder as 1 to 10, they will reduce the Friction to $\frac{1}{30}$ Part; and if again the Axis of the Wheel be to the Wheel as 1 to 10, the Wheel will reduce it to $\frac{1}{300}$ Part; and if the Axis of this Wheel be laid on the

that in all, a certain Power is applied to produce an Effect of much greater Moment; and that, in the last Place, it is known, that the greatest Effect or Perfection of the Machine is then, when it is set to work with *four Ninths* of that Charge which

Perimeter of another Wheel, the Friction will be reduced to a still lesser Part of the Weight, and so you may proceed to diminish the Friction *ad infinitum*. Hence Wheels applied in this Manner are called FRICTION-WHEELS.

13. Besides what we have now premised, somewhat farther is necessary to be understood to diminish Friction by *Wheel-Carriages*. It was before observ'd that Friction arose chiefly by lifting the Body over the prominent Parts of the Plane on which it moved; now if we can

Pl. X. contrive to move the Body along without lifting or sustaining its Weight, we shall move it without much Friction; and this may be done by laying the Body on any moveable circular Subject, as Rollers, Wheels, &c.

Fig. 5. Thus let A B be the Section of an heavy Body laid on a Roller E F, upon the Plane C D, and drawn by the Power P; 'tis evident when A B moves, the Asperity of its Surface will lay hold on that of the Roller, and move it likewise; and 'tis as plain, that when the Body A B is drawn against the prominent Parts of the Roller, they immediately give way, and make no Resistance; thus the perpendicular Diameter *a b* yields into the Situation *e f*, and *c d* succeeds in its Place. By this circular Motion of the Roller, its prominent Parts below do only descend, and move upon, or over, and are not drawn against the fix'd prominent Parts of the Plane, and so receive no Resistance from them; hence the Body A B is convey'd along without being lifted up, in the Manner as a Wheel is moved by a Pinion, without any considerable Resistance. And this is the true Foundation of the *Theory of Wheels*, or Doctrine of *Wheel-Carriages*.

which is equivalent to the Power, or will but just keep the Machine *in Equilibrio* (XL).

THE

(XL) 1. To demonstrate this Proposition, I shall chuse a Water-Wheel A D E F, driven round by a Current of Water G A, striking the lower Float-Boards A in a perpendicular Direction, in the Manner of an undershot Mill. Now if the Wheel be not loaded or charged with any Weight, but moves freely on the Gudgeons of its Axis C, then the Water, coming on the Floats, will put the Wheel in Motion, and acting upon it continually will soon accelerate its Motion so far, as to give it a Velocity equal to its own. Plate X, Fig. 6.

2. But if the Axle of the Wheel C be charged with a Weight P, which it is obliged to raise, this will give Resistance to the Wheel, and diminish its Velocity, or cause it to move slower than the Water; as the Weight P is increased, the Motion of the Wheel will be proportionably retarded; till the Weight P, coming to have an equal *Momentum* with the Water, the Wheel will lose all its Motion, or be reduced to a State of *Equilibrium*.

3. Now let F = Force of the Water, V = its Velocity, v = Velocity of the Wheel, P = Weight that holds the Wheel *in Equilibrio*, z = Weight raised by the Wheel in Motion. Then the Difference of those Velocities, *viz.* $V - v$ will be that with which the Water strikes the Wheel; and since the Force of Striking Fluids is always as the Square of the Velocity, (as will hereafter be shewn) and Causes are proportional to their Effects, we shall have $\overline{V-v}^2$ always proportional to z; and when $v = o$, we shall have $z = P$; and then V^2 will be as P, so that it will always be $V^2 : P :: \overline{V-v}^2 : z$; and so $\frac{V\sqrt{z}}{\sqrt{P}} = V - v$ and $v = \frac{V\sqrt{P} - V\sqrt{z}}{\sqrt{P}}$.

4. But

THE *Common* JACK is a compound Engine, where the Weight is the Power applied;

4. But from the Principles of Mechanics, we have the Radius of the Wheel AC ($=R$) to the Radius of the Axle CB ($=r$) as the Velocity of the Wheel v to the Velocity of the Weight z; that is, $R : r ::$ $\frac{V\sqrt{P}-V\sqrt{z}}{\sqrt{P}} : \frac{rV}{R}\times\frac{\sqrt{P}-\sqrt{z}}{\sqrt{P}}$ = Velocity of z.

Which Velocity of the Weight being multiplied by the Weight z, gives an Expression $\frac{rV}{R}\times\frac{z\sqrt{P}-z\sqrt{z}}{\sqrt{P}}$ for the Effect of the Engine.

5. This Expression is to be determined to a *Maximum* by making the Fluxion of $\frac{z\sqrt{P}-z\sqrt{z}}{\sqrt{P}}$ equal to Nothing, (the Part $\frac{rV}{R}$ being constant, we neglect.) Let $\frac{z\sqrt{P}-z\sqrt{z}}{\sqrt{P}}=y$, then $z\sqrt{P}-z\sqrt{z}=z\times P^{\frac{1}{2}}-z^{\frac{3}{2}}=y\times P^{\frac{1}{2}}=y$, (because $P^{\frac{1}{2}}$ is constant) therefore $\dot{z}P^{\frac{1}{2}}-\frac{3}{2}z^{\frac{1}{2}}\dot{z}=\dot{y}=0$; hence $\dot{z}P^{\frac{1}{2}}=\frac{3}{2}z^{\frac{1}{2}}\dot{z}$, therefore $P^{\frac{1}{2}}=\frac{3}{2}z^{\frac{1}{2}}$; whence $P=\frac{9}{4}z$, and $z=\frac{4}{9}P$; that is, the Weight z, when the Machine is in its greatest Perfection, is equal to $\frac{4}{9}$ of the Weight P, that will keep it *in Equilibrio*. *Q. E. D.*

6. Hence, if in the Expression $v=\frac{V\sqrt{P}-V\sqrt{z}}{\sqrt{P}}$ we substitute the Value of z, *viz.* $\frac{4}{9}P$, we shall have $v=\frac{V\sqrt{P}-V\sqrt{\frac{4}{9}P}}{\sqrt{P}}$, or $v\sqrt{P}=V\sqrt{P}-V\sqrt{\frac{4}{9}P}=V\times\frac{1}{3}\sqrt{P}$, therefore $v=\frac{1}{3}V$. That is, the Velocity of the Wheel (v) is equal to the *Third Part* of the Velocity of the Water, when the Machine is in the greatest Perfection.

7. When

plied; the Friction of the Parts, and the Weight with which the Spit is charged, is the Force to be overcome; and a steady, uniform Motion, by Means of the Fly, is the

7. When the Wheel is kept *in Equilibrio* by the Weight P, we have $F : P :: CB\ (=r) : CA\ (=R)$ by the Principles of Mechanics; whence we have $\frac{FR}{r} = P$; whence $z = (\frac{4}{9}P =)\ \frac{4}{9}\frac{FR}{r}$; which Values of z and P, substituted in the Expression $\frac{rV}{R} \times \frac{z\sqrt{P} - z\sqrt{z}}{\sqrt{P}}$ will give $\frac{4}{27}VF$ for the Exponent of the greatest Effect of the Engine.

8. To illustrate this by Example; suppose G A a Body of Water issuing from an Aperture in the Pen-Stock *one Foot Square*, and four Feet below the Surface of the Water-Head; in this Case the Force of the Water $F = 250$ *lb.* and the Velocity $V = 16$ Feet *per* Second (as will appear hereafter); whence $\frac{4}{27}VF = \frac{4 \times 250 \times 16}{27} = 592{,}6$, and this is the greatest Effect of the Engine; now if $R : r :: 8 : 1$; then $P = \frac{FR}{r} = 2000$; and $z = \frac{4}{9}P = 888{,}9$ *lb.* And its Velocity will be $\frac{1}{8}$ of $\frac{1}{3}$ of 16, or $\frac{2}{3}$ of a Foot *per* Second. To prove the Truth of all that has been said, one need only take z either greater or lesser than $888{,}9$ *lb.* and then the Expression for the Effect of the Engine $\frac{rVz}{R} \times \frac{\sqrt{P} - \sqrt{z}}{\sqrt{P}}$, will give a Number in each Case less than $592{,}6$; which therefore is the *Maximum*, as above found.

the End or Intention of the Machine (XLI).

In

(XLI) 1. The FLY is the only Part I ſhall here take Notice of in the Chimney-Jack, and ſhall explain its Nature and Uſe both there and in other Mechanical Machines. For it may be applied to any Sort of Engines to very good Purpoſes, which have a quick and circular Motion, and where the Power or Reſiſtance acts unequally in the different Parts of a Revolution.

2. The Uſe of the Fly is to facilitate the Motion of Engines, by accumulating and retaining the Power communicated to it, and exerting it gradually and equally in each Revolution of the Machine; whence it comes to paſs that the Motion of the Machine is rendered very nearly uniform and of an equal Tenour in all Parts of the Revolution, and therefore more eaſy, pleaſant, and convenient to be acted by the impelling Power. For neither the Strength of Men, nor other Powers or Reſiſtances affecting the Engine, can or do act equally and uniformly in every Part of a Revolution. Thus Meat on a Spit gives always more Reſiſtance on one Part than another, by which Means the Motion would be ſo irregular and jolting, that it would ſoon become ſhaken and looſed from the Spit, and ſo not be carried round, were not this Irregularity prevented by the Fly.

3. The Fly does not add any new Power to an Engine, as ſome have imagined; as is evident for the following Reaſons. (1.) The Fly has no Motion but what it receives at firſt from the Machine. (2.) A Degree of Force is always neceſſary to maintain the Motion of the Fly, which muſt be ſupplied from the Machine. (3.) The Friction of the Pivot, Screw, *&c.* of the Fly is a Reſiſtance to the impreſs'd Force, and muſt abate it. (4.) The Air likewiſe makes Reſiſtance to the Weights at the End of the Fly. Upon all which Accounts it is eaſy to underſtand, that the Fly, inſtead of adding, does very much decreaſe or leſſen the Power impreſſed on the Machine.

4. The

IN CLOCKS, WATCHES, *&c.* the Power is the Weight or Spring; the Force to

4. The best Form for a Fly is that of a heavy Wheel or Circle, of a fit Size; for this will meet with less Resistance from the Air; and being continuous, and the Weight ever where equally distributed through the Perimeter of the Wheel, the Motion will be more easy, equable, and regular. In this Form the Fly is most aptly applied to the perpendicular Drill, where it not only gives Weight and Regularity of Motion, but contributes to keep the Drill upright by its Centrifugal Power.

5. In this Form it is also best applied to a *Windlass* or common *Winch*, where the Motion is pretty quick; for when a Man turns the bended Handle of the Winch, his Strength is not, nor can be equally exerted in every Part of the Revolution; for in pulling upwards from the lower Quarter, he can exercise more Power than in thrusting forward in the upper Quarter, where, of course, Part of his former Force would be lost, were it not accumulated and conserved in the equable Motion of the Fly. By this Means a Man may work all Day in drawing up a Weight of 40 *lb.* whereas 30 *lb.* would create him more Labour in a Day without the Fly.

6. The Fly is sometimes made use of to increase the Force of Mechanical Engines, as of the Lever and Screw, in that for *Stamping* of Money in the Mint. Here the Power exerted by the Man is accumulated by the large Weights at each End of the Lever; the Lever increases and communicates it to the Cylinder, upon which it is fixed; the Screw does again increase or condense it upon the Medal, by Means of which the Impression of the Image is made.

7. The great Power of this (or any other Machine of this Kind) may be thus computed. Suppose the Arms of the Fly 15 Inches each, or the Length of the Lever 30 Inches, and each Weight to be 50 *lb.* and the Diameter of the Cylinder at the Screw to be *one Inch.* If each Stroke be made in a Half-Circle, which will be four Feet, and in half a Second of Time, the Velocity (being

to be overcome is the Friction of the numerous Parts, which are chiefly a Combination of Wheels and Axles, whose Use is to divide a large Portion of Time, as a *Day* or *Hour*, into very minute equal Parts, as *Minutes*, *Seconds*, &c. and to point out those Divisions by an equable Motion of a proper Hand or Index round a graduated Circle (XLII).

IN

ing equably accelerated by the continued Action of the Man) will at the Instant of the Stroke be at the Rate of eight Feet *per* Second; therefore 8×100 *lb.* $= 800$ *lb.* for the *Momentum* of the Lever or Fly. But since the Diameter of the Screw is but $\frac{1}{30}$ of the Length of the Lever, we shall have 30×800 *lb.* $= 24000$ *lb.* for the *Momentum* of the Engine, which is equal to that of 100 *lb.* falling 120 Feet; which prodigious Force is still farther increased for coining larger Pieces of Money, by increasing the Weights and Arms of the Fly.

(XLII) 1. I shall here shew the Principles of Watch and Clock-Work, and how to calculate the Numbers for the Movements, in as clear and compendious a Manner as I possibly can. In all *Automata*, or Machines of Clock-Work, there is a natural Agent or Principle of Motion; which, by acting on one Part, gives Motion to that, and all the other Parts depending upon it, and thus becomes the *Primum Mobile*, or first Mover to the whole Machine.

2. In common *Clocks* and *Watches* this is of two Sorts, *viz.* A SPRING or a WEIGHT; either of which may be made to act with any determinate Force; the Spring by its *Elasticity*, and the Weight by its *Gravity*. In these Machines this Force is required to be such as will overcome the *Vis Inertiæ* and Friction of all the Parts in Motion; which in Watches is very inconsiderable, but in Clocks is much greater, and that in proportion as they are more compounded.

3. The

to be overcome and
ngerous Parts, wh
nature of W
to divide a large
or Ha
Mamma,
Division by
Hand or instr
(M F.)

M
at the par
Mamma
Law, at the
Mamma,
too failing
farther
MILK

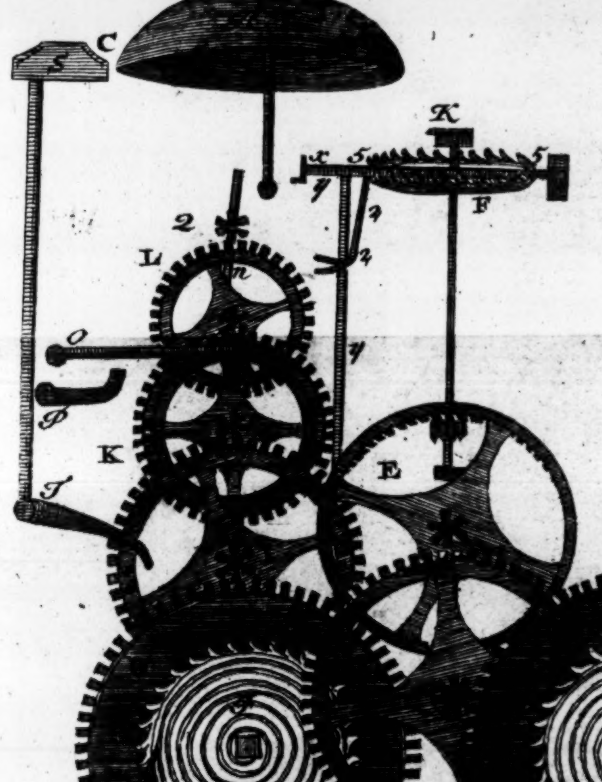


Fig.1.p.191.

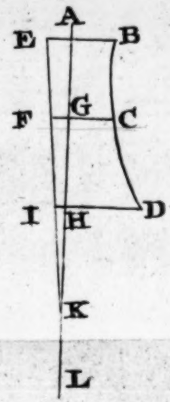


Fig.2.p.196.

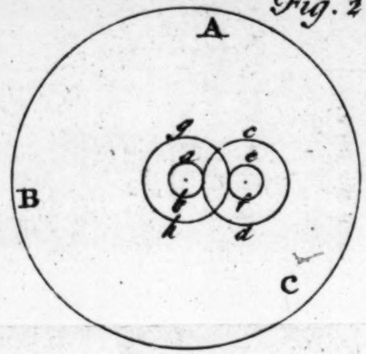
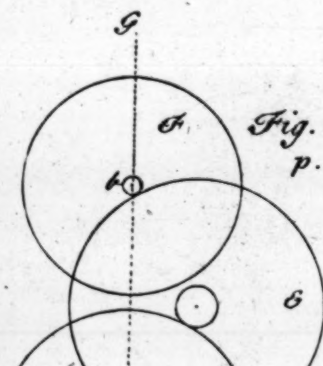


Fig.4.
p.198.



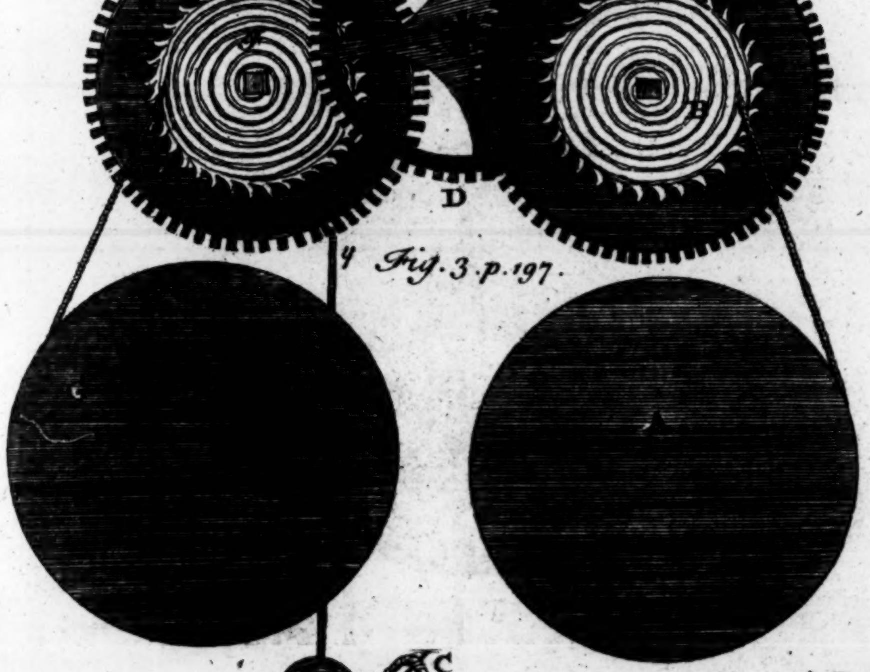


Fig. 3. p. 197.

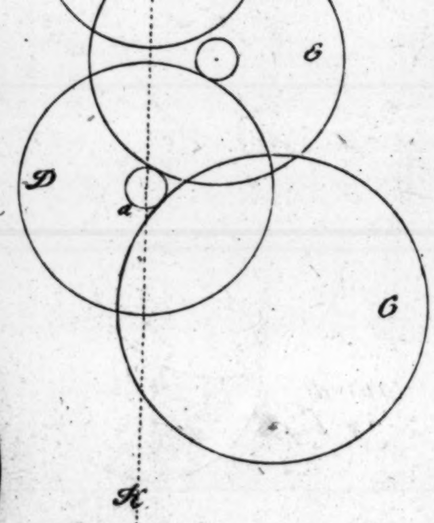


Fig. 6. p. 234.

Fig. 5. p. 234.

3. The Manner in which a Weight acts upon the Cylinder, about which the Line or Cord (to which it hangs) is wound, is easy to be understood by all; but the Action of the Spring coil'd up within the Cylindric Barrel or Box of a Clock or Watch, is somewhat more nice and mysterious; and the Manner how it acts upon the Fusee always with an equal Force by means of the Chain, and the proper Figure of the Fusee for that Purpose, is next to be explain'd.

4. The Chain being fix'd at one End to the Fusee, and at the other to the Barrel, when the Machine is winding up the Fusee is turn'd round, and of course the Barrel; on the Inside of the Barrel is fix'd one End of the Spring, the other End being fix'd to an immoveable Axis in the Centre. As the Barrel moves round, it coils the Spring several Times about the Axis, thereby increasing its elastic Force to a proper Degree; all this while the Chain is drawn off the Barrel upon the Fusee; and then when the Instrument is wound up, the Spring by its elastic Force, endeavouring constantly to unbend itself, acts upon the Barrel, by carrying it round; by which means the Chain is drawn off from the Fusee, and thus turns the Fusee, and consequently the whole Machinery.

5. Now as the Spring unbends by Degrees, its elastic Force, by which it affects the Fusee, will gradually decrease; and therefore unless there were some Mechanical Contrivance in the Figure of the Superficies of the Fusee to cause that as the Spring is weaker, the Chain shall be removed farther from the Centre of the Fusee, so that what is lost in the Spring's Elasticity is gained in the Length of the Lever; I say, unless it were for this Contrivance the Spring's Force would always be unequal upon the Fusee, and so would produce an unequable Motion of the Parts of the Machine.

6. The Figure of the Curve, which shall form the Superficies of the Fusee by a Revolution about its Axis, may be investigated as follows. Let BCD be the Curve, AL the Axis of the Fusee produced; let D be the Point where the End of the Chain is fix'd on the Fusee when the Watch is down, or the Spring uncoil'd, and B the Point where it touches it when the Pl. XI. Fig. 1.

Spring

Spring or Machine is wound up. From the Points B and D let fall the Perpendiculars to the Axis B A and D H; in which produced, let there be taken A E and H I proportional to the Force or Strength of the Spring, when the Chain is at B and D. Through E, I, draw the Right Line E I K intersecting the Axis somewhere in K; and from any Point C in the Curve, draw C F perpendicular to the Axis in G; then will F G be as the Strength of the Spring when the Chain is as G.

7. Now, since the Force acting on the Fusee ought always to be uniformly the same; and this Force being always as the Strength of the Spring exprefs'd by F G, and the Distance at which the Chain acts from the Axis of the Fusee conjointly: Therefore the Force at any Point C will be as the Rectangle $FG \times GC$, and since this is a given Quantity it may be made $FG \times GC = ab$, and so we have $FG = \frac{ab}{GC}$.

8. Therefore to determine the Equation of the Curve B C D, let $KH = a$, $HI = b$, $HG = x$, and $GC = y$. Then because of the similar Triangle H K I and G K F, we have $HK : HI :: GK : FG = \frac{ab}{y}$; that is, $a : b :: a + x : \frac{ab}{y}$; whence we have $aa = ay + xy$, which is the Equation of the Curve, and shews it to be of the *Hyperbolic Kind.*

9. The Form or Figure of the Fusee being thus determined by the given Force of the Spring in the Points B and D, and the Points G taken at Pleasure successively from H towards A; we next consider that it is acted upon, or put into Motion, by an uniform Force; by which means also the *Great Wheel*, which is fix'd to it, is put into Motion, and that drives the Pinion of the *Centre Wheel*, which *Centre Wheel* drives the Pinion of the *Third Wheel*, and this drives the Pinion of the *Contrate Wheel*, and this the Pinion of the *Balance Wheel*, which plies the two Pallets on the Axis of the Balance, and keeps the Balance in Motion.

10. The Balance in a Watch is instead of the Pendulum in a Clock; both serving to govern the Motion of the whole Machinery. To this Balance is fix'd a small Steel spiral Spring which regulates the Motion thereof,

thereof, and makes it equable; whence its Name of *Regulator*. N. B. *The Reader will do well to have his Eye upon the internal Parts of the Watch as he reads this Description.*

11. When the Watch is wound up, the Chain from the Spring exerts a Force upon the Fusee, which gives Motion to all the Parts of the Machine in the following Manner; which will be easy to understand when the Number of Teeth in each Wheel, and Leaves in the Pinions which they drive, are specified. And these in Modern Thirty-Hour Watches are as follows.

	Teeth.		Leaves.
Great-Wheel	48	———	12
Centre-Wheel	54	———	6
Third-Wheel	48	———	6
Contrate-Wheel	48	———	6
Balance-Wheel	15	———	2 Pallets.

12. Hence 'tis easy to understand how often any one Wheel moves round in the Time of one Revolution of that which drives it. Thus the Great-Wheel on the Fusee, having 48 Teeth, and driving the Centre-Wheel by a Pinion of 12, must cause the Centre-Wheel to move round 4 Times in one Turn of the Fusee. And so for all the rest, as below.

12)48(4 = Turns of the Centre-Wheel.
6)54(9 = Turns of the Third-Wheel.
6)48(8 = Turns of the Contrate-Wheel.
6)48(8 = Turns of the Balance-Wheel.

13. Whence it follows that the Turns of each of those Wheels respectively in one Turn of the Fusee, will be had by multiplying those several Quotients together successively, as follows:

1 Turn of the Fusee or Great-Wheel.
$4 \times 1 = 4$ Turns of the Centre-Wheel.
$9 \times 4 \times 1 = 36$ Turns of the Third-Wheel.
$8 \times 9 \times 4 \times 1 = 288$ Turns of the Contrate-Wheel.
$8 \times 8 \times 9 \times 4 \times 1 = 2304$ Turns of the Balance-Wheel.

14. The Balance-Wheel having 15 Teeth, and each striking a Pallet twice in one Revolution, there will be

30 Strokes upon the Axis of the Balance, which we call the *Beats* of the Balance; and therefore there will be $2304 \times 30 = 69120$ Beats in one Turn of the Fusee or Great-Wheel.

15. The Centre-Wheel is that which we must have the principal Regard to in the *Division of Time*; the Wheels beyond this, towards the Balance, serving only to multiply the Strokes of the Balance, and cause it to move with an insensible Power, and be thereby subject to a more perfect Regulation. But the Centre-Wheel is that upon which both the *Hour and Minute-Hand* is moved or carried round upon the Face of the Watch to indicate the Time, *viz.* the Hour of the Day, or Minute of the Hour.

16. Since the Time of the Watch's going is 30 Hours, and the Minute Hand, and consequently the Centre-Wheel, goes round once in an Hour, the said Centre-Wheel will have 30 Turns in the Time of the Watch's going; and because it has four Turns in one of the Fusee, therefore $4)30(= 7\frac{1}{2} =$ the Number of Turns of the Fusee in winding up the Watch. Whence $69120 \times 7{,}5 = 518400 =$ the Number of Beats during the whole Time of the Watch's going.

17. If then we divide 518400 by 30, we shall have the Quotient $17280 =$ Number of Beats in an Hour, which is call'd the TRAIN of a Watch; and it is said to be a *swifter or slower Train* as the Number of Beats in an Hour is greater or lesser. If we divide the Train 17280 by 3600, the Seconds in an Hour, the Quotient will be almost 5; that is, there will be near 5 Beats *per* Second in such a Watch.

18. From this Analysis of a Watch, it will be easy to form an Idea of the Manner of Calculation for the Numbers of the Teeth and Leaves for the several Wheels and Pinions throughout the Work; and I shall endeavour to facilitate and illustrate this by an Example of the Numbers of a Watch whose *Train* is 14400, and which therefore will beat Quarter-Seconds, because such a one will be useful for many Philosophical Purposes, as well as the common Measure of Time.

19. The Time which this Watch shall go, may be 32 Hours; then $14400 \times 32 = 460800 =$ the Beats of

of the Balance in the whole Time. Suppose the Number of Turns in the Fusee be 8; then 8)460800(=57600 = the Beats in one Turn of the Fusee. Again, let the Number of Teeth in the Balance-Wheel be 15, there will be 30 Beats in one Turn of this Wheel; then 30)57600(=1920, which will be the Number arising from the continued Multiplication of all the Quotients of the Wheels, divided by the Pinions they drive from the Great-Wheel to the Balance-Wheel, as will be easy to understand by *Art.* 12, 13.

20. The Business is now to break this Number into 4 convenient small Numbers, which multiplied together shall make the same Number 1920. This may soon be done by a few Trials; thus suppose I take the Number 4 for one of them; then 4)1920(480. This Number, 480, I plainly see can be divided by 6 without a Remainder; therefore 6)480(= 80; and as I plainly see that 80 = 8 × 10, therefore the four Numbers sought are 4, 6, 8, and 10. For these, multiplied together, make 1920, *viz.* 4×6×8×10 = 1920.

21. Having thus got the Quotients, it will be very easy to find what large Numbers, divided by small ones, will produce the said Quotients; thus 12)48(= 4; wherefore, if I allow 48 Teeth to the Great-Wheel on the Fusee, it must drive a Pinion of 12 on the Centre-Wheel.

22. In the next Place, for the Quotient 6, I chuse the Numbers 54 and 9; thus 9)54(= 6; which shews that the Teeth of the Centre-Wheel may be 54, and it must then drive a Pinion of 9 on the Third-Wheel. Or, if instead of 54 and 9, I rather make Choice of 48 and 8, it will answer the same End; for 8)48(=6, as before.

23. Then for the Quotient 10, 'tis easy to see, that 50 and 5 will produce it, *viz.* 5)50(=10. That is, the Third Wheel having 50 Teeth must drive a Pinion of 5 on the Contrate-Wheel. If the said Wheel has 40, or 60 Teeth, and drive a Pinion of 4 or 6, the same Number of Turns will be obtained.

24. Lastly, for the Quotient 8, we have the Numbers 48 and 6; for 6)48(= 8; or 7)56(= 8; or 5)40(= 8. Whence if the Contrate-Wheel be al-

low'd 40, or 48, or 56 Teeth, it must drive a Pinion of 5, 6, or 7 Leaves on the Balance-Wheel.

25. By this Means all the Wheels and Pinions are determined, and adjusted in the Body of the Watch from the Fusee to the Balance; but all that we have hitherto done shews only the *Minutes of an Hour*, and *Seconds*, or *Quarter Seconds* of a Minute; but nothing has yet been mention'd relating to the Mechanism for shewing the *Hour of the Day*, of which we shall treat in the next Place.

Plate XI. Fig. 2. 26. This Part of the Work lies conceal'd from Sight between the upper Plate of the Watch-Frame and the Dial-Plate; and since few People have the Opportunity of viewing this Work, I have here represented it by a Figure; wherein ABC is the uppermost Side of the Frame-Plate, as it appears when detached from the Dial-Plate; the Middle of this Plate is perforated with a Hole receiving that End of the Arbor of the Centre-Wheel which carries the Minute-Hand; on this End of the Arbor, near the Plate, is fix'd a Pinion *ab* of 10 Teeth; this is call'd the *Pinion of Report*; it drives a Wheel *cd* of 40 Teeth; this Wheel *cd* carries a Pinion *ef* of 12 Teeth, and this drives a Wheel *gh* with 36 Teeth.

27. As in the Body of the Watch the Wheels every where drive the Pinions, and so quicken or increase the Motion; here, on the contrary, the Pinions drive the Wheels, and by that Means decrease the Motion, which is here necessary; for the Hour-Hand, which is carried by a Socket fix'd on the Wheel *gh*, is required to move but once round, while the Pinion *ab* moves 12 Times round. To this End the Motion of the Wheel *cd* is $\frac{1}{4}$ of the Pinion *ab*, because 10)40(=4. Again, while the Wheel *cd*, or the Pinion *ef*, goes once round, it turns the Wheel *gh* but $\frac{1}{3}$ Part round, for 12)36(=3. Consequently the Motion of *gh* is but $\frac{1}{3}$ of $\frac{1}{4}$ of the Motion *ab*; but $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, that is, the Hour-Wheel *gh* moves once round in the Time that the Pinion of Report on the Arbor of the Centre or Minute-Wheel makes twelve Revolutions, as requir'd.

28. Having thus shewn the Nature and Manner of the Mechanism of a *Watch*, the Structure of that Part of

of a CLOCK, which is concern'd in ſhewing the Time, will eaſily be underſtood, eſpecially as it is repreſented in the Draught. The Mechaniſm of a Clock conſiſts of *two Parts*, one to *ſhew the Time*, the other to *report it*, by ſtriking the Hour upon a Bell. How each of theſe is effected, I ſhall ſhew by the following Steps.

29. Each Part is actuated or moved by *Weights*, as in common Clocks, or *Springs* included in Boxes or Barrels, as A; this Cylinder moves the Fuſee B, and the great Wheel C, (to which it is fix'd) by the Line or Cord that goes round each, and anſwers to the Chain of the Watch. The Method of Calculation is here nearly the ſame as before. For ſuppoſe here the Great-Wheel C goes round once in twelve Hours; then if it be a *Royal Pendulum* Clock, ſwinging Seconds, we have $60 \times 60 \times 12 = 43200$ Seconds or Beats, in one Turn of the Great-Wheel. Plate XI. Fig. 3.

30. But becauſe there are 60 Swings or Seconds in one Minute, and the Seconds are ſhewn by an Index on the End of the Arbor of the Swing-Wheel (which in thoſe Clocks is in an horizontal Poſition) therefore 'tis neceſſary that the Swing-Wheel ſhould have 30 Teeth. Whence 60)43200(=720, the Number to be broken into Quotients for finding the Number of Teeth for the other Wheels and Pinions, as before, in *Art.* 20.

31. But ſince the Minute-Wheel D goes 12 Times round in one Turn of the Great-Wheel C, we know one of the Quotients muſt be 12; whence 12)720(= 60; and 60 is compounded of 6×10, or, $8 \times 7{,}5$. So that either 6 and 10, or 8 and $7\frac{1}{2}$ are the two other Quotients; but the latter Numbers are preferable to the former, as affording a greater Uniformity of the Parts and Eaſe in the Execution. Therefore 8)60(=$7\frac{1}{2}$; hence if we give 60 Teeth to the Middle or Minute-Wheel D, it drives a Pinion of 8 on the Contrate-Wheel E.

32. The other Quotient 8 will give us 8)64(=8, or 7)56(=8; that is, if we give 64 Teeth to the Contrate-Wheel E, it muſt drive a Pinion of 8 on the Ar-

bor of the Swing-Wheel. But if only 56 be allow'd, then it muſt drive a Pinion of 7, and theſe laſt Numbers are now generally uſed. Thus much for the *Time Part*, as far as the Minutes and Seconds: As for the Pinions and Wheels to ſhew the Hours, they are the ſame as was before ſhewn in the Watch, *Art*. 26, 27.

33. In Spring Clocks the Diſpoſition of the Wheels in the Watch Part is ſuch as is here repreſented in the Figure, where the Swing Wheel F is in an horizontal Poſition, the Seconds not being ſhewn there by an Index, as is done in the large Pendulum Clocks, according to the foregoing Calculation: Whence in theſe Clocks the Wheels are diſpoſed in a different Manner, as repreſented in *Fig*. 4. where C is the Great-Wheel, D the Centre or Minute-Wheel, both as before; but
Fig. 4. the Contrate-Wheel E is plac'd on one Side, and F the Swing-Wheel is plac'd with its Centre in the ſame perpendicular Line GH with the Minute-Wheel, and with its Plane perpendicular to the Horizon, as are all the others. Thus the Minute and Hour Hands turn on the End of the Arbor of the Minute-Wheel at *a*, and the Second-hand on the Arbor of the Swing-Wheel at *b*.

34. It remains now that we give an Account of the Machinery of the *Striking-Part* of a Clock. Here as in the *Watch-Part*, the *Primum Mobile* is a large Spring, in the *Spring-Barrel* G; but in long Pendulums it is a Weight, as is well known. This by its Cord and Fuſee moves the *Great Wheel* H; that gives Motion to the *Pin-Wheel* I; that continues it to the *Detent* or *Hoop-Wheel* K; and that to the *Warning-Wheel* L; which at laſt is ſpent on the *Flying-Pinion* Q, which carries the *Fly* or *Pan*; and by its great Velocity it meets with great Reſiſtance from the Air it ſtrikes, and by this Means bridles the Rapidity of the Clock's Motion, and renders it equable.

35. All theſe Wheels are *quieſcent* or motionleſs, unleſs when at the Beginning of each Hour the Detent O is lifted up, by which Means the Work is unlock'd, and the Whole put into Motion by Virtue of the Spring in

in the Box G. During this Motion the Pins *e*, *e*, *e*, *e*, of the Pin-Wheel I, take the Tail of the Hammer T, and carrying it upwards remove the Head of the Hammer *s* from the Bell R; then being let go by the Pin it is made by a ſtrong Spring to give a forcible Stroke upon the Bell, and this is repeated as often as the Hour requires, by Means of a Contrivance in another Part.

36. This conſiſts of moveable Wheels, and ſeveral Levers, and other Parts, which cannot be underſtood by a bare Deſcription, or even by a Repreſentation in a Draught, ſo well as any Perſon may have an Idea of them by taking off the Face or Dial-Plate of a late-made Eight-Day Clock; for, within 8 or 10 Years paſt, great Improvements have been made in this Part of the Mechaniſm. Of which, perhaps, I may give a more ample Account in another Place, the Scope of my preſent Deſign not admitting it here.

37. To the Invention of Mr. *Maurice Wheeler* we owe the curious Contrivance of a *Clock deſcending on an Inclined Plane*; the Theory of which is worth any Gentleman's Knowledge, and may be ſeen in N° 161. of the *Philoſophical Tranſactions*. Alſo the Clock itſelf may be ſeen at *Don Saltero*'s Coffee-houſe in *Chelſea*.

38. But ſince many People may have a Curioſity to be acquainted with this odd Kind of Machinery, who have neither of the Opportunities abovementioned, I will here preſent them with a ſhort Account of this Clock. DE is the inclined Plane on which the Clock A B C deſcends; it conſiſts externally of a Hoop, and two Sides or Plates ſtanding out beyond the Hoop about $\frac{1}{8}$ of an Inch all round, and indented that the Clock may not *ſlide*, but turn round as it moves down. Plate X. Fig. 7.

39. One of theſe Plates is inſcribed with the 24 Hours, which paſs ſucceſſively under the Index, which is always in a Poſition perpendicular to the Horizon, and ſhows the Hour on the Top of the Machine: For this Reaſon the lower Part of the Index is heavieſt, that it may preponderate the other, and always keep it upright as the Movement goes on.

40. For the internal Part or Mechaniſm, let LEDQ be the external Circumference of the Hoop, and *ff* the

Frame-Plate on which is placed the Train of Wheel-Work 1, 2, 3, 4. which is much the ſame as in other Clocks, and is governed by a Balance and Regulator as in them. But for a Spring and Fuſee, there needs none in this Clock, their Effects being otherwiſe anſwered, as we ſhall ſee.

41. In this Machine the great Wheel 1 is placed in the Centre, or upon the Axis of the Movement, and the other Wheels and Parts towards one Side, which therefore would prove a Biaſs to the Body of the Clock, and cauſe it to move, even on a horizontal Plane, for ſome ſhort Diſtance; this makes it neceſſary to fix a thin Plate of Lead at C on the oppoſite Part of the Hoop, to reſtore the *Equilibrium* of the Movement.

42. This being done, the Machine will abide at reſt in any Poſition on the horizontal Plane HH; but if that Plane be changed into the Inclined Plane DE, it will touch it in the Point D; but it cannot reſt there, becauſe the Centre of Gravity at M acting in the Direction MT, and the Point T having nothing to ſupport it, muſt continually deſcend and carry the Body down the Plane.

43. But now if any Weight P be fixed on the other Side the Machine, ſuch as ſhall remove the Centre of Gravity from M to the Point V in the Line LD, which paſſes thro' the Point D, then it will reſt upon the Inclined Plane, as we have ſhewn in the Caſe of the *Rolling Cylinder*; for this Inſtrument is founded on the ſame Principle.

44. If now the Weight P be ſuppoſed not fixed, but ſuſpended at the End of an Arm, or *Vectis*; which Arm or Lever is at the ſame Time faſtened to a Central Wheel 1, moving on the Axis M of the Machine, which Wheel by its Teeth ſhall communicate with the Train of Wheels, &c. on the other Side; I ſay, in this Caſe if the Power of the Weight P be juſt equal to the Friction or Reſiſtance of the Train, it will remain motionleſs all one as before when it was fixed; and conſequently the Clock alſo will be at reſt on the Inclined Plane.

45. But ſuppoſing the Weight P has a Power ſuperior

rior to the Resistance of the Train, it will then put it into Motion, and of course the Clock likewise, which will then commence a Motion down the Plane, while the Weight P, its *Vectis* PM, and the Wheel 1, all retain constantly the same Position they at first have when the Clock begins to move.

46. Hence 'tis easy to understand that the Weight P may have such an intrinsic Gravity as shall cause it to act upon the Train with any required Force, so as to produce a Motion in the Machine of any required Velocity, as suppose such as shall carry it once round in twenty-four Hours; then if the Diameter of the Plates A B C be four Inches, it will describe the Length of its Circumference, *viz.* 12,56 Inches in one natural Day; and therefore if the Plane be of a sufficient Length, such a Clock may go several Days, and would be a *perpetual Motion*, if the Plane were of an infinite Length.

47. Let S D be drawn through M perpendicular to the Inclined Plane in the Point D. Also let L D be perpendicular to the horizontal Line H H, passing through D. Then is the Angle HDE=LDS=DMT. Whence it follows, that the greater the Angle of the Plane's Elevation is, the greater will be the Arch D T, and consequently the farther will the common Centre of Gravity be removed from M; therefore the Power of P will be augmented, and of course the Motion of the whole Machine accelerated.

48. Thus it appears, that by duly adjusting the intrinsic Weight of P at first to produce a Motion shewing the mean Time as near as possible; the Time may afterwards be corrected, or the Clock made to *go faster or slower* by raising or depressing the Plane, by means of the Screw at S. The Angle to which the Plane is first raised is about ten Degrees.

49. We have given the Theory of a Clock moving *down an inclined Plane*: Let us now consider how a Clock may be made *to ascend* on the same inclined Plane. To this End let ABD be the Machine on the Inclined Plane EDE, and let it be kept thereon at Rest, or *in Equilibrio*, by the Weight P, at the End of the Lever P M. The circular Area C F is one End Plate X. Fig. 8.

End

IN the ORRERY and COMETARIUM, the whole Machinery is a Compages of Wheels of various Sizes, ſuitably contrived and adapted to produce circular and elliptical Motions of Bodies repreſenting the *Planets* and *Comets*, in ſuch Periods of Time as are exactly proportional and correſpondent to the reſpective Motions of the Heavenly Bodies which they repreſent; alſo their ſeveral Phaſes, Poſitions of the Orbits, and other Affections, ſo as to be a perfect *Microcoſm*, or *Solar Syſtem* in Miniature (XLIII, XLIV).

IN

End of a Spring Barrel in the Middle of the Movement, in which is included a Spring, as in a common Watch.

50. To this End of the Barrel the Arm or Lever P M is fixed upon the Centre M; and thus when the Clock is wound up, the Spring moves the Barrel, and therefore the Lever and Weight P, into the Situation *P* M. In doing this, the Centre of Gravity is conſtantly removed farther from the Centre M of the Machine, and therefore it muſt determine the Clock to move upwards; which it will continue to do, ſo long as the Spring is unbending itſelf; and thus the Weight and its Lever PM will preſerve the Situation they firſt have, and do the Office of a Chain and Fuſee. This is the Contrivance of M. *de Gennes*. See *Phil. Tranſ.* N° 140.

(XLIII) It would be too great an Undertaking here to give an Account of the Mechaniſm of the larger Sort of Orreries, which repreſent the Movements of all the heavenly Bodies; nor indeed can it be done either by Diagram or Deſcription, to render it intelligible to the moſt diſcerning Reader; but inſtead of that, I ſhall exhibit

[illegible]

[illegible] is

[illegible] of various [illegible]

and adapted to produce [illegible]

[illegible] of Bodies

Planets and *Comets*, in

[illegible] exactly resembling

[illegible] to the [illegible]

[illegible] Bodies which they

their [illegible] and Phases. [illegible]

and other Affections [illegible]

[illegible] *Microcosm*, or *Star* [illegible]

[illegible]

[illegible] which [illegible]

[illegible] upon the [illegible]

[illegible] the Sun,

therefore [illegible] and [illegible]

[illegible] the [illegible]

[illegible] removed further [illegible]

[illegible] and [illegible]

[illegible]

Fig. 1. p. 203.

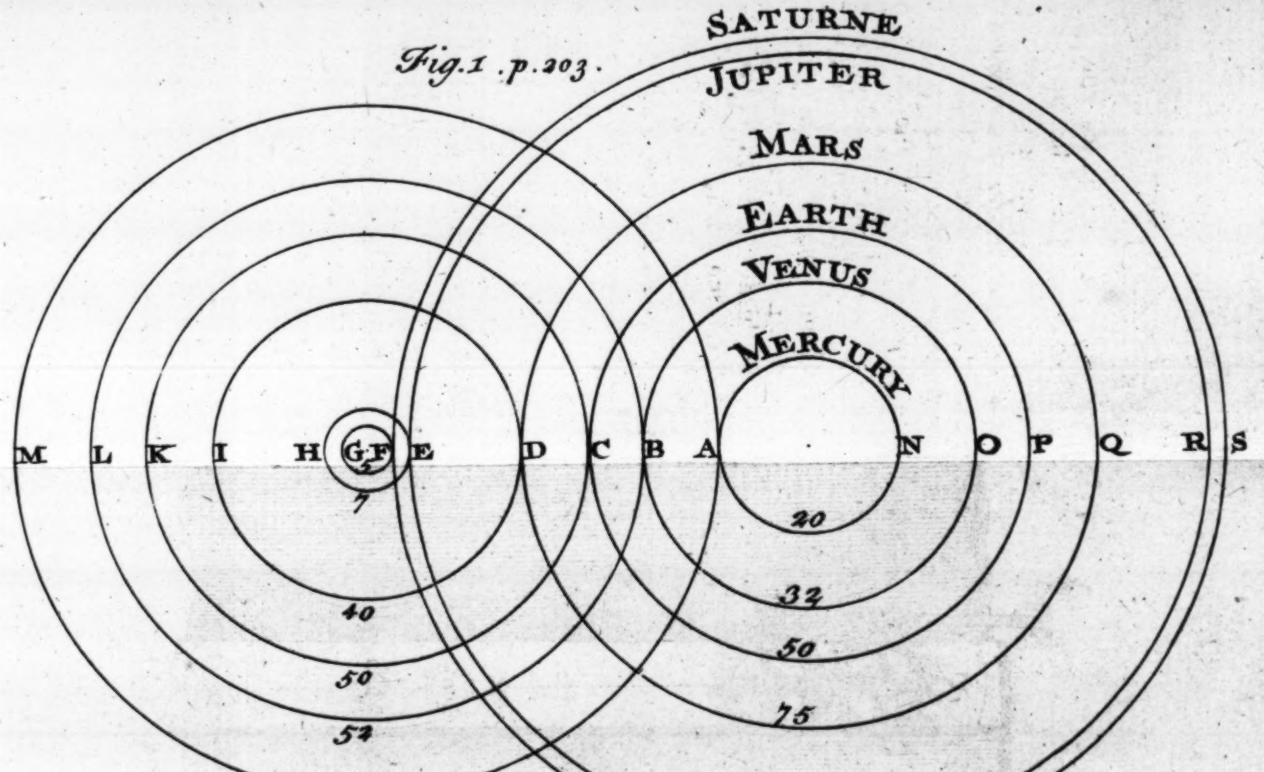

SATURNE
JUPITER
MARS
EARTH
VENUS
MERCURY
M L K I H G F E D C B A N O P Q R S
5
7
40
50
54
20
32
50
75

Fig. 2.
p. 205 & 251.

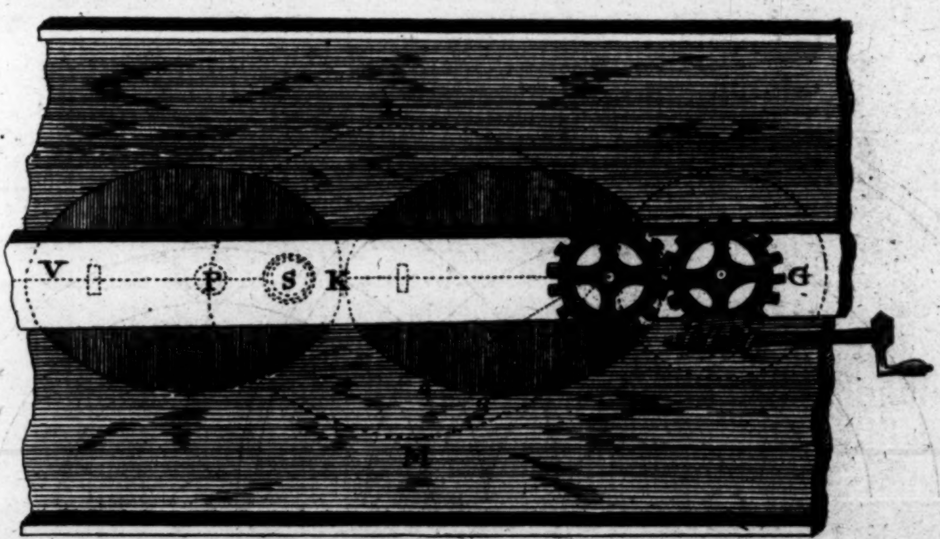


exhibit an Idea of the Theory and Structure of an useful, concise, and portable *Planetarium*, which any Gentleman may have made for a small Expence, and will exhibit very justly the Motions of all the primary Planets about the Sun by Wheel-Work; and those that have Secondaries or Moons, may have them placed about their Primaries, moveable by the Hand, so that the whole shall be a just Representation of the *Solar System*, or true State of the Heavens, for any given Time of the Year.

2. In order to this we must first compare, and find out the Proportion, which the Periodical Times or Revolutions of the Primary Planets bear to that of the Earth; and they are such as are expressed in the Tablet below, where the first Column is the Time of the Earth's Period in Days, and Decimal Parts; the second that of the Planets; the third and fourth are Numbers in the same Proportion to each other,

As 365,25	: 88	☿	:: 83	: 20,	for *Mercury*.
365,25	: 224,7	♀	:: 52	: 32,	for *Venus*.
365,25	: 686,9	♂	:: 40	: 75,	for *Mars*.
365,25	: 4332,5	♃	:: 7	: 83,	for *Jupiter*.
365,25	: 10759,3	♄	:: 5	: 148,	for *Saturn*.

3. If now we suppose a Spindle or Arbor with six Wheels fixed upon it in an horizontal Position, having the Number of Teeth in each, corresponding to the Numbers in the third Column, *viz.* the Wheel A M of 83 Teeth, BL of 52, CK of 50 (for the Earth,) DI of 40, EH of 7, and F G of 5; and another Set of Wheels moving freely about an Arbor, having the Number of Teeth in the fourth Column, *viz.* A N of 20, B O of 32, C P of 50 (for the Earth,) D Q of 75, E R of 83, and F S of 148; then if those two Arbors of *fixed and moveable Wheels* are made of the Size, and fixed at the Distance from each other, as here represented in the Scheme, the Teeth of the former will take those of the latter, and turn them very freely when the Machine is in Motion. Pl. XII, Fig. 1.

4. These Arbors, with their Wheels, are to be placed in a Box of an adequate Size, in a perpendicular Position;

Position; the Arbor of fixed Wheels to move on Pivots at the Top and Bottom of the Box; and the Arbor of moveable Wheels to go through the Top of the Box to a proper Height, on the Top of which is to be placed a round Ball, gilt with Gold, to represent the Sun. On each of the moveable Wheels is to be fixed a Socket or Tube ascending above the Top of the Box, and having on the Top a Wire fixed, and bent at a proper Distance into a Right Angle upwards, bearing on the Top a small round Ball representing its proper Planets.

5. If then on the lower Part of the Arbor of fixed Wheels be placed a Pinion of Screw-Teeth, a Winch turning a Spindle, with an Endless-Screw, playing in the Teeth of the Arbor, will turn it with all its Wheels; and these Wheels will move the others about with their Planets in their proper and respective Periods of Time very exactly. For while the fixed Wheel C K moves its equal CP once round, the Wheel A M will move A N a little more than four Times round, and so will nicely exhibit the Motion of *Mercury*; and the Wheel F G will turn the Wheel F S about $\frac{1}{29,5}$ round, and so will represent the Motion of *Saturn*. And the same is to be said of all the rest.

6. If on the Top of the Box be placed a Circle with the Signs of the Ecliptic, including all the Planets, it will be easy by this Machine to represent the Motions and various Phænomena of the heavenly Bodies, by those who have Skill in such Things. Having thus shewed the Reason, Structure, and Parts of the portable *Planetarium*, the rest must be left to the Fancy and Pleasure of the Gentleman, and the Ingenuity of the Workman.

(XLIV) 1. I shall here give an Account of the Mechanism of that Instrument, which I call a COMETARIUM; for though Dr. *Desaguliers* has (the first of any I know of) described it under the Character of Part of a PLANETARIUM, and by it represented the Motion of the Planet *Mercury*; yet I conceive it is much more adapted to represent the Motion of a Comet than a Planet, because none of the Planets describe

I Orbits

Orbits sensibly *Elliptical*, whereas those of the Comets are very much so. I have therefore altered the Doctor's Contrivance, and adapted it to the Motion or Theory of the Comet of the Year 1682, whose Period is $75\frac{1}{2}$ Years; and have moreover altered the Mechanism of the Parts, by which the Instrument is rendered much more elegant and useful.

2. The Structure and Rationale of this Instrument is as follows. When the Lid is taken off the Box, the internal Parts appear as in the Figure. NO and QT are two Elliptic Wheels turning each other about their Foci I and S, by means of a Cat-Gut String in a Groove on their Edges, crossing at K. These oval Wheels are fixed in Arbors or Axes which pass through the same Focus S and I in each; the Oval NO is moved by the circular Wheel I, fixed also upon the same Axis, but above it on the Bar or long Piece GV; which Wheel is itself moved by another equal Wheel G, and that by an Endless-Screw, turned by a Winch, on the Outside of the Box. All which is evident in the Figure. Pl. XII. Fig. 2.

3. The Perimeter of the Oval QT, where it touches that of NO, will have a Velocity always proportional to the Distance from I, that is, in the Points K, 4, 3, 2, 1, &c. the Velocities will be as the Lines IK, I 4, I 3, I 2, I 1, &c. which may be considered as Levers acting upon and moving the Oval QT in those Points. Now if the Ovals are such that SK is to SV, or IK to I 1, as 6 to 1, then will the Point K have 6 Times the Velocity turned by the Lever IK, as the Point V will have when it has made $\frac{1}{2}$ a Revolution, or is come under the Point S, where it is turned by the Lever I *s*, than in the Situation I S.

4. If we take SP=SK=IS, and upon the Point S, *s*, as Foci, you describe the Ellipsis PLIM, that will represent the Orbit of the Comet, or the Figure of the Groove on the Lid of the Box, in which a round Brass Ball, representing the *Comet*, is made to slide along on a Piece of Wire, called the *Radius Vector*, fixed at one End into the Top of the Arbor at S, where we may suppose

IN WATER-MILLS, the *Momentum* of the falling Water is the Power; the Force to be overcome is the great Attrition of the two Stones in grinding the Corn, &c. which is effected wholly by a Complication of Wheels and Axles. A Query may here be put, Why, since the Power constantly acts upon the Wheel, the Motion of the Wheel should be equable, and not accelerated: The Answer is, The Increments of Velocity keep rising, till their *Momentum* is equall'd by the Resistance of the

suppose the Sun to be, and is accordingly represented by a silvered Plate at Top.

5. The Place of the Comet at P is called the *Perihelion*, as being there *nearest the Sun*; as I is its *Aphelion* or Point of *greatest Distance*. Since S P = S K, the Velocity of the Comet will be in the Point P equal to that of the Point K; and were the Comet's Aphelion at *s*, its Velocity there would be equal to that of the Point V when under *s*, *viz.* six Times less than before; but since the Comet's Aphelion is at I, and since the greater Arch described in the same Time must have a greater Velocity, the Velocity of the Comet at I will be about $\frac{1}{3}$ of that at P.

6. The silvered Ellipsis on the Lid of the Box is divided into 100 Parts, shewing the *Anomaly* of the Comet; and about the Axis of the Wheel G is placed a Silver Circle, divided into $75\frac{1}{2}$ equal Parts, representing the Years or Period of the Comet, with a proper Index, pointing to those Divisions, by which Means it is easy to shew the several Particulars relating to the Theory of elliptic Motions, whether of a Planet or Comet.

the Machine; after which Equilibrium the Wheel goes on with an uniform Motion (XLV).

(XLV) The Mechanism of a Water-Mill depends upon the following Principles.

1. The Action or Power of the Water which drives the great Wheel. Here it will be necessary to determine its Force issuing out of the Aperture of the Sluice or Pen-Stock, and also the Velocity of its Motion when the Height is known; and the Height necessary to produce a given Velocity.

2. As to the Force of Water issuing through the Sluice, that is, its momentary Impulse, we shall shew from Hydrostatic Principles, that it is always proportional to the Altitude of the Water above the Centre of the Hole through which it passes. And since we know by Experiment that a cubic Foot of Water weighs very nicely 1000 Ounces *Averdupois*, or 62,5 *lb.*; if we find the Area of the Aperture in Feet and Parts, and multiply that by the Number of Feet the Water has above the Centre, and lastly, you multiply that Product by 62,5 *lb.* this last Product will be the Force of the Water express'd in Pounds *Averdupois*.

3. For Example; suppose the Width of the Sluice 12 Inches, or 1 Foot, and that it is drawn up to the Height of three Inches, or 0,25 of a Foot, the Area of the Aperture will then be $1 \times 0{,}25 = 0{,}25$; if the Height of the Water be 7,5 Feet above the Orifice, then $7{,}5 \times 0{,}25$ will be 1,875, which multiplied by 62,5 gives 116,1875, or about 116,2 *lb.* for the instantaneous Pressure of the Water on any Obstacle.

5. To find the Velocity, and consequently the Quantity of Water expended at the Orifice in a given Time, we must consider that a Body falls 16,2 Feet in the first Second, and acquires a Velocity which in an horizontal Direction is at the Rate of 32,4 Feet *per* Second; now let S be any other Space and the horizontal Velocity V; then we shall have $16{,}2 : S :: \overline{32{,}4}^2 : V^2$; therefore $\overline{32{,}4}^2\, S = 16{,}2\, V$, and so $\frac{\overline{32{,}4}^2}{16{,}2} \times S =$ V^2,

V^2, that is $\sqrt{64,8\,S} = V$. Now since $S = 7,5 =$ Height of the Water above the Centre of the Orifice, therefore $\sqrt{64,8 \times 7,5} = 22$ *per* Second, the Velocity of the issuing Water as required.

6. This Velocity might also have been found by finding the Time of the Fall thro' 7,5 Feet; thus 16,2 : $\overline{60}^{2\,'''}$: : 7,5 : $t^2 = 1666,6$, the Square Root of which is 40''',8; now since the Velocity of spouting Water is uniform, there will pass twice 7,5 or 15 Feet in 40''',8; and therefore if 40''',8 : 15 F. : : 60''' : 22 F. and a little more, the same as before.

7. If the Velocity be given, and it be required to find the Height of the Fall necessary to produce it; we have from the foregoing Theorem ($\sqrt{64,8S} = V$,) $S = \frac{V^2}{64,8}$; so that if the given Velocity be 22 Feet *per* Second, then $V=22$, and $S = \frac{V^2}{64,8} = 7,5$ nearly, the Height of the Water above the Centre of the Aperture. For the Velocity of Water is the same in the Fall thro' any Space or Height, and in the Orifice of a Sluice at the same Depth below the Surface.

8. The Quantity of Water issuing out is thus determined for any given Time; since a Column of Water $= 22$ Feet is produced in 1 Second, we shall have $60 \times 22 = 1320$ Feet in 1 Minute, or 158400 Inches; and the Area of the Aperture being $12 \times 3 = 36$ Inches, we have in that Column $158400 \times 36 = 5702400$ Cubic Inches; which divided by 282, gives 2022 Gallons, or 32 Hogsheads 6 Gallons, which spouts on the Wheel in a Minute.

9. If we now suppose this Column of Water flowing into the Buckets of an Over-shot Wheel, which is 16 Feet in Diameter, and has 30 Buckets on its Periphery, (as is the Case of that at the *Bar-Pool*, by the Abbey in *Nun-Eaton* in *Warwickshire*, which is reckoned the best in *England*) then the Water will act on the Wheel by two Forces, *viz.* of Impulse and Weight. The Impulse or Stroke, were it made in a Tangent-Direction, and perpendicular to the Sides of the Buckets, would be equal to 116 *lb.* (by *Art.* 4.) But since it

runs

runs some Distance in a *Trough* before it comes to the Wheel, and the Buckets are placed not at Right Angles, but nearly at the Angle of 45 Degrees with the Circumference of the Wheel, and so the Water must strike them very obliquely; I say, on these Accounts we must abate more than one Half of the Force of Water, and may take about 50 *lb.* for a Medium.

10. The Weight of the Water is more or less in twelve or thirteen of these Buckets on the Fore-part of the Wheel, but most of all on that Bucket at the End of the horizontal Diameter, because there no Part of the Water rests upon the Perimeter of the Wheel or Sides of the Bucket, but gravitates with its whole Weight in a Tangent to that Part of the Wheel. What that Weight in that Bucket is, may be thus found. It was shewn that when the Engine is in the greatest Perfection, the Velocity of the Wheel ought to be equal to $\frac{1}{3}$ of that of the Water. (See *Annotat.* XL. 6.) Now since the Diameter is 16 Feet, the Circumference will be 50,3; and the Velocity of Water being 1320 Feet *per* Second, $\frac{1}{3}$ of that is 440, which divided by the Circumference gives $8\frac{7}{10}$ Revolutions of the Wheel in a Minute. The Aperture of the Sluice gives 2022 Gallons *per* Minute, which divided by 8,7 gives 232,4 Gallons, which again divided by 30, (the Number of Buckets) is 7,7 Gallons for a Bucket.

11. Now a Gallon of Water weighs 10,2 *lb.* (for 1728 : 62,5 :: 282 : 10,2); therefore 7,7 × 10,2 = $78\frac{1}{2}$ *lb.* for the Weight of Water in each Bucket. But as Part of this Water runs out of the Buckets in the lower Quarter of the Wheel, and what remains gravitates in various Degrees of Obliquity to the Radii of the Wheel, as does all the Water in the full Buckets above the horizontal one, we must allow for the total Weight about 450 *lb.* (as will be found near the Truth by Calculation) to which if we add the 50 *lb.* for Impulse, the whole Force of the Water on the Wheel will be 500 *lb.*

12. If the Cog-Wheel be 3,5 Feet Radius, or 7 Feet Diameter (as in the Mill above mentioned) then, as 7 : 16 :: 500 *lb.* : 1143 *lb.* = the Force of the Cogs to turn the Wallower or Trundle, which Force is applied to overcome the Resistance arising from the

Weight of the Stone, the Friction of the Geer in general, and the great Friction of the Stones and Corn in grinding.

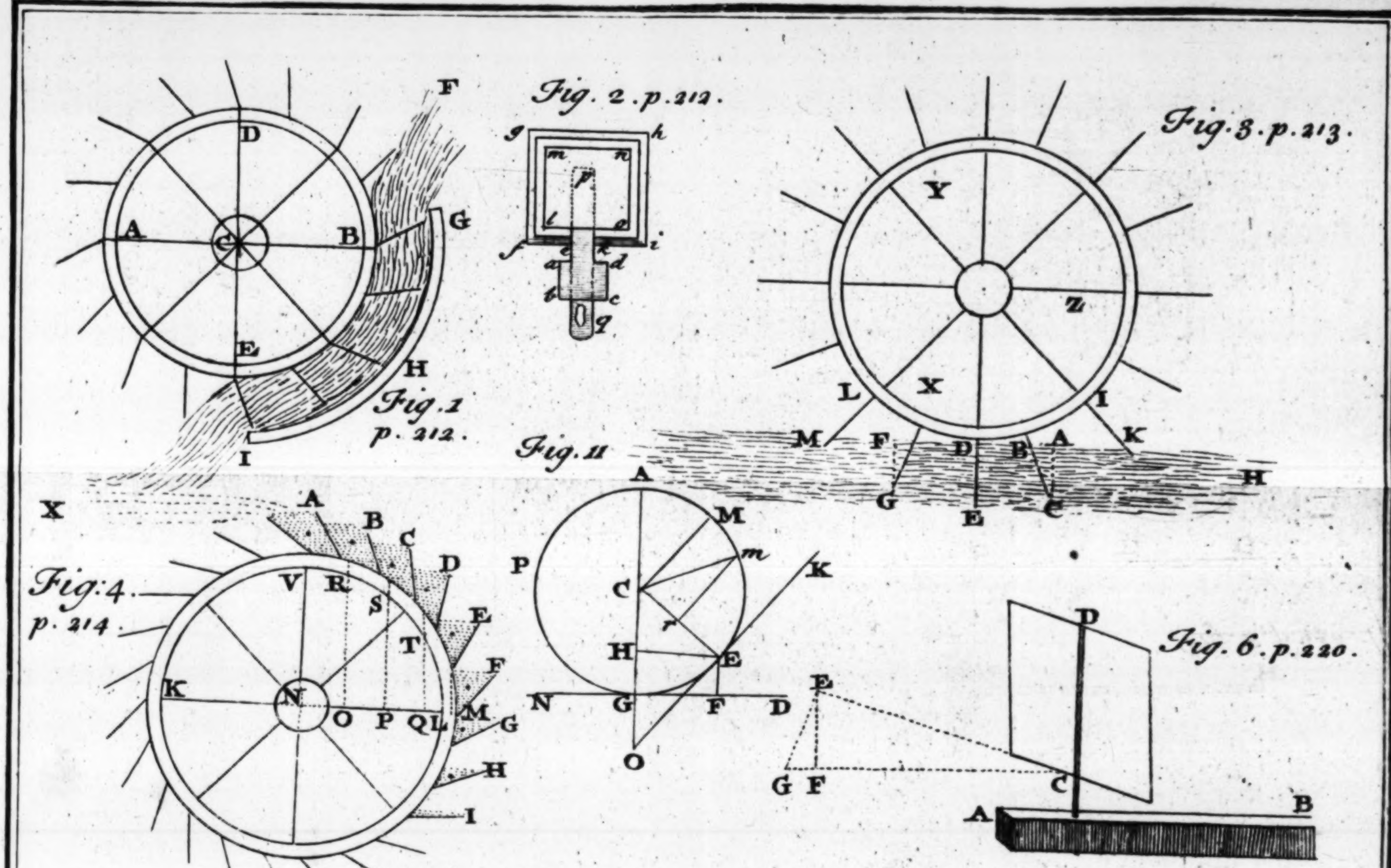
13. Suppose the Diameter of the Trundle 1,5 Foot, or 18 Inches, and that of the upper Stone 6 Feet; then as 6 : 15 :: 1143 : 285,7 *lb.* = the Force at the Periphery of the Stone. If there be 48 Cogs in the Cog-Wheel, and nine Rounds in the Trundle, then $\frac{48}{9} = 5,33 =$ the Turns or Revolutions of the Trundle for one of the Water-Wheel; wherefore since the Wheel makes 8,7 in a Minute, the Trundle, and of course the Stone, will make (8,7×5,33=) 46½ Turns in a Minute. The Circumference of the Stone is 18,84 Feet; whence 18,84×46,5=876 Feet *per* Minute, for the Velocity of the Stone's Periphery.

14. Suppose the Stone contain 22,5 Cubic Feet, or its Weight 1912 *lb.* then the *mean Velocity* of the Stone being that Point of the Radius ⅔ of its Length, consequently ⅔ of the Velocity 876 at the Periphery, *viz.* 584, will be the mean Velocity of the Stone, which multiplied by 1912, *viz.* 1912×584=1116608 *lb.* would be the Expression of the *Momentum* of the Stone *per* Minute, were it to press upon the Corn with its whole Weight, which it does not; for nearly the whole Weight being supported by the Pivot of its perpendicular Spindle, a very small Part thereof is concerned in the Triture of the Corn, for that is principally effected by the violent Rotation of the Stone producing a Centrifugal Force.

15. This circular Motion of the Stone brings the Corn out of the Hopper by Jerks, and causes it to recede from the Centre to the Circumference in a special Manner; the Corn, while whole, causing the Stone to rise a little higher above the fixed Stone than it would otherwise do, begins to be crushed by the Weight of the Stone gently pressing upon it, and the more so, as it approaches the Circumference, where being quite reduced to Flour, it is thrown out of the Mill by the Centrifugal Force of the Stone through a Hole for that Purpose.

16. When I say, *the Stone presses gently, and but with a very small Part of its Weight on the Corn*, it is implied,

Vol.I. P.211. Plate XIII.



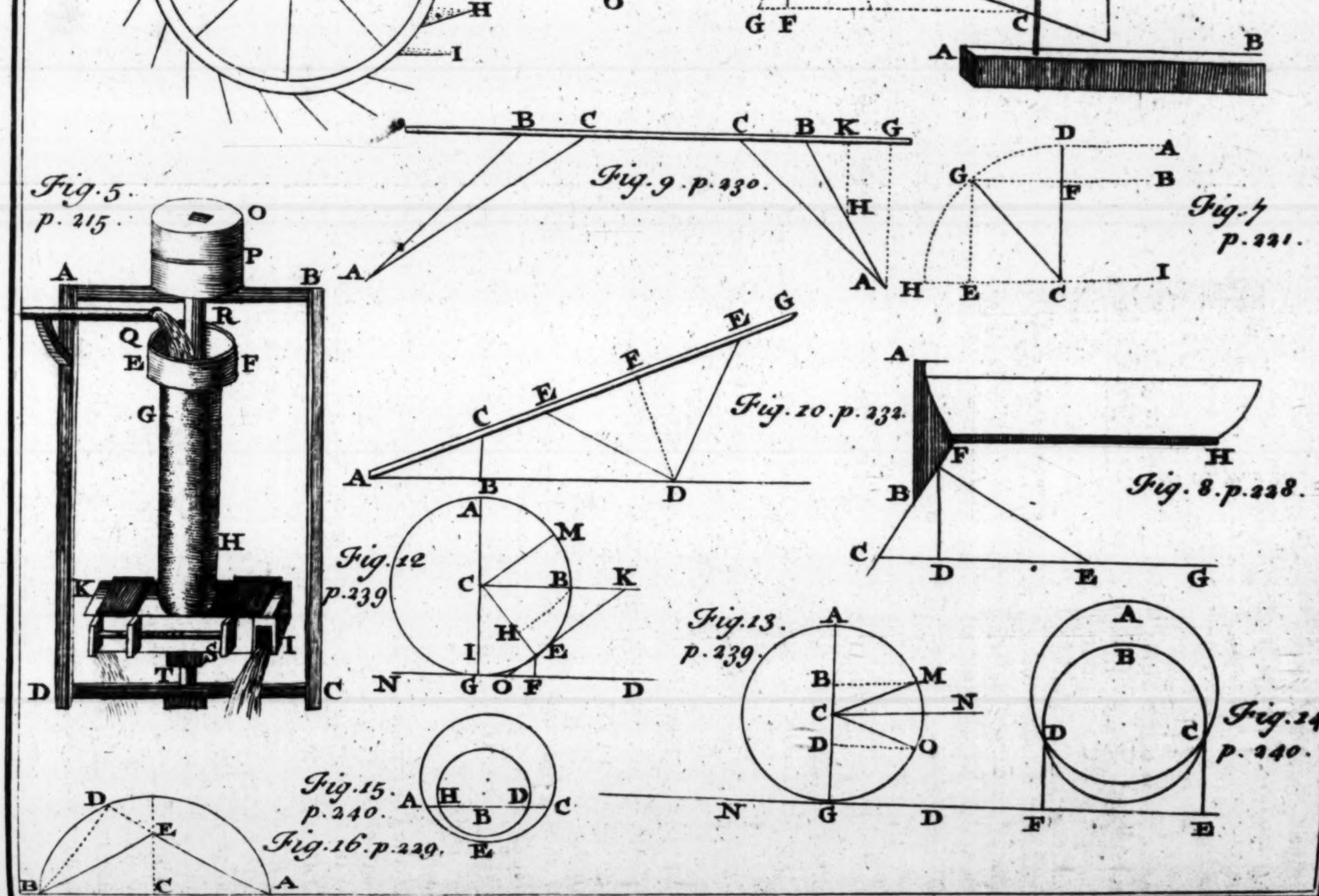


Fig.5. p.215.
Fig.9. p.230.
Fig.7 p.221.
Fig.10. p.232.
Fig.8. p.228.
Fig.12 p.239
Fig.13. p.239.
Fig.14 p.240.
Fig.15. p.240.
Fig.16. p.229.

implied, that the Piece of Wood which ſupports the Stone on the Pivot of its Spindle muſt be ſomewhat elaſtic or ſpringy, in order to admit of an eaſy and ſmall Degree of Motion upwards and downwards, as the Stone is more or leſs reſiſted by the Grains of Corn, by which Means the Surfaces of the two Stones will have always a varying Diſtance, and ſo be adapted for Trituration and Comminution of the Corn in any Degree; whereas, if the ſupporting Piece were perfectly rigid or fixed, the Stones muſt always be at an equal Diſtance, and the Flour very coarſe, becauſe it can't be affected by the upper Stone after the whole Grain has receiv'd its firſt Impreſſion, or ſuffered its firſt Comminution. And the nicer this *Point of Support* is adjuſted, ſo much the more nice and exact will be the Work done by the Mill, or ſo much the better will the Flour be that is produced thereby.

17. As the Water acts upon an *Overſhot-Mill* both by Impulſe and Weight, ſo does it likewiſe on a *Breaſt-Mill*, or that where the Water comes upon the Breaſt, or middle Part of the Wheel; and here, though the Weight of the Water on the Wheel is not ſo great as before, being contained in the Buckets of the lower Quarter only, yet the Impulſe of the Water is much greater, the Height of the Water being increaſed nearly the Semidiameter of the Great-Wheel, all other Things being equal.

18. If the Height of the Water remains the ſame, the Aperture of the Pen-ſtock muſt be enlarged to nearly twice the Area, that the Force of Water may be the ſame; and in this Caſe there will be twice the Expence of Water as before; ſo that as much more Water is neceſſary for a Breaſt-Mill than for an Overſhot one, every Thing elſe being the ſame.

19. Since the Spout of Water is in the Curve of a Parabola, the Orifice through which it iſſues ſhould not be juſt againſt the Middle of the Wheel, for then, if it were very near the Wheel, great Part of the Force would be ſpent in ſtriking the Wheel directly againſt its Axis, and ſo would make the Motion round more difficult; and if it were not very near the Wheel, the Spout would reach only ſome of the lower Buckets,

and none go to those on the Middle, where the Effect would be rather greater. Therefore the Orifice of the Pen-stock ought to be situated some little Height above the Middle or Horizontal Diameter of the Wheel.

20. In order that the Water may have the greatest Effect on the Wheel, Dr. *Barker* has constructed it with 24 Ladle-Boards, (instead of Buckets) of 18 Inches Square; these Ladle-Boards pass through a quadrantal Channel of the same Dimensions nearly, or a little more than 18 Inches square, that the Motion may be free. The Water entering on the Top of this Channel is kept wholly on the Boards, till it comes to the Bottom, where it goes off without interrupting the Motion of the Wheel. The Contrivance is very useful, and may be easily apprehended by the Scheme, where A D B E represents the Wheel with its Ladle-Boards, C the Axis, F the Jet or spouting Water, GHI the square Channel or Trough in which the Ladle-Boards with the Water descend to I, where the Water is discharged from the Wheel.

Pl.XIII. Fig. 1.

21. In *Fig.* 2. you have a Section of the Sole or Perimeter of the Wheel, and the Channel with the Ladle-Boards in it; thus *a b c d* is the Sole of the Wheel, *f g h i* is the Section of the Channel open on the Part next the Wheel by a narrow Slit, in which *e k* is the Part called the Tongue, which projects from the Sole to fill up that Slit, that the Water may not run through it; *l m n o* is one of the Ladle-Boards put on upon the Supporter *p q*, which goes through the Tongue and Sole of the Wheel, with a Hole behind at *q*, through which a Key or Wedge, like a Piece of Wood, being driven, fastens it with the Board to the Wheel. Thus you see by this Means how great the Force of the Water must be on the Wheel, and how much less will suffice here than in the common Way.

Fig. 2.

22. As to an *Undershot-Wheel*, it is evident there can be only the Force of the Impulse from the Water on such a Wheel; and therefore the Height of Water remaining the same, there must be a larger Aperture of the Pen-stock, that so a greater Quantity of Water may come upon the Float-Boards in the same Time, to

to have an equal *Momentum*, or to produce the same Effect as in the *Overshot* or *Breast-Wheel.* Whence a greater Expence of Water will be made here, than in any other Sort of Mill, and can only be supplied for a Constancy by a River; and where there is not a Want of Water, this is the easiest, cheapest, and most simple Structure a Mill is capable of.

23. In this Case, the Float-Boards are to be placed perpendicular to the Sole or Periphery of the Wheel, because when they come into a vertical Position at the Bottom or lowest Part of the Wheel, they will then receive the horizontal Impetus of the Water directly, and therefore with the greatest Force. If the Water-Course be sufficient to cover the Float-Boards, that is enough; all that is more, runs waste, either by the Sides or under the Floats, or both.

24. Let XYZ be an Undershot-Wheel, and HC the Water coming upon it in the Direction HC. Now if there were so few Float-Boards, that when one, as DE, became vertical, the next on each Side IK, LM, should but just touch the Surface of the current Water, then would the Water strike the said Float-Board DE with all its Force. But when the two Floats K and E come into the Situation C and G, then will the Water strike but a Part of the Float C, *viz.* BC, and that too obliquely, which is to be estimated by AC, the Sine of the Angle of Incidence; and since we suppose the Section of the Water equal to the Area of a Float-Board, it is plain the Float C will intercept the Water from the Float G, so that none can strike it; and therefore the Force of Water will be every where less than upon the Float-Board in the Site DE. So that the Force upon the Wheel will always be fluctuating between AC and DE, which will be the two Extremes. Pl. XIII. Fig. 3.

25. If the Float-Boards are so many, that while one is vertical as DE, others on each Side, as C and G, are also partly in the Water-Way; then in this Case it is best to have the Stream of Water larger than the Floats, that some may run beside, and fill up the Space between, that so the Floats C, E, G, may all be impell'd at once; for the Back-Water, here, having a greater Velocity than that of the Wheel, will still keep

the Float G forward; and though it be an oblique Force, it will always avail ſomething ſo long as it touches the Float; and cannot be eſteemed a Negative Quantity, as ſome have aſſerted.

26. Since many may be curious to know how the Force of Impulſe and Gravity of the Water in the Buckets of an *Overſhot-Wheel* is to be computed or eſtimated to a Mathematical Exactneſs, I ſhall here give the Method, and illuſtrate it by a Scheme. Let ABC, *&c.* be the Buckets of an Overſhot-Wheel, (having twenty-four in all) inclined to the Periphery in an Angle of forty-five Degrees: Thoſe with Dots or Points repreſent the Buckets with the different Quantities of Water they contain, among which the largeſt Dots in the Middle repreſent the Centres of Gravity of the ſeveral Bodies of Water. Now ſuppoſe the Water comes upon the uppermoſt Bucket A in the horizontal Direction XA, though by its Curvature at entering the Bucket it cannot ſtrike the Side of the Bucket directly, yet, ſince the Side of the Bucket obſtructs and ſuſtains the ſaid whole horizontal percuſſive Force, and that under an Inclination of forty-five Degrees, we may conclude that half that percuſſive Force is ſpent in turning about the Wheel.

27. As to the Force ariſing from every deſcending Bucket of Water, it may be eaſily determined by finding the Bulk of the Water in each Bucket, and multiplying that in the perpendicular Diſtance of the Line of Direction of the Centre of Gravity from the Centre of Motion. Hence, with reſpect to the firſt Bucket A, ſince its Centre of Gravity acts in the Direction VN, that is, perpendicularly on the Centre of Motion, that Product, or the Weight of that Bucket of Water, will avail nothing to move the Wheel round, as being wholly ſupported on the Axis of Motion.

28. But the Water in the ſecond Bucket B gravitating in the Direction R O, at the perpendicular Diſtance N O from the Centre, if we multiply its Maſs or Weight into the Diſtance N O, we ſhall have its *Momentum* or Force to move the Wheel; and ſo of any other. Here we may obſerve, that as the Water in the Buckets decreaſes, the Diſtances increaſe from the Centre, in the upper deſcending Quadrant; and ſince the

the Diſtances increaſe much faſter than the Quantities decreaſe, the Forces will increaſe till we come to the horizontal Bucket F, which is half full of Water; from thence it decreaſes to the Bucket I, where all the Water runs out, as being parallel with the Horizon, or Diameter K M. And hence we ſee four Buckets on the lower deſcending Quadrant carry no Water, when they are inclined to an Angle of 45 Degrees.

29. I ſhall here ſubjoin a Specimen of the Computation, ſuppoſing the Wheel and Buckets ſuch as in the Scheme, and the Quantities of Water, and Diſtances of their Centres of Gravity, the ſame as in the Table below.

Water in the Bucket.		*Diſtances from the Centre* N.		*Momenta of each Bucket.*
A = 1,00	×	O		= 0,00
B = 0,9	×	NO	= 0,3	= 0,27
C = 0,8	×	NP	= 0,6	= 0,48
D = 0,7	×	NQ	= 0,85	= 0,595
E = 0,6	×	NL	= 0,95	= 0,57
F = 0,5	×	NM	= 1,00	= 0,5
G = 0,4	×	NM	= 1,00	= 0,4
H = 0,2	×	NL	= 0,55	= 0,19

The Total of all the Buckets = 3,005

That is, the *Momentum* of Water in all the Buckets is equal to the *Momentum* of three Times the Water contained in the Bucket A, and hanging at the End of the horizontal Diameter K M.

30. I ſhall conclude this Theory of *Water-Mills*, with a Deſcription of Dr. *Barker*'s new-invented Mill, of the moſt ſimple Structure of any yet made, performing its Effect without any Wheel, Trundle, Cog, or Round; the Nature of the Machine, and Manner of its Operation, will be eaſy to underſtand from the following Account of its ſeveral Parts.

31. ABCD is an upright Frame ſtanding on a proper Baſe; EF is the wider Part of GH an upright hollow Pipe or Tube, fixed at the Bottom to an horizontal Square Trunk IK; which Trunk, together with the Tube, is fixed to an upright Spindle or Axis RS, by means of a Nut and Screw at S. The lower End of Pl. XIII. Fig. 5.

 the

the Axis on a fine Point moves in the Pivot Hole in the Part of the Frame at T; on the upper Part of the Frame is a Hole thro' which the Spindle passes, as also thro' the round circular Piece P fix'd on the said Frame; on the upper Part of the Spindle is fix'd another round circular Piece O, which represents the upper moveable Stone of the Mill. Q is a Spout of Water filling the Tube or Trunk, and giving Motion thereto, and consequently to the Axis and upper Stone, by the horizontal Jets of Water from each End of the Trunk IK, thro' Holes made at each End on contrary Sides.

32. While the Holes continue stopped, the Trunk will be at rest, because then the Pressure is equal over all the Parts: but when the Holes are open, the Pressure of the Water (by its having Liberty to issue out) will be less on that Part where the Hole is, than on the other Part opposite to it; which stronger Pressure will prevail, and carry round the Trunk and Tube with the Axle and Stone, in a contrary Direction; and each Hole contributes to produce this Motion, which will be greater or lesser in Proportion to the *Momenta* of the Jets of Water, or greater or lesser Aperture of the Holes.

33. For it is easy to understand, that the Power of this Machine is derived from, or depends upon, three Things: (1.) The Velocity of the Spouting Water; (2.) The Quantity thereof; and (3.) The Distance at which the Water spouts from the Axis of Motion. The two first make the *Momentum* arising from the Pressure of the Fluid, which is proportional to the Altitude, or Height of the Tube; the last is of a Mechanical Nature, for the Trunk is in this respect exactly of the Nature of the *Lever*.

34. In the Lecture of Hydrostatics it will be shewn, that the Velocity of the Spouting Fluid will be as the Square Root of the Altitude of the Fluid; whence this Part of the Force will be *in the Subduplicate Ratio of the Height of the Tube*. The Quantity of the spouting Fluid will be also in the same Ratio, while the Apertures are the same; but if these vary their Magnitude, it will be directly as the Aperture, the Altitudes being given. Therefore the whole *Momentum* arising from these

these hydrostatical Principles will be *as the Altitude of the Tube, and Sum of the Apertures conjointly*; and if this be multiplied by the Distance of the Aperture from the Centre or Axis, we shall have the Expression of the whole Force of the Machine.

35. That is, if $H =$ the Height of the Fluid, $A =$ Sum of both the Apertures, and $D =$ Distance of each from the Axis; then we shall have the *Momentum* M expressed by $M = H \times A \times D$. Hence we see that the Trunk IK is analogous to the great Wheel of a common Water-Mill, whose Force is in the same Manner computed from the Height of the Fluid H, the Aperture of the Sluice A, and the Distance or Radius of the Wheel D.

36. For both in the common Mill and this, if the Aperture A and Distance D from the Centre be the same, the Force of the Jet will vary with the Height of the Fluid H. Also if the Height of the Fluid H and Length of the Trunk or Diameter of the Wheel D remain the same, the Force will be as the Aperture A, or as the Quantity of the Fluid flowing out in a given Time. Lastly, if the Height of the Fluid H, and the Aperture A continue the same, the Force will be as the Diameter D of the Wheel, or Length of the Trunk of this new Mill.

37. I shall now give a Calculation of the Power of this Machine; and in the first Place, let us suppose the Height of the Tube to be 9 Feet, and always full of Water; the Velocity of the Spouting Water will be the same that a Body will acquire in falling 9 Feet, *viz.* an uniform Velocity of 18 Feet in the *Time of the Fall*; which Time is thus found, as $16 : 9 :: 1^2 : \frac{9}{16} =$ the Square of the Time, whose Square Root is $\frac{3}{4}$, that is, $\frac{3}{4}$ of a Second. Therefore say, as $\frac{3}{4} : 18 :: 1 : 24$; the uniform Velocity is at the Rate of 24 Feet *per* Second, in the Spouting Water, from 9 Feet Altitude.

38. Let the Aperture at each End of the Trunk be 6 Inches Square, then in both there will be 72 square Inches, or half a square Foot in Area. The Water issuing out in a Second at both Orifices will be equal to a Column 24 Feet long, and $\frac{1}{2}$ a Square Foot in Base; therefore the whole Water will be equal to 12 Cu-

bic Feet. The Weight of one Cubic Foot is $62\frac{1}{2}$ *lb.*; wherefore $62\frac{1}{2} \times 12 = 750$ *lb.* is the Force acting on the Trunk, supposing the Depth of Water only one Foot; but since it is at the Depth of 9 Feet, we have $9 \times 750 = 6750$ *lb.* acting upon the extreme Parts of the Trunk *per* Second.

39. Let us now suppose the Length of the Trunk to be 6 Feet; then the Motion is made by a Power of 6750 *lb.* acting at the End of a Lever three Feet long; consequently $6750 \times 3 = 20250$ *lb.* the *Momentum* of the Wheel *per* Second.

40. Now the Velocity of the Trunk at the Orifices will be the same as that of the Water nearly, while the Mill is not charged; but that of the Water is 24 Feet *per* Second, and the Circumference which each Orifice describes in 18,84, or 19 Feet nearly; therefore the Trunk turns round a little more than once *per* Second. But when the Mill is charged with the Stone, Corn, &c. the Velocity will not then be so great; and we have shewn (*Annot.* XL. 6.) that it ought not to be above $\frac{1}{3}$ Part of that of the Water, *viz.* at the Rate of 8 Feet *per* Second; but let us suppose it a little more, *viz.* $9\frac{1}{2}$ Feet, then will the Trunk move round once in two Seconds.

41. If we suppose the Stone 6 Feet in Diameter, its Velocity in the Periphery will be the same with that of the Trunk; *viz.* $9\frac{1}{2}$ Feet *per* Second; but $\frac{2}{3}$ of this is the mean Velocity of the Stone, which is therefore but $6\frac{3}{10}$ Feet *per* Second. Suppose the Weight of the Stone 1912 *lb.* and that $\frac{1}{3}$ of this be allowed for Friction, *viz.* 637 *lb.* then $6,3 \times 637 = 4015$ *lb.* the Moment of the Stone by its Resistance arising from $\frac{1}{3}$ of its Weight; but this is only when the Stone is first put into Motion; after it is in Motion, this *Momentum* is greatly lessen'd by the Centrifugal Forces; and therefore if we allow 4000 *lb.* for the *Momentum* arising from the Weight of the Stone, Attrition of the Corn, &c. it will be but $\frac{1}{5}$ Part of the *Momentum* of the Trunk or Mill, which we shewed was 20250 *lb.*

42. Here we have supposed the Mill to carry the largest Stone that is used in any Mill, and that the Power of the Mill is five Times greater than the Resistance to be

IN WIND-MILLS the Mechaniſm is the ſame, only the Vanes or Sails are to be conſidered as a *Wheel on the Axle*, actuated by the Power of the Wind. In this Machine we have only to conſider the Poſition of the Sails, and the Power of the Wind. As to the former, if the Sails ſtand *right* before the Wind, it cannot affect them at all; if they ſtand *direct* to the Wind,

be overcome; the Quantity of Water therefore in the Jets may be diminiſh'd a *Fifth Part*, and conſequently the Apertures; which therefore, inſtead of being 72 Square Inches, need be only 15 or 16. If therefore at each End of the Trunk there be an Aperture of four Inches long and two wide, the Jets from the Altitude of nine Feet, at three Feet from the Axis, will have a *Momentum* ſufficient for turning the largeſt Stones.

43. I ſhall only obſerve further, that there are many mechanical Purpoſes to which this Invention might be made ſubſervient, as it has ſo great a Power of Motion in ſo eaſy and ſimple a Structure: And alſo, that the Water-Wheel of a common Mill, if placed in a horizontal Situation, and the Sluice ſo ordered as to throw a Side-Jet in a Tangent-Direction on the Ladle-Boards, ſuch a Wheel would be in the ſame Circumſtances nearly as the Trunk of this New Mill, and therefore might be made to anſwer all the ſame Intentions, and capable of nearly all the ſame Advantages.

44. I need not obſerve to the Reader, that whatever Quantity of Water is expended at the Jets, as that muſt be ſupplied at the Top or Ciſtern of the Tube, ſo the Stream which ſupplies it muſt be as much longer than the Jets, as its Velocity is leſs; becauſe what is deficient in one Reſpect muſt be made up in the other. There are ſeveral other Circumſtances and Particulars, which may deſerve to be conſider'd, when it ſhall be found to anſwer any valuable End, beyond that of mere Speculation.

Wind, the Mill will be blown down, at least the Sails can have no Power to move round; they must therefore be placed *oblique* to the Wind, and that under an Angle of 54 Degrees, and 44 Minutes, for the greatest Advantage (XLVI).

(XLVI) The internal Parts of a Wind-Mill are the same with those of a Water-Mill nearly, of which I shall take no farther Notice, but shall confine myself to the Theory of the Sails in regard of their Position, Motion, and Figure.

1. In regard of the Position of the Sails, we must consider that if they are placed *Direct to the Wind*, or at Right Angles to the Axis of the Mill, they will receive the whole Force of the Wind, which in this Case will tend to blow them forward, and consequently to blow down the Mill; which Position of Course cannot be admitted.

2. If the Sails are set *Right to the Wind*, or parallel with the Axis of the Mill, 'tis plain that in that Position the Wind cannot act upon them at all, and therefore they cannot be turn'd round, nor the Mill put into Motion; which Position of the Sails must likewise be rejected.

3. Since neither the *Direct* nor *Right Position* of the Sails will do, an *Oblique Position* must, as there can be no other. Now to shew that an Oblique Position of

Pl. XIII. Fig. 6. the Sails will turn the Mill, let AB be the Axis, CD a Sail, and its Angle of Obliquity (*viz.* that which it makes with the Axis) be ECG; then if GC be the Force of the Wind in the Direct Position of the Sail, GE will be the Force of the Wind in its oblique Position (as being the Sine of the Angle of Incidence GCE). But the Force GE is resolvable into two others, EF and GF; of which the latter, being parallel to the Axis, avails nothing in turning the Sails about it; but the other, EF, being perpendicular thereto, is wholly spent

spent in compelling the Sail to turn round; which was the Thing to be shewn.

4. The Force of the Wind on the Sail will be as the Square of the Sine of Incidence, or as $\overline{GE}^2$; for the Force of each single Particle of Air will be as the Sine GE, (by *Annot.* XXIV. 6.) and it will be also as the Number of Particles which strike it at the same Time, which Number of Particles is also as the Sine of Incidence GE. For let CD represent the Section of the Sail in a direct Position, and CG the same in an oblique Position, 'tis plain the Number of Particles striking it in the former Case will be to the Number striking it in the latter as CD to CF, which is equal to GE the Sine of Incidence; for all the Particles between AD and BF will not come upon the Sail in the oblique Position CG. Since then the Force of the Wind on the Sail is on two Accounts as GE, it will be as the Square of the said Line GE.

Pl. XII. Fig. 7.

5. If we suppose the Velocity of the Wind to vary, the Force thereof will be as the Square of the Velocity; for the greater the Velocity, the greater will be the Stroke of each single Particle, and also the greater will be the Number of Particles coming upon the Sail in the same Time; the Force will be therefore as the Squares of the Velocity.

6. Again, if the Area of the Sail be variable, the Force of the Wind will be directly as the Area or Superficies of the Sail, because the Number of Particles of the Air coming upon it will always be proportional thereto, and consequently the Force with which they strike it. Hence, if A, S, and V represent the *Area*, *Sine* of Incidence, and *Velocity* of the Wind on one Sail, and *a*, *s*, *v*, those on another; the Force compelling the former to turn round will be to that compelling the latter, as $A \times S^2 \times V^2$ to $a \times s^2 \times v^2$.

7. When the Area of the Sail and its Position in respect of the Wind continue the same, the Force which turns the Sail will be as the *Squares of the Velocity*; and since the Wind scarce ever blows with one uniform Velocity, but varies with almost every Blast, the Force upon the Sail will be much more variable and unequal; and therefore the Action or Working of a Wind-Mill cannot

cannot be so equal, uniform, and steady as that of a Water-Mill, whose Power is always of the same Tenor, while the Jet of Water is so.

8. If the Area of the Sail and the Velocity of the Wind be supposed constant, the Force of the Wind in the Direct Position will be to that in the Oblique one as $\overline{GC}$ to $\overline{GE}^2$, as we have before shewn; and it has been also shewn that that Part of the Force which turns the Sail is represented by EF, when GE is the whole Force: But $GE : EF\ (:: GC : CE) :: \overline{GE}^2 : \frac{CE \times \overline{GE}^2}{GC}$ = to the Force which turns the Sail, when the whole Force is represented by $\overline{GE}^2$, as is here the proper Expression of it.

9. This Expression $\frac{CE \times \overline{GE}^2}{GC}$ begins from Nothing, when the Angle of Incidence begins to be oblique, and increases with the Obliquity of the said Angle to a certain Number of Degrees; because that Part of the Force which is parallel to the Axis becomes lesser in proportion to that which is perpendicular to it; but after it has pass'd this Limit, it again decreases, and becomes nothing, when the Angle of Incidence vanishes; as is easy to understand by considering that the Quantity of Wind on the Sail does in this Case continually decrease.

10. There is therefore one certain Position of the Sail, in which the Force of the Wind is greatest of all upon it, or a *Maximum*; and to find it, put Radius $GC = a$, $EC = x$, and we have $\overline{GE}^2 = aa - xx$, and consequently the Force $\frac{CE \times \overline{GE}^2}{GC} = \frac{aax - xxx}{a}$, which must be a *Maximum*: Therefore its Fluxion $a a\dot{x} - 3xx\dot{x} = 0$; whence $aa = 3xx$, and so $x = \sqrt{\frac{aa}{3}}$, which in Logarithms is $\frac{20,000000 - 0,477121}{2} =$ 9,761439, which is the Logarithm Sine of the Angle 35° 16′ = Angle CGE; and therefore the Angle ECG is

is equal to 54° 44′, when the Force of the Wind is a *Maximum*, as required.

11. The Angle now found, is only that which gives the Wind the greateſt Force to put the Sail in Motion, but not the Angle which gives the Force of the Wind a *Maximum* upon the Sail when in Motion. What this Angle is Mr. *MacLaurin* has ſhewn in his Book of *Fluxions*, and which I have further explained in a Treatiſe, intitled, *New Principles* of GEOGRAPHY and NAVIGATION.

12. Mr. *Parent* has alſo ſhewn that an Elliptic Form of the Sails is better than the Parallelogram or long Square; and that the beſt Poſition of the Sail is not that which is common, *viz.* with its longeſt Side or Diameter parallel to the Axis of the Sail; but on the contrary, it ought to be perpendicular to it; that is, they ought to be of ſuch a Form, and placed in ſuch a Manner, as repreſented in the Figure; and after the four Sails B, C, D, E, are thus placed in the Axis or Arm A, they are then to be turn'd about, and fix'd under the proper Angle of Obliquity above-mention'd. Pl. X. Fig. 9.

13. There are three Things yet wanting to the Perfection of a Wind-Mill. The *Firſt* is, ſome Contrivance in the Nature of a *Fly* to regulate the Motion of the Train, under the irregular and unequal Impulſe of the Wind. The *Second* is, ſome other Contrivance to ſupply the Hopper or Stones with more or leſs Corn, in Proportion to the greater or leſs Strength of the Wind. And, *Thirdly*, a Method of altering the Angle of the Sail's Obliquity from its *Maximum* of 54° 44′ at the Beginning of the Motion to its *Maximum* when in Motion.

14. By means of an *Anemoſcope* (hereafter to be deſcribed) it will be eaſy to prove by Experiment what Form of the Sails, that is, whether Rectangular or Elliptical, whether the Vertical or Horizontal Poſition of the longeſt Diameter, and what Angle of Obliquity is beſt; alſo whether the Surface of the Sails ſhould be plain or concave, with many other Things of this Kind. But this muſt be done by as many particular Anemoſcopes, or in other Words, you muſt have an Anemoſcope for every particular Experiment, and all diſpoſed together in one common Frame; the Reaſon is evident, becauſe

because they all require the same Strength of Wind, which cannot otherwise be had.

15. That these Things may be better understood, I shall premise the following Definition of some Geometrical Lines and Figures, which are absolutely necessary to a compleat Knowledge of the modern Mechanical Philosophy.

Pl. IX. Fig. 6. 16. I take it for granted, that the Reader knows, that if on any Point C, taken in the Right Line AB, a Circle ADEF be described, the Point C is call'd the *Centre*, and AE the *Diameter* of the Circle: To which I shall add, that AC, or CE, is call'd the *Radius* of the Circle, which is the same Thing as the *Semidiameter*.

17. If the Circle be divided into four equal Parts, AD=DE=EF=FA, by the two Diameters AE, DF; then each of the Areas ACD, DCE, ECF, FCA, are call'd *Quadrants*, or Quarters of the circular Space; and the Parts of the Circle AD, DE, EF, FA, are call'd *Quadrantal Arches*, or Quarters of the Circle.

Fig. 7. 18. Every Circle, great or small, is supposed to be divided into 360 equal Parts, call'd *Degrees*; consequently each Quarter, AD, DE, &c. will contain 90 of those Degrees, as is evidently represented by the large Half-Circle of *Fig*. 7.

19. If two Lines BC and FC meet in a Point C, the Space FCB included between them is call'd an *Angle*; and the Measure of that Angle is the Number of Degrees contain'd in an Arch EI of a Circle described on the Angular Point C, and included between the said two Lines BC and FC. Thus the Angle in the Figure contains 40 Degrees.

20. If a Line, as GC, be drawn through the 90th Degree on the Point C, it will make the Angle on one Side GCB equal to the Angle GCA on the other Side, because each is equal to 90 Degrees. Such an Angle is called a *Right Angle*; and the Line GC is then said to be *perpendicular* to the Line AB.

21. The Angle FCB, which is less than a Right Angle or 90 Degrees, is call'd an *Acute Angle*; and the Angle HCB, which is greater than a Right Angle or 90 Degrees, is call'd an *Obtuse Angle*. Again; the Arch

Arch I D is called the *Complement* of the Arch E I to a Quadrant E D, or 90 Degrees; and the Arch A K is called the *Supplement* of the Arch E K to a Semicircle E D A, or 180 Degrees.

22. If from the Point I be let fall the Perpendicular I L to the Line or Radius E C, then is that Line I L called the *Sine* of the Angle I C E or F C B. In the same Manner the Line I M is the Sine of the Complement Arch I D, or Angle F C D. But instead of *Sine-Complement*, we say, in short, *Co-Sine*: Thus we say that I L is the *Sine*, and I M the *Co-Sine*, of the Angle I C E. The Angle F C B is called the *Inclination* of the Line F C to the Line B C; and the Angle F C D is the *Inclination* of the Line F C to the Perpendicular D C: That is, F C is inclined to B C in an Angle of 40 Degrees, and to D C in an Angle of 50 Degrees. Hence I L and I M are called the *Sines of Inclination* respectively.

23. Hence, when we say, *The Force of a direct Stroke is to that of an oblique one as Radius is to the Sine of Inclination*, we only mean, that those Quantities are to each other as the Radius I C to the Sine I L, or I M, according as the Inclination is 40 or 50 Degrees. Also, when it is said that *the centrifugal Force decreases from the Equator towards the Poles, in proportion of Radius to the Co-Sines of the Latitude*; no more is meant than this, that if the Radius C E represent the said Force in the Equator E, and E I be any given Latitude, then will I M, the Co-Sine of the Latitude, represent the Force in that Latitude: Or, the Force decreases with the Length of the Line I M, as the Point I moves on from E to D.

24. In several Books we have Tables of Numbers which express the Length of the Sine of every Degree and Minute of the Quadrant, in such equal Parts as the Radius C E or C I contains 100000. And since it is of the greatest Use to know the Proportion of Radius to the Sine of every Degree, I have here subjoined a Table thereof, and a Specimen of its Use.

Deg.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.
SINE of 1	1745	19	32556	37	60181	55	81915	73	95630
2	3483	20	34202	38	61566	56	82903	74	96126
3	5233	21	35836	39	62932	57	83867	75	96592
4	6975	22	37460	40	64278	58	84804	76	97029
5	8715	23	39073	41	65605	59	85716	77	97437
6	10452	24	40673	42	66913	60	86602	78	97814
7	12186	25	42261	43	68199	61	87461	79	98162
8	13917	26	43837	44	69465	62	88294	80	98480
9	15643	27	45399	45	70710	63	89100	81	98768
10	17364	28	46947	46	71933	64	89879	82	99026
11	19080	29	48480	47	73135	65	90630	83	99254
12	20791	30	50000	48	74314	66	91354	84	99452
13	22495	31	51503	49	75470	67	92050	85	99619
14	24192	32	52991	50	76604	68	92718	86	99756
15	25881	33	54463	51	77714	69	93358	87	99862
16	27563	34	55919	52	78801	70	93969	88	99939
17	29237	35	57357	53	79863	71	94551	89	99984
18	30901	36	58778	54	80901	72	95105	90	100000

25. The Uſe of this Table will be obvious from two or three Examples. It was obſerved, that the Power is to the Weight it ſuſtains on any Inclined Plane IC, as the Height of the Plane IL to the Length thereof IC; that is, as the Sine of the Plane's Inclination to the Radius. Suppoſe the Angle of Inclination ICE = 40 Degrees, then will the Sine IL be equal to 64278, and the Radius CI = to 100000, which Numbers are as 64 to 100; therefore 100 Pounds will be ſuſtained on the Inclined Plane by a Power equal to 64 Pounds nearly.

26. Again; ſince EC = 100000 repreſents the *centrifugal Force* under the Equator, then will IM = 76604 (the Sine of 50 Degrees, and Co-Sine of 40) be as the ſaid Force in the Latitude of 40 Degrees: Which Numbers are as 1000 to 766; and ſuch is the Proportion of the Forces in thoſe two Places.

27. In the ſame Manner, if the Radius CD = 100000 expreſs the Force of any *direct Stroke*, then will the Sine IL = 64278 be expreſſive of the Force of an *oblique Stroke* in the Direction FC, every Thing elſe being equal.

28. Again; ſince the Force of a direct Stroke is expreſs'd by CD = 100000, if it were required to find the Angle of Obliquity, ſuch that the Force of the Stroke

Stroke shall be but one fourth Part so great, or 25000, look in the Table for the Number nearest to this, and you will perceive it to lie between 14 and 15 Degrees, and therefore about 14 Degrees and a Half is the Angle required.

29. In the last Place: It was said, That *the Force of the Wind on the Sail is proportional to the Squares of the Sines of the Angle of Incidence.* This may be illustrated by Numbers, thus: If the Sail be turned to the same Wind, first under an Angle of 45 Degrees, and then under an Angle of 30 Degrees; the Sine of the first Angle is (by the *Table*) 70710, and of the latter 50000, the Squares of which are 4999904100 and 2500000000, which Squares are as 50 to 25 or as 2 to 1; and therefore the Power of the Wind is twice as great upon an Angle of 45 Degrees, as upon an Angle of 30.

30. Because the Square of the Sine of 45 Degrees is 5000000000, twice that Square will be 10000000000, which is equal to the Square of Radius 100000; it is evident the Sum of the Squares of the Sines of any two Angles above 45 Degrees will be greater than the Square of Radius; and therefore the Force of the Wind upon two oblique Sails, in that Case, will be greater than upon one Sail set direct before the Wind.

31. After the same manner, the Table of Sines may be applied to Calculation in any other Case of the like Nature, where the *Proportion of Radius and Sine of an Angle* is required to be expressed or stated in Numbers. And since each Degree is divided into 60 equal Parts or *Minutes*, therefore the Sine of any Number of Degrees and Minutes also may be easily found by the foregoing Table, by those who understand the Rule of Proportion.

32. I shall only further observe here, that as I L is the *Sine*, and L M the *Co-sine*, of the Arch I E: so if on the extreme Point E of the Radius C E we raise a Perpendicular which shall cut the Line C I, continued, in F, then is the said Perpendicular E F called the TANGENT of the Angle I C E, and the Line F C the SECANT of the same Angle. In like manner the Perpendicular D G is the *Tangent* of the Angle D C I, and

IT is to be obſerved, that in order to turn a Ship about in the leaſt Time, or with the greateſt Celerity poſſible, the Rudder ought to make an Angle with the Stern of 54 Degrees, 44 Minutes: And alſo, that this is the Angle which the *Gates of a Lock* upon a River ought to make with the Sides of the River, in order to reſiſt the Water with the greateſt poſſible Force. (XLVII).

I SHALL

G C the *Secant* thereof; and therefore D G is the *Co-Tangent*, and G C the *Co-Secant* of the Angle I C E. I thought it neceſſary to acquaint the Reader with theſe Definitions, becauſe they ſometimes occur in Treatiſes of this Kind.

Pl. XIII. Fig. 8. (XLVII) 1. If A B F be the Rudder of a Ship, A H placed in the oblique Situation F C, and the Water ſtriking againſt it in the Direction G C; let C E be Radius, then the Sine of the Angle of Incidence will be F E, and ſo the Force of the Water againſt the Rudder will be as $\overline{FE}^2$; but E F is reſolvable into the two Forces F D and D E, of which the latter is parallel, the other perpendicular to the Direction of the Ship's Courſe, and therefore F D is the only Part of the Force that compels the Ship to turn round. But E F is to F D (:: C E : C F) :: $\overline{EF}^2 : \frac{CF \times \overline{EF}^2}{CE}$, that is (putting $CE = a$, $CF = x$) as $\frac{aax - x^3}{a}$, and ſo x will be found (by the Method *de Maximis & Minimis*) equal to $\sqrt{\frac{aa}{3}}$; and therefore the Angle of Incidence $ECF = 54^\circ\ 44'$, as before, when the Force of the Water againſt the Rudder to turn the Ship is a *Maximum*.

I SHALL conclude this Lecture with a few Words concerning WHEEL-CARRIAGES, the

2. After a like Manner we determine the Angle of Position of the Gates AE, BE, of a Lock upon a River, (*viz.* the Angle BAE = ABE) in which the said Gates shall resist the Pressure of the Water with the greatest Force possible. For if upon AB we describe the Semicircle ADB, and continue AE to D; then the Pressure of the Water against the Gate AE will increase with the Length of the Gate, and the Resistance of the Gate will decrease as the Pressure increases, and therefore it will be on this Account inversely as the Length of the Gate, or as $\frac{1}{AE}$.

3. Again, the Resistance will be diminish'd as the Length of the Gate increases, inasmuch as the Strength of the Gate will be diminish'd in that Proportion, therefore it will be on this Account also as $\frac{1}{AE}$, consequently the Resistance of the Gate on both these Accounts will be as $\frac{1}{\overline{AE}^2}$. Pl. XIII. Fig. 16.

4. Join BD and EC perpendicular to AB; then we have $\overline{AE}^2 : \overline{AC}^2 :: \overline{AB}^2 : \overline{AD}^2$; and here, because $\overline{AD}^2$ decreases as $\overline{AE}^2$ increases, we shall have the Resistance expressed by $\overline{AD}^2$.

5. But this Resistance is augmented by the Oblique Pressure of the other Gate, which let be represented by BE; this Oblique Force BE is resolvable into two Forces BD and DE; which latter, as it is parallel to the Gate AE, avails nothing, but the other BD being at Right Angles thereto, is wholly spent in resisting it; therefore the whole Resistance the Water meets with from the Gate AE, is as $\overline{AD}^2 \times BD$.

6. This Expression, therefore, is to be determined to a *Maximum*; in order to which, let $AB = a$, $BD = x$, and then $\overline{AD}^2 = aa - xx$, and so $\overline{AD}^2 \times BD = aax$

the whole Doctrine whereof (as it ſtands on a *Mathematical Theory*) may be reduced

$= aax - x^3$, whoſe Fluxion $aa\dot{x} - 3xx\dot{x} = 0$, gives $x = \sqrt{\frac{aa}{3}}$, which ſhews the Angle BAE = $35^\circ, 16'$, as in the Examples above.

Pl. XIII. Fig. 9. 7. Since we are upon the Subject of *Maximums*, I ſhall here add Examples of two or three other Caſes of the ſame Kind, which it is hoped will be acceptable to the Curious, and yet not beſides the Purpoſe of Mechanical Gentlemen. Let BB be a Piece of Wood placed horizontally, and ſupported by the Pieces AB, AB, which make a given Angle ABC with the former; it is required to find the Poſitions of two other Pieces AC, AC, given in Length, ſuch that they ſhall ſupport the Piece BB with the greateſt Force poſſible.

8. The Pieces AC, are fixed in A and C ſo as not to ſlip, they are ſuppoſed to have no conſiderable Weight. Then make $BH = \frac{1}{2}AC$, and from the Points A and H draw the Lines AG, HK, at right Angles to BB. If AC expreſſes the abſolute Strength of the Piece AC, then AG will expreſs the Strength with which it ſupports the Piece BB, as being perpendicular thereto. Now AG multiplied by the Lever (or Diſtance) BC from the Centre of Motion B, (which expreſſes the *Momentum* or Force of the Piece AC) ought to be a *Maximum*.

9. To this End, put $AC = a$, $GA = x$; alſo $HK = n$, and $KB = m$; then $GC = \sqrt{aa - xx}$, and becauſe of the ſimilar Triangles HKB, AGB, we have HK : KB :: AG : GB $= \frac{m}{n}x$, and ſo $BC = \sqrt{aa - xx} - \frac{m}{n}x$, and

$AG \times BC = x\sqrt{aa - xx} - \frac{m}{n}xx$, whoſe Fluxion made equal to nothing is $\dot{x}\sqrt{aa - xx} - \frac{xx\dot{x}}{\sqrt{aa - xx}} - \frac{2m}{n}x\dot{x}$ $= 0$.

duced to the following Particulars, *viz.* (1.) Wheel-Carriages meet with leſs Reſiſtance

10. Therefore $\sqrt{aa-xx} - \frac{xx}{\sqrt{aa-xx}} - \frac{2m}{n} x = o$:

Whence $aa - 2xx - \frac{2m}{n} x \sqrt{aa-xx} = o$.

And ſquaring each Part $\Big\}$ $a^4 - 4a^2x^2 + 4x^4 - \frac{4m^2}{n^2} a^2 x^2 + 4\frac{m^2}{n^2} x^4 = o$.

And multiplying by n^2 $\Big\}$ $n^2 a^4 - 4n^2 a^2 x^2 - 4m^2 a^2 x^2 + 4n^2 x^4 + 4m^2 x^4 = o$.

But it is $4n^2 + 4m^2 = aa$, becauſe $\overline{BH}^2 = \overline{HK}^2 + \overline{KB}^2$,

Therefore $n^2 a^4 - 4n^2 a^2 x^2 - 4m^2 a^2 x^2 + a^2 x^4 = o$.

And dividing by a^2 $\Big\}$ $n^2 a^2 - 4n^2 x^2 - 4m^2 x^2 + x^4 = o$.

Whence again $n^2 a^2 - a^2 x^2 + x^4 = o$, (becauſe $-4n^2 - 4m^2 = -aa$)

Hence by Tranſpoſition $\Big\}$ $x^4 - a^2 x^2 = -a^2 n^2$.

And compleating the Square $\Big\}$ $x^4 - a^2 x^2 + \frac{1}{4} aa^4 = \frac{1}{4} aa^4 - a^2 n^2$.

But it is $\frac{1}{4} aa^4 - a^2 n^2 = aam^2 + a^2 n^2 - a^2 n^2 = a^2 m^2$.

Therefore $x^4 - a^2 z^2 + \frac{1}{4} aa^4 = a^2 m^2$.

And extracting the Square Root. $\Big\}$ $x^2 - \frac{1}{2} aa = \pm am$.

Wherefore $x^2 = \frac{1}{2} aa \pm am$; and ſo $x = \sqrt{\frac{1}{2} aa \pm am}$.

11. Hence 'tis evident if $m\,(=BK) = o$, in which Caſe the Angle ABC is a right one, then $x = \sqrt{\frac{1}{2} aa}$, and therefore the Angle ACB will then be half a right one, or 45 Degrees.

12. Since the two Values of x, *viz.* $\sqrt{\frac{1}{2} aa - am}$ and $\sqrt{\frac{1}{2} aa + am}$ being ſquared, and the Sum of thoſe Squares are equal to the Square of the Radius, *viz.* $aa - am + \frac{1}{2} aa + am = aa$, therefore the two Angles, of which x is the Sine, are Complements to each other. Thus ſuppoſe the Angle ABC obtuſe, and $= 120$ Degrees, or the Angle ABG $= 60$ Degrees; then BK $= m = \frac{1}{4} a$, and ſo AG $= x = \sqrt{\frac{1}{2} aa - am} = \sqrt{\frac{1}{2} aa - \frac{1}{4} aa} = \frac{1}{2} a$,

sistance than any other. (2.) The larger the Wheels the easier is the Draught of the Carriage,

$\frac{1}{2}a$, therefore the Angle ACB is $=$ 30 Degrees. But if the Piece AC be placed on the other Side AB, then $x=\sqrt{\frac{1}{2}aa+am}=\sqrt{\frac{1}{4}aa}$, and so the Angle A C B would in that Case be $=$ 60 Degrees, and consequently equal to the Angle A B G.

13. If the Angle A B C, instead of being obtuse, were acute, and the Complement to this, we should have $AG \times BC = x\sqrt{aa-xx} = \frac{m}{n}xx$ a *Maximum*, which would, in Fluxions, give the same Value for x as before, *viz.* $\sqrt{\frac{1}{2}aa \mp am}$, which if substituted for x in the above Expression, will give $\frac{2ann + aam \mp 2amm}{2n}$, where 'tis plain if $x = \sqrt{\frac{1}{2}aa-am}$, the greatest Force will be $\frac{2an^2+a^2m-2am^2}{2n}$; but if $x=\sqrt{\frac{1}{2}aa+am}$, then the greatest Force will be $\frac{2an^2+a^2m+2am^2}{2n}$: But $x=\sqrt{\frac{1}{2}aa-am}$ is the Sine of 30°, and $x=\sqrt{\frac{1}{2}aa+am}$ is the Sine of 60°: therefore the Piece BB is supported with the greatest Force by the Piece AC when placed on that Side B on which the acute Angle is.

Pl. XIII. Fig. 10. 14. If A G be a Piece of Wood of an indefinite Length, and fixed in A, so as to make a given Angle G A D with the horizontal Line A D; let it be required to find the Position of another Piece D E, given in Length, such that it shall support the Piece A G with the greatest possible Force. To this End make AC $=\frac{1}{2}a$, and from B let fall the Perpendicular C B; then since the Angle A is given, the Ratio of C B to A B is given also, which let be as n to m; that is, let C B $=n$, and A B $=m$; from the Point D draw the Perpendicular D F $(=x)$ to the Piece A G. Then if D E $(=a)$ expresses the whole Force of the Piece D E pressing

Carriage. (3.) A Carriage upon four large Wheels, of equal Size, is drawn with

ing perpendicularly, D F will expreſs that with which it ſupports A G; therefore D F multiplied by the Diſtance from the Fulcrum A, or Lever A E, ought to be a *Maximum*.

15. Now from the ſimilar Triangles A C B, A D F, we have C B : B A :: D F : F A $= \frac{m}{n} x$, alſo F E $= \sqrt{aa - xx}$, therefore A E $= \sqrt{aa - xx} + \frac{m}{n} x$, which multiplied by D F $= x$, is A E × D F $= x \sqrt{aa - xx} + \frac{m}{n} xx$, whoſe Fluxion made equal to nothing will give $x = \sqrt{\frac{1}{2} aa \pm am}$, as before. If the Angle A D E be acute, the Point E will fall between F and A, and we ſhall have A E $= \frac{m}{n} x - \sqrt{aa - xx}$ and A E × D F $= \frac{m}{n} xx - x\sqrt{aa - xx}$, which fluxed will give the ſame Value of x as before found.

16. In each Caſe, it is plain $x = \sqrt{\frac{1}{2} aa + am} =$ Sine of 60°, muſt be the *Maximum*, which if ſubſtituted in the Expreſſion $x \sqrt{aa - xx} + \frac{m}{n} xx$, (where the Angle A D E is oblique) will give $\frac{2ann + aam + 2aam}{2n}$ = Force of D E which ſupports the Piece A G. But if $\sqrt{\frac{1}{2} aa + am}$ be put for x in the other Expreſſion $\frac{m}{n} xx - \sqrt{aa - xx}$, we ſhall have for the ſaid Force (when the Angle is acute) $\frac{aam + 2amm - 2aan}{2n}$, which is leſs than the other Forces; and therefore the latter Poſition is leſs advantageous than the former,

with less Force than with two of those Wheels, and two of a lesser Size. (4) If the Load be laid on the Axle of the larger Wheels, it will be drawn with less Force

17. I cannot conclude this Speculation of the *Maxima* and *Minima* of Quantities, without observing to the Reader, that though the Method of discovering them by Fluxions is a Part of Knowledge which the *Mathematicians* have but lately acquired, and which they esteem the Sublimity of human Science, yet this very Thing was imparted to the *Insect Tribe* at the first Creation of Things; for by this very Method it is that Bees construct the Cells of their Combs in which they deposit their Honey.

Plate XI. Fig. 5. 18. Each Cell consists of six plain Sides, which are all *Trapeziums*, but equal to each other. The Bottom of the Cell is contrived with three Rhombus's H K D I, D E F I, and F I H G, so disposed as to constitute a Solid Angle at I, under the three equal Angles D I H, D I F, and H I F; each of which is double the *Maximum* Angle of $54^\circ - 44' = DIK = DKI$. Hence it comes to pass, that a less Quantity of Surface is sufficient to contain a given Quantity of Honey, than if the Bottom had been flat, in the Proportion of 4658 to 5550, as I have found by Calculation; that is, $\frac{892}{4658}$, or $\frac{1}{5}$ Part of the Whole, so far as the Figure of the Ends of the Cells extends in each, which fifth Part of Wax and Labour saved amounts to a vast deal in the whole Structure of the Comb. And if those Creatures knew their Advantage, they could not more nicely keep up to the Rules of this sublime Geometry.

Fig. 6. 19. The last Thing among the *Maxima*, that I shall mention, is, that if a Chain A B C be suspended by its two Ends, it will sink down in such a Manner, by its Gravity, as to form the Curve A B C, called the *Catenaria*, which if inverted, would exhibit the best Form for an Arch of any other whatsoever. For the Demonstration of this, we must refer the Reader to the Inventor. Dr. GREGORY, in *Philos. Trans.* N° 231.

Force than if laid on the Axis of the lesser Wheels; contrary to the common Notion of *loading Carriages before*. (5.) The Carriage goes with much less Force on *Friction-Wheels*, than in the common Way; all which will be confirmed by Experiments (XLVIII) (XLIX.)

(XLVIII) The THEORY of Wheel-Carriages is as follows: Let A P G E M be a Wheel, N D the horizontal Plane on which it moves, E F the Height of an Obstacle over which it is to be drawn; the Wheel arriving at the Obstacle, and touching the Top E, stands upon the Point G, and presses it with its whole Weight. Draw O E K, a Tangent to the Wheel in the Point E, and meeting the vertical Diameter A G produced, in O. Draw the Radius E C, and E H perpendicular to A G; and M C, *m r*, perpendicular to C E, and consequently parallel to the Tangent O K. Lastly, draw the Radius C *m*.

2. Since the Wheel gravitates in the Direction C O, let C O express its Weight pressing the Point G; this may be resolved into two others C E and O E; of which C E presses the Top of the Obstacle, and is wholly sustained by it; the other Weight O E draws the Wheel down in a Direction parallel to the Tangent O K. Now let $W = CO$ Weight of the Wheel, $R =$ Radius, $H = EF$, the Height of the Obstacle, and $x = OE$; then since $OE : CO :: HE : CE$, we have $x : W :: HE : R$, whence $x = \frac{W \times HE}{R}$; but, from the Nature of the Circle, $HE = \sqrt{AH \times HG} = \sqrt{AH \times EF} = \sqrt{2RH - H^2}$; therefore $x = \frac{W \times \sqrt{2RH - HH}}{R}$.

3. A Force just equal to this Weight, *x*, and acting in Opposition to it, that is, drawing the Wheel upwards in the Direction C M parallel to E K, will just be able

able to make the Wheel rest on the Top of the Obstacle at E, without suffering any Part of its Weight to rest on the horizontal Plane at G.

4. Now this Force must be increased if it acts in any other Direction but that of C M; for let it draw the Wheel in the Direction C *m*, between M and E, then the Force may be resolved into two others C *r*, and *r m*, of which C *r* draws the Wheel directly against the Top of the Obstacle E, and so is destroyed by equal Re-action of the Point E; what therefore remains to draw it upwards in a Direction parallel to O K, is *m r*, which is less than C *m* or C M; and to be made equal thereto, (as it must be to support the Wheel on the Top of the Obstacle E) it must be increased in the Ratio of C *m* to *r m*, which let be as R to S (or as *Radius to the Sine* of the Angle which the Direction of the Force makes with C E). But it is plain, the Force *r m* cannot be increased, but the whole Force C M must be increased in the same Proportion; that is, when *r m* becomes $\frac{R}{S} \times r\,m$, CM will become $\frac{R}{S} \times CM = \frac{R}{S} \times$

$$\frac{W \times \sqrt{2RH - H^2}}{R} = \frac{W \times \sqrt{2RH - H^2}}{S}.$$

5. In order that the Wheels may be drawn over the Obstacle F E, it is necessary the Direction of the Force should lie between C E and C A; for if it were in the Direction C E, it could only draw the Wheel upon or against, but not over the Obstacle; and if it acted in the Direction C A, it would not make it press against the Obstacle, and consequently, could never draw it over.

6. Let $F = \frac{W \times \sqrt{2RH - H^2}}{S}$ = the Force sufficient to sustain the Wheel on the Top E of the Obstacle, it is evident if W, R, H, continue the same, F = $\frac{1}{S}$; that is, the Force will always be less as the Sine of the Angle E C *m* is greater, till *r m* = C M, when the said Force will be least of all.

7. If W and H be given, or always the same; then $F = \frac{\sqrt{2R-1}}{R}$ (for here we suppose the Force applied to draw in the most advantageous Direction, *viz.* CM, where S becomes equal to R.) If therefore the Radii of four Wheels be 1, 2, 3, 4, then will $\frac{\sqrt{2R-1}}{R}$ be 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{5}}{3}$, $\frac{\sqrt{7}}{4}$, or as the Numbers 1000, 866, 745, 661. From hence it is evident how much less Force is necessary to draw a large Wheel over any Obstacle than a lesser one, when the Weight of the Wheels are the same.

7. If the Height of the Obstacle H be indefinitely small and given, in which Case the Tangent O K will coincide with the horizontal Line N D, and the Point E with the Point G, very nearly; and the Direction of the Force be parallel to N D; then because H^2 is inconsiderable we reject it, and the Expression for the Force will be $F = \frac{W \times \sqrt{2R}}{R}$, (for H is given, and therefore not express'd). And if W be also given, the Force will be $F = \frac{\sqrt{2R}}{R}$, or $F = \frac{\sqrt{R}}{R}$, because 2 is a given Quantity; but $\frac{\sqrt{R}}{R} = \frac{1}{\sqrt{R}}$; therefore $F = \frac{1}{\sqrt{R}}$; that is, in case of rough uneven Surfaces the Force to draw the Wheel will be inversely as the Square Root of the Radius or Diameter of the Wheel. Thus if three Wheels are in Diameter as 1, 4, 9; the Force to draw them will be as 3, 2, 1.

8. If $H = o$, that is, if the horizontal Plane on which the Wheel moves be perfectly smooth or plain, then the Quantity $\frac{W}{S} \sqrt{2RH - H^2} = o$; whence it appears that no Force is required to move an heavy Body on an horizontal Plane which is perfectly even.

9. If the Height H of the Obſtacle be proportional to the Radius of the Wheel, that is, if H be as R, and the Force draw in a Direction parallel to O K; then because $\frac{\sqrt{2RH - H^2}}{R} = \frac{\sqrt{2RR - RR}}{R} = \frac{\sqrt{R^2}}{R} = 1$, therefore F = W, *or the Force will be proportional to the Weight of the Wheel.*

10. If the Direction of the Force be parallel to the horizontal Plane, that is, if C m be parallel to N D, then becauſe the Angle m C E is (in that Caſe) equal to the Angle C E H, their Sines will be equal, that is, $rm = CH = R - H$; therefore the Expreſſion of the Force (*Art.* 6.) will become $F = \frac{W \times \sqrt{2RH - H^2}}{R - H}$; and if the Height H be given it will be $F = \frac{W \times \sqrt{2R - 1}}{R - 1}$.

11. From the Expreſſion $F = \frac{W \times \sqrt{2RH - H^2}}{S}$, we have this Equation $\frac{F}{W} = \frac{\sqrt{2RH - H^2}}{S}$, which gives the following Analogy $F : W :: \sqrt{2RH - H^2} : S$. That is, *The Force is to the Weight as the Sine of the Angle* E C H (*viz.* E H) *is to the Sine of the Angle* m C E, *which the Line of Direction makes with the Line* E C.

12. If the Obſtacle is capable of being depreſs'd or borne down by the Wheel; the larger the Wheel the greater will be the Force to do this; for ſince C E repreſents the whole Force with which the Wheel bears upon the Obſtacle, and this is reſolvable into the two Parts C H and H E, of which the former C H being parallel to E F tends to preſs it down, it will be expreſs'd by R — H, and ſince H is given, the depreſſing Force will be as R — 1, and therefore will increaſe with R, or the Radius of the Wheel.

13. If the Obſtacle be ſuch, as that it can neither be ſurmounted nor depreſſed, but muſt be driven forward, the Force to do that will be expreſſed by HE = $\sqrt{2RH}$

2

$\sqrt{2RH - H^2}$, which, ſince H is given, will be as $\sqrt{R-1}$; but $\sqrt{R-1}$ will be greater in Proportion to R when R is ſmall than when it is greater. Thus if $R = 2$, then $\sqrt{R-1} = 1 = \frac{1}{2}R$; but if $R = 5$, then $\sqrt{R-1} = 2$, which is leſs than $\frac{1}{2}R$; and if $R = 10$, then $\sqrt{R-1} = 3$, which is leſs than $\frac{1}{3}R$; ſo that in this Reſpect ſmall Wheels have the Advantage of large ones. But this Caſe ſeldom happens.

14. The principal Advantage of ſmall Wheels is, that in them the Line of Traction is not parallel to the Horizon as C K, but inclined thereto in a certain Angle, as C M, making with the Horizon the Angle M C K; now if C M be parallel to the Tangent O K, the whole Force will be employed to draw the Wheel over the Obſtacle E F; whereas, if the Line of Traction were parallel to the Horizon, the Line C K might then expreſs the Force, which being reſolved into the two Forces C E and K E, ſhews that the Part C E draws the Wheel directly upon the Obſtacle, and is therefore loſt by its Re-action; and only the Part K E remains to draw the Wheel over the ſaid Obſtacle; and conſequently the horizontal Direction is not the beſt, unleſs upon a ſmooth and even Plane, where no Obſtacles and Aſcents are to be ſurmounted. Pl. XIII. Fig. 12.

15. From what we have ſaid, it is evident that a ſmall Wheel, whoſe Radius is K E, and the Line of Traction parallel to O K, is equivalent to a large Wheel whoſe Radius is C K, and the Line of Traction parallel to the Horizon N D; but E K : C K :: H B : C B :: C I : C E; that is, the Radius of the ſmaller Wheel is to that of the larger, as the Co-ſine of the Angle E C G to Radius.

16. Though the Force employed be never wholly ſpent in drawing, but when the Direction is C N, parallel to the Plane on which the Carriage moves; yet if it be applied in that oblique Direction C M, where the Breaſt of the Horſe is higher than the Axle of the Wheel C, in which Caſe only the Part B M is employed in drawing, the other Part C B is not however wholly loſt, but is acting contrary to the Gravity of the Carriage, and by that Means leſſens ſomewhat of Fig. 13.

the

the Weight of the Load, by lifting it (as it were) along, for in this Caſe the Horſe not only *draws*, but alſo *carries* along (in ſome meaſure) the Load.

17. On the contrary, if the Axle of the Wheel be higher from the Plane than the Breaſt of the Horſe, that is, if the Power be applied in the oblique Direction C O, then the Part D O draws the Load along, but the Part C D acting perpendicular on G draws the Load directly againſt the Plane, and thereby increaſes the Weight of the Load, or the Difficulty of drawing it; and is therefore the worſt Direction in which the Force can be any how applied in drawing. Hence it follows, that (*cæteris paribus*) where the Wheels of a Carriage have their Radius equal to the Height of the Horſe's Breaſt, or Traces, the Draught will be eaſieſt of all; and Wheels, whoſe Radius's are leſs than that, are better than thoſe Wheels whoſe Radius's exceed it.

Pl. XIII. Fig. 14. 18. A ſmall Wheel B D C will deſcend farther down between two Obſtacles D F and C E than a larger Wheel A D C, as is evident from the Figure; and therefore the Draught is more difficult, and ſubject to greater Shocks or Jolts, with the ſmall Wheel, inaſmuch as its Axis, and conſequently the Weight of the Load, muſt be raiſed to a greater Height in order to get from between them.

19. Alſo in ſoft or yielding Ground, a ſmall Wheel will ſink deeper than a larger Wheel charged with the ſame Weight. Thus ſuppoſe A B C be the Plane of the Road, which is ſo ſoft as to permit the ſmall Wheel
Fig. 15. to ſink down to E, then the Weight muſt overcome the Reſiſtance of as much Earth as the Wheel in ſinking has diſplaced, that is, as much as is equal to the Segment H E D; if now the larger Wheel were to ſink to the ſame Depth, it muſt overcome the Reſiſtance of ſo much Earth as is equal to the Segment A E C, which is greater than H E D, which is impoſſible, becauſe the ſame Weight can overcome but an equal Reſiſtance in either Wheel, therefore the larger Wheel will not ſink ſo deep as the ſmaller, and ſo will be drawn more eaſily.

20. Since the Ends of the Axles, and the Holes in the Naves of large and ſmall Wheels are equal, and ſince

ſince, in paſſing along, the ſmall Wheel (to meaſure the ſame Length of Road) muſt turn round upon its Axis oftener than a large one; it follows, that there will be a greater Quantity of Friction in the ſmall Wheel than in the larger, and that in the ſame Proportion as it is leſs, or as its Velocity is greater. Hence on Account of this, and ſeveral other like Cauſes, ſmall Wheels are much more ſubject to be out of Repair, to be at Fault, and to be worn quite out, than larger ones.

21. Next to the Conveniency mentioned *Art.* 14. that of turning the Carriage in a ſmaller Compaſs, with ſmall Wheels, than can be done by large ones, has made them more neceſſary in Waggons and Coaches, for becauſe of their Smallneſs they can be brought near to, and partly under the Sides of the Carriage; and ſo their Axles lying more obliquely under the Bed of the Carriage, admit it to be turned about with greater Eaſe.

(XLIX) 1. To conclude theſe Mechanical Lectures, I ſhall give the Reader a ſhort View of the famous Controverſy that has ſo long (about 80 Years) ſubſiſted between the *Engliſh* and *French* Philoſophers on one Side, and the *Dutch*, *Germans*, and *Italians* on the other. The Subject was, *Whether the Force of Bodies in Motion, ſtriking each other, be proportional to the Simple Velocity of the Motion, or to the Square of the Velocity?* The *Engliſh* and *French* maintain the former; the other Gentlemen the latter.

2. This Diſpute firſt commenced between Mr. *Huygens* and the Abbot *Catalan*, about the Force of Oſcillating Bodies; it continued ſome Time between theſe two Gentlemen; at laſt, another Subject of the ſame Kind engaged the ſaid Abbot with the famous *Leibnitz*, who is to be eſteemed the firſt Author, that plainly declared, in expreſs Words, *That the Forces of Bodies were as their Maſſes multiplied by the Square of the Velocity.* *Catalan*, and afterwards Mr. *Papin*, anſwered *Leibnitz*; he replied again, and ſeveral Papers were written on the Subject.

3. It then became a Matter of general Enquiry, and the Philoſophers of every Nation began to conſider which Side to be of, and whether they ſhould declare

for the *old* or for the *new Opinion*. However, they did here, as they do in Religion, go by a whole Nation together; the common Herd of Philosophers following the Dictates of their Leaders. Thus *Leibnitz*, *Polenus*, *S'Gravesande*, and *Muschenbroek* lead the *German* and *Dutch*; *Papin*, *Mairan*, &c. the *French*; and *Pemberton*, *Eames*, *Desaguliers*, *Clarke*, &c. the *English*.

4. In this Controversy also, as in those of Religion, the Opponents disputed with very great Warmth, wondered at each other's Slowness of Apprehension, or Backwardness of Belief; and I wish I could say, that they had always observed such an impartial, free, and generous Behaviour and Stile of Expression, as the Dignity of Philosophy demands. We should then perhaps have found Dr. *Desaguliers* better employed than in blaming Dr. *Samuel Clarke* for *uncivil Treatment*, for *rude and impertinent Expressions*, &c. in regard to those who defend the new Opinion of the *Square of the Velocity*.

5. As it usually falls out in other Cases, so here when Men find themselves pressed with Difficulties and Absurdities in their Schemes and Notions, they have recourse to the Subtilties of Metaphysical Distinctions, though seldom to any good Purpose. Thus when it appeared too plainly by all Experiments, and even to common Sense, that the natural Force of Bodies was proportional to the simple Velocity and Mass of Matter conjointly; we were told it was necessary to distinguish the Force of Bodies into two Kinds, *viz.* the *Vis Viva* or *Living Force*; and the *Vis Mortua*, or the *Dead Force*.

6. By this *Vis Viva*, or Living Force, we were to apprehend that which resulted from the visible Action of one Body upon another, as that of a falling Weight; but the *Vis Mortua* or Dead Force was to be understood of that which was destroyed by a contrary Agent, as a Weight in one Scale of a Balance is kept from descending by a Counter-Poise in the other Scale. But unluckily for the Author of this Refinement, both those Forces appear, even by the Balance, to be in the Ratio *of the simple Velocity into the Mass*.

7. Thus

7. Thus for the Living Force, if on the proportional Balance you place 4 *lb.* at the Distance of 6 Inches on one Side, and 2 *lb.* at the Distance of 12 Inches on the other Side, and if the Balance be put into Motion, they will each of them have a *Vis Viva*, or an active Force, because they will keep the Beam in Motion for some Time, till, by militating, they murder or kill each other; and then surely enough they become *Vires Mortuæ*, or *Dead Forces.*

8. But let us see how they exerted their Power while living. One (A) acted against the other (B), with the Gravity of every Particle in a Mass of 4 *lb.* and with the Velocity in each Stroke that 6 Inches Distance could give; but since all allow the Weight to be as the Mass of Matter, and the Velocity as the Distance from the Centre of Motion, therefore all the Force which A exerted was as $4 \times 6 = 24$. In the same Manner it is shewn the whole Force of B acting against A, was as $2 \times 12 = 24$; that is, in each Case, the Force was as the Velocity into the Mass of Matter. And because they were equal they destroyed each other.

9. But had those Combatants A and B been armed with Forces proportioned to the Squares of the Velocities, that of A would have been but $4 \times 6 \times 6 = 144$; whilst B would have had a Force equal to $2 \times 12 \times 12 = 288$, by which he would have demolished A at one single Stroke, and been the surviving Conqueror.

10. Let us now consider those two Bodies A and B as dead, and see what Forces they exert (*Verbo detur Venia*) in that State. In order to this they must be hung upon the Arms of the Balance, one on each Side, till they are dead, *i. e.* motionless; but this will not happen till their Distances from the Centre of Motion become reciprocally proportional to their Masses of Matter; and then it is plain the Case is the same as before; for the dead Force of A will be as $4 \times 6 = 24 = 2 \times 12 =$ dead Force of B. Whereas if these Forces were as the Masses into the Square of the Velocities, A of 4 *lb.* and B of 2 *lb.* ought to die at the Distances 6 and 8,4 Inches from the Centre respectively; but if the Experiment be tried there, such strong Symptoms of Life

will be found in A, as manifest the Falsity of this Hypothesis.

11. Being driven from this Subterfuge, they seek another in a critical Distinction between *Force* and *Pressure*. *Pressure*, say they, is the Power with which Bodies act by Means of Instruments; thus a Weight in one Scale acts against the Weight in another by Pressure; but the Power by which Bodies act on each other alone, is properly called *Force*, as when one Stone strikes another by falling on it, or a Hammer strikes an Anvil. The former they allow is proportional to the Velocity and Mass of Matter conjointly; but the latter, they say, is as the Mass multiplied by the Square of the Velocity.

12. But this *Eclarcissement* boots them as little as the former, if they mean a momentary Impact or Stroke, or such whose Effect is produced in a Moment of Time. For in such a Case we say, the Stroke is in Proportion to the Mass of Matter, and also to the Degree of Velocity, and therefore as both conjointly; but we deny there is any other Source of Power from whence a Body can derive any Force for producing a momentary Effect: Nor have any of those Gentlemen been able to shew there is, though some (bewildered in the Labyrinth) have attempted it; but in how weak and ridiculous a Manner may be seen in a Piece intituled *De Conservatione Virium vivarum*, &c. And it is worth the Reader's while to see the jocular Confutation of this ludicrous Piece by *Phileleutheris Londinensis*.

13. If they say, they would not be understood of momentaneous Effects, but such as are produced in Time, then they have no Antagonists. And it is plain from their Experiments, that this is their Meaning after all. For the Experiments which they so much insist on, are such as are made by letting Bodies fall on soft Substances, as Clay, Butter, &c. or by the Action of Bodies on yielding Springs, or springy Bodies; in all which Cases, the Effect is not produced instantaneously, but in Time; the Clay takes Time to recede, the Spring to bend, &c.

14. If the Time be taken into Consideration, then every Body must know that the Effect will be proportional

tional to the Intensity of the Cause, and the Time of its Continuance: Thus, for Example, the Effect of Gravity on Bodies left to themselves causes them to descend; their Descent therefore will be proportionate to the Power of Gravity upon them, and therefore greater in those which fall freely, than in others which descend on Inclined Planes in a given Time, where Part of the Force of Gravity is destroyed by the Reaction of the Plane. But since the Action of Gravity is not instantaneous, but continual, therefore its Effect, *i. e.* the Descent of the Body, will be greater in Proportion to the Time of its continuing to act on the Body, separately consider'd from its intrinsic Force. But this Force is as the Velocity produced by it in a given Time; also the Time is as the Velocity, when the Force is given; therefore the Effect, *viz.* the Descent of the Body, is as the Square of the Velocity.

15. Or in Symbols thus; let $E =$ the Effect produced by any Power $= P$, acting in any Space of Time $= T$, upon any Body $= Q$, moving with any Degree of Velocity V. Then it is plain, E will be as $Q =$ Mass of Matter in the Body, when P, T, and V, are given; also the Effect E will be as the Intensity of the Power P, when T, Q, and V are given; again, when P, Q, T, are given, E must be as V; and lastly, we shall have E as T, when P, Q, V, are given. Therefore when neither of these are given $E : Q \times P \times V \times T$; and in case of any different Effect, let $e : q \times p \times v \times t$.

16. Then supposing the Bodies equal, *viz.* $Q = q$, we have $E : e :: P\,V\,T : p\,v\,t$. Thus the same or two equal Bodies, descending on two Planes unequally inclined, will descend thro' Spaces, which will be as the different Powers of Gravity P and p, as the different Velocities V and v in any Point of Time, and as the different Times T and t of their Descent, conjointly.

17. If not only the Bodies Q and q, but also the Powers P and p which actuate them, be supposed equal, or $P = p$; then $E : e : T\,V : t\,v$. Thus in equal Bodies falling freely by Gravity, the Effect, or the Space which they describe, will be as the Time T and t of the Falls, the Velocities V and v at any Point of Time in

in the Fall. And because in this Case the Velocity is always proportional to the Time, that is, $T : t :: V : v$; if we multiply each Ratio by the same Ratio $V : v$, the Analogy will still be the same, *viz.* $TV : tv :: V^2 : v^2$; whence $E : e :: V^2 : v^2$; or the Effect of falling Bodies in describing the Space, will be as the Square of the Velocities.

18. If we suppose the Times given, that is, $T = t$, then (the rest as before) we have $E : e :: V : v$; that is, the Forces or Effects of equal Bodies, falling in equal Times, are proportional to the *simple Velocities*.

19. If $V = v$, or the Velocities given; then $E : e :: T : t$. Thus if two equal Bodies lying on an horizontal Plane, receive a Stroke each from Springs of equal Force, then will the Velocities be equal in every Part of the Motion, and the Effects E and *e*, in this Case, being the Spaces described, will be as the Times of their Motion.

20. Hence we have seen every possible Case wherein the Forces of Bodies or their Effects can be supposed to vary; and it is plain there is none where the Force is as the Square of the Velocity but where the Time is concerned; or where some one of the Factors Q, P, V, T, is proportional to the Velocity; thus in spouting Water, because Q is always as V, therefore, tho' T be given, E will be as QV, or as V^2; for P also in this Case is given, the Power of Gravity which gives Motion to the Particles of the Fluid being always constant.

21. Nor have they any Experiment which shews the Force or Effects of Bodies proportional to the Squares of the Velocities, but where the Time in which the Effect is produced ought to be considered. Thus in their famous Experiment of Cavities, formed in soft Clay by falling Bodies, it is true, those Cavities are the Effect of the falling Bodies, and proportional to the Squares of the Velocity. But what is this to the Purpose, unless they will say, those Pits are instantaneously produced, which I believe none will pretend to do?

22. It is evident, this Cavity must be proportional to the Quantity of Matter put in Motion in the Clay, by

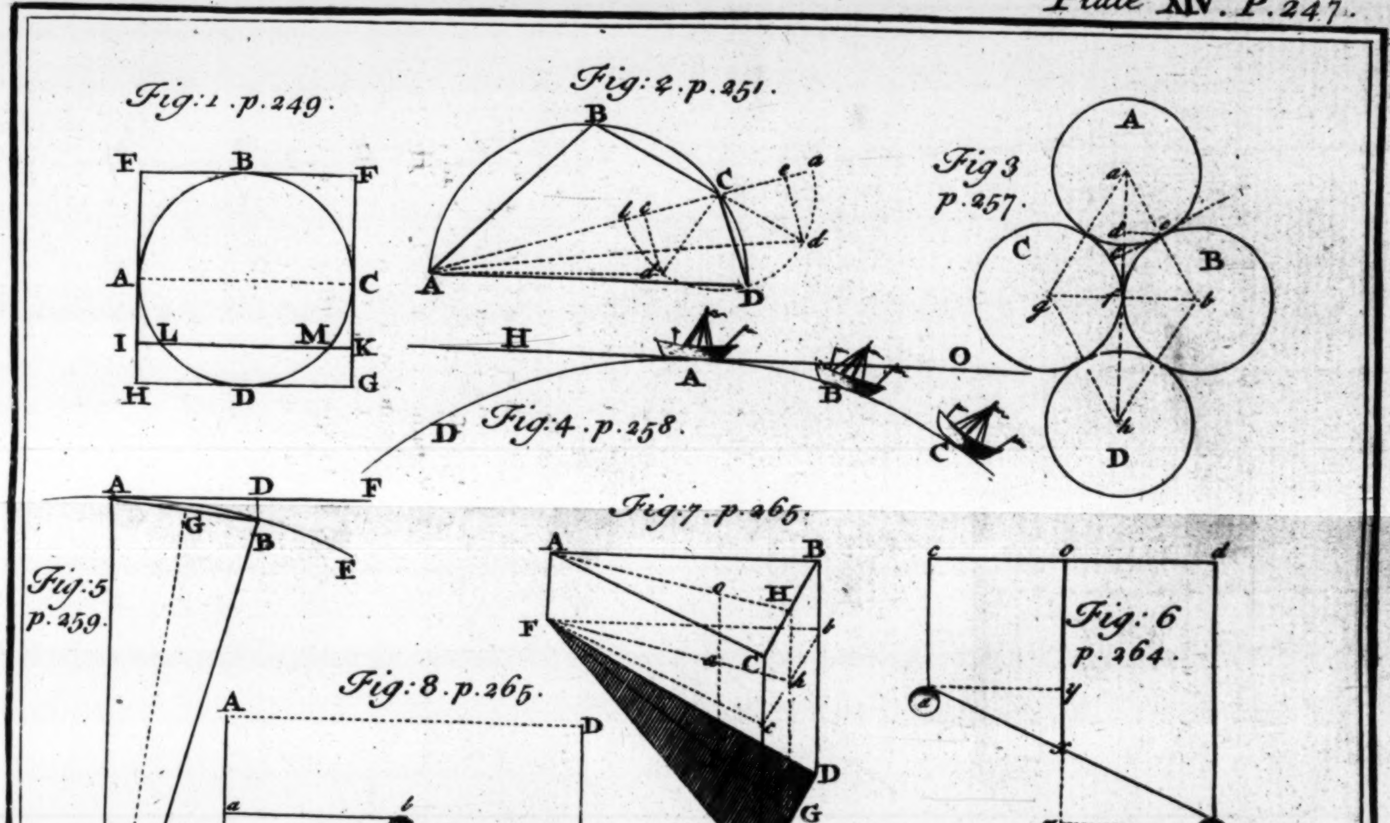
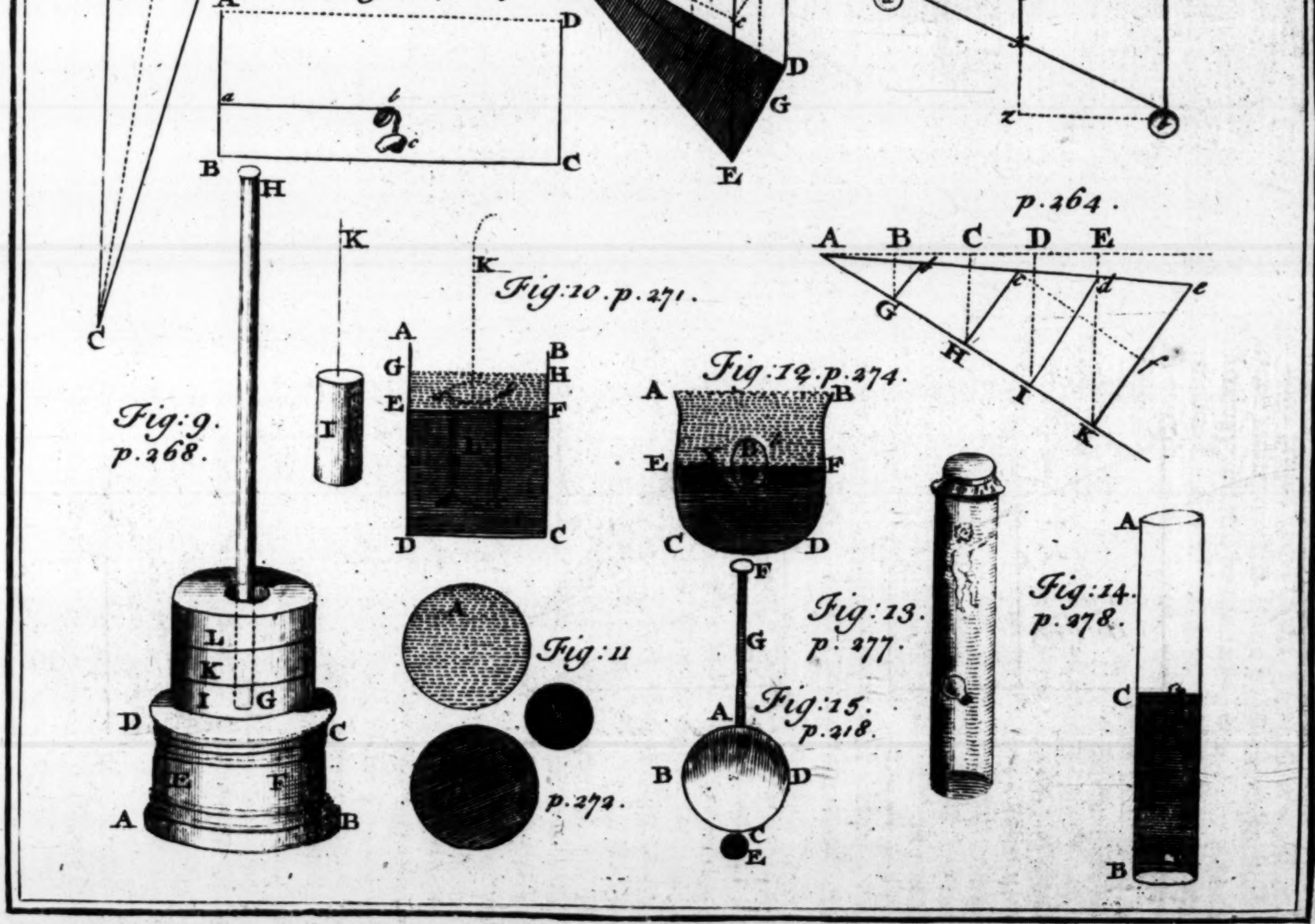


Fig: 1. p. 249.
Fig: 2. p. 251.
Fig 3 p. 257.
Fig: 4. p. 258.
Fig: 5 p. 259.
Fig: 6 p. 264.
Fig: 7. p. 265.
Fig: 8. p. 265.


p.264.
Fig:9.
p.268.
Fig:10.p.271.
Fig:11
p.272.
Fig:12.p.274.
Fig:13.
p.277.
Fig:14.
p.278.
Fig:15.
p.218.

by the falling Body; but this is proportional to the Velocity on two Accounts; the firſt is, that every Particle which comes in Contact with the Striking Body, receives a Stroke proportional to the Velocity, therefore alſo the Number of Particles which each of theſe can move, will be as the Force of the Stroke, or as the Velocity of the falling Body. Secondly, among yielding Particles, the Number which the falling Body can apply to in a given Time, will be as the Velocity; conſequently the whole Number of Particles which can be moved, both immediately by the Body itſelf, and by the Motion communicated to the Particles, will be as the Square of the Velocity; and therefore the Cavity will be in the ſame Ratio alſo.

23. We hence obſerve, the ſame Method of Reaſoning may be applied to ſoft and yielding Subſtances, as has always been uſed in the Caſe of unelaſtic Fluids; and for this Reaſon only, that in every Caſe of yielding Particles, the Effect of a Body in ſtriking them is not deſtroyed momentaneouſly, but in Time, which Time (*cæteris paribus*) will be as the Velocity; and therefore E being as T V, and T, in all theſe Caſes, being as V, it will be univerſally for all ſoft and yielding Subſtances $E : V^2$.

24. Hence we may obſerve that as Bodies are more or leſs yielding or ſoft, ſo the Effect will be more or leſs approaching to the Ratio of the Square of the Velocity. This Conſideration is of great Uſe in mechanical Affairs. Thus a Hammer with a double Velocity will produce much more than a double Effect in driving a Nail; the ſame may be ſaid of a Rammer driving a Pile: Hence the Reaſon why a ſmall Hammer will by its Velocity do more Execution upon red hot Iron, than the large Hammer by its *Momentum*. In ſhort, all theſe Things are ſo plain, and eaſy to be underſtood, that it is ſurprizing to think how ſuch a Diſpute could ſubſiſt ſo long, and occaſion ſo much to be ſaid about it.

LECTURE IV.

Of HYDROSTATICS *in general.* *Of the* PARTICLES *of a* FLUID, *the* FORM, SIZE, *&c.* *The* HYDROSTATIC LAWS *of* FLUIDS. *Of the* GRAVITY *and* PRESSURE *of* FLUIDS. *The* HYDROSTATIC PARADOX *demonstrated.* *The* RATIONALE *of the* SWIMMING *and* SINKING *of* BODIES *expounded.* *Of* ABSOLUTE, RELATIVE, *and* SPECIFIC GRAVITIES *of* SOLIDS *and* FLUIDS. *The* NATURE *and* USE *of the* HYDROMETER *or* WATER-POISE. *The* NATURE, USE, *and* NEW STRUCTURE *of the* HYDROSTATIC-BALANCE. *The* COMMON BALANCE *improved on* HYDROSTATIC PRINCIPLES. *A large* TABLE *of* SPECIFIC GRAVITIES. *The* QUANTITY *of* PRESSURE *how estimated.* *The* CENTRE *of* PRESSURE *calculated.* *Various* HYDROSTATIC PROBLEMS *in* GEOMETRY, MECHANICS, PHILOSOPHY, *&c.* *The* USEFULNESS *of* HYDROSTATIC SCIENCE. *The* THEORY *at large relating to the* MOTION *of* BODIES

DIES *in resisting* MEDIUMS, *as* AIR, WATER, *&c.*

HYDROSTATICS is that Part of Philosophy which treats of the Nature, Gravity, and Pressure of Fluids. Fluid is a Substance whose Particles yield to the least partial Pressure, or Force impress'd. And hence 'tis reasonable to infer, that those Particles must be exceeding small, smooth, round, and ponderous Bodies; and observe all the Laws of Motion and mutual Action in common with those of all other Matter (L). The following are the Laws and Properties of Fluid Bodies.

I. *All*

(L) 1. That the Characters of a Fluid intended in this Definition are all of them necessary, will appear from what follows. The first is, that they are *exceeding small*; for that the Smallness of the Particles conduces to *Fluidity*, is evident from hence, that the Points of Contact between Spheres are in Proportion to their Diameters, and therefore grow less with the Spheres, though not in the same Proportion. And the less the Points of Contact, the less will be the Cohesion, and of course the greater the Disposition of the Particles to *Fluidity*. Pl. XIV. Fig. 1.

2. To illustrate what I have said, let ABCD be a Sphere inscribed in a Cylinder EFGH; it is demonstrated by Geometers, that the Superficies of the Sphere is equal to the curved Surface of the Cylinder, and that the Superficies of any Segment of the Sphere LDM is equal to the correspondent Surface IKGH of the Cylinder; if therefore the Line IK move on till it coincides

I. *All Fluids are incompressible, except Air*; or, *they cannot by any Force be compress'd into*

cides with HG, the Superficies of the Segment of the Sphere LDM will become the Point of Contact with the Plane HG, and that of the Segment of the Cylinder IKGH will become the Periphery of the Base of the Cylinder; but this Periphery of the Base will be as the Diameter HG of the Cylinder, which is equal to the Diameter of the Sphere AC; therefore the Point of Contact of the Sphere with the Base of the Cylinder, or the Plane which it touches, will be proportional to its Diameter: whence it follows, that the Point of Contact between two Spheres will be in Proportion to their Diameters.

3. The *second Characteristic* of the Particles of a Fluid is, that they are *smooth*; by this means they become lubricous, and apt to slide or move by each other with the greatest Facility and Freedom; and therefore their Disposition to *Fluidity* is proportionably promoted and augmented; for want of this Quality, though the Particles of Matter were possess'd of all others mentioned in the Definition, yet would they never constitute a Fluid, since nothing is a greater Obstruction thereto, than Asperity or Roughness of the Superficies.

4. A *third Characteristic* is their *Roundness or Sphericity*; for the more spherical the Particles, the fewer Points they touch each other by, and the less the Points of Contact are; on both which Accounts the Attraction of Cohesion will be diminished, and their Disposition to Fluidity or Volubility will be increased.

5. The *fourth Characteristic* in the Definition of the Particles of a Fluid is, that they are *ponderous or heavy Bodies*. By this Property, Fluids do not only gravitate in common with all other Sorts of Bodies, but also they derive from hence a Power or Force of Pressure peculiar to themselves; and by that means act upon Solids in a very different Manner than that in which Solids act upon each other, as will be shewn farther on.

into a less Space than what they naturally possess; as is proved by the *Florentine Experiment* of filling a Globe of Gold with Water, which, when press'd with a great Force, causes the Water to transude or issue through the Pores of the massy Gold, in Form of Dew, all over its Surface (LI).

II. *All*

6. The surprising Subtilty or Minuteness of the Particles of Fluids deserves our farther Speculation and Remark. So small are they, as to escape the Sight assisted by the best of Glasses; that they freely pass through the Pores of the densest Matter, as *Gold*, &c. that they become invisible in Vapour; and so light as to rise in Air, yea, in Air that is greatly rarified and attenuated. And it is well known, that Water has its Particles so very subtile as to pass by Ways that Air will not, and perhaps is exceeded by nothing in this wondrous Property, but the Particles of Fire and ardent Spirits.

(LI) 1. This Experiment must be made with a Globe, for this Reason, because a Sphere contains a greater Quantity of Matter under the same Superficies, than a Body of any Figure whatsoever. This may be made appear in the following Manner; let AB, BC, CD, and AD be four Lines, of which the three first are of a given Length, and the fourth variable; and let it be required to dispose them in such a manner as to comprehend the greatest Area possible. Pl. XIV. Fig. 2.

2. Let AB and BC make a given Angle ABC; then will the triangular Space ACB be given. Upon C, with the Radius CD, describe the Semicircle *a* D *e*, then 'tis plain the other Part of the Area, or the Triangle made by the Lines AC, CD, and AD, will be greater or lesser, according to the Position of the Line CD; for as 'tis made CD or *cd*, the variable Line will be AD or A*d*; and the Triangle form'd will be ACD or AC*d*, whose Area will be $\frac{AC}{2} \times CD$, or

c d,

II. *All Fluids gravitate, or weigh, in proportion to their Quantity of Matter*; and that not only in the Air, or *in Vacuo*, but *in proprio*

cd, (cd being drawn perpendicular to AC produced.)

3. Now, 'tis evident this Triangle will be greatest, when cd is a *Maximum*, that is, when it becomes CD; and consequently the Triangle ACD is the greatest possible when CD makes a right Angle with AC; and in this Case the Point C is in a Semicircle described on AD as a Diameter. Therefore, also the other Angle B, being in a Semicircle described on the same Diameter AB, will make the other Part of the Trapezium a *Maximum*; and so the whole Trapezium ABCD inscribed in a Semicircle, will be greater than any other, whose three Sides AB, BC, CD, are the same.

4. Since what has been demonstrated of the Trapezium, is true of any other Polygonal Figures (because they may be resolved into *Trapezia*) and since the Sides of a Polygon, when infinitely small, do coincide with the Circle; therefore the Circle is the most capacious Figure, or contains the greatest Area under the same Periphery.

5. For Example; suppose a String C were disposed into the Form of a Circle; then as $22 : 7 :: C : \frac{7C}{22}$ = Diameter of the Circle; the Radius therefore is $\frac{7C}{44}$ and since the Area of a Circle is equal to the Periphery multiplied into half the Radius, therefore $C \times \frac{7C}{88} = \frac{7CC}{88}$ = Area of the Circle. Again, suppose the same String disposed in the Form of a Square; the Side would be $\frac{C}{4}$, and the Area $= \frac{CC}{16}$; hence the Area of the Circle

proprio Loco; or, a Fluid weighs the ſame communicating with a Quantity of that Fluid, as *in Vacuo*; which all Philoſophers (till very lately) have denied (LII).

III. *From*

Circle would be to that of the Square as $\frac{7CC}{88}$ to $\frac{CC}{16}$, or as $\frac{7}{88}$ to $\frac{1}{16}$, that is, as 14 to 11.

6. Now ſince a Sphere may be conſidered as made up of circular Areas, it plainly follows, that a Sphere will contain a greater Quantity of Matter under the ſame Surface than any other Solid. Thus a Sphere will be found by Calculation to be more than twice as big as a Cube of the ſame Surface; and therefore the Water contained in the Globe muſt, when the Figure of the Globe was altered, be either compreſſed into a leſs Space, or make its Way through the Pores of the Globe, as in the Experiment we find it will; which therefore evinces its *Incompreſſibility*.

7. Some Philoſophers are inclined to think that Water is not abſolutely incompreſſible, or that the Particles thereof do not touch each other; but are kept at a Diſtance by a centrifugal Force ſuperior to any Force we can apply in compreſſing them. But becauſe this is an *Hypotheſis* that has ſcarce any thing more than bare Poſſibility to recommend it, at leaſt nothing is offered to render it probable or neceſſary; therefore we are not to admit it among the Principles of the *Newtonian* Philoſophy.

(LII) 1. That Fluids gravitate, or are heavy, in the ſame Manner with Solids, is evident, becauſe the Earth's Attraction, which is the Cauſe of Gravity, equally affects the Particles of all Sorts of Matter; and therefore excites the ſame Endeavour or Tendency towards the Centre of the Earth in the Particles of a Fluid, as in thoſe of a ſolid Body; and this is what we call their *Abſolute Gravity*.

2. Now ſince in Fluids of the ſame Kind, as Water, all the Particles are reaſonably ſuppoſed equal and alike

III. *From the Gravity of Fluids arises their Pressure, which is always proportional thereto*; and since we may suppose all the Particles of a Fluid to have equal Bulk and Weight, the Gravity of the Fluid, and consequently its Pressure, *will be always proportional to the Altitude or Depth thereof.*

Whence

alike in all Circumstances, they will be all equally affected by Attraction, and therefore have among themselves an equal Tendency towards the Earth's Centre. Whence, since they gravitate equally, if they are equally obstructed in their Descent (as by the Bottoms of Vessels, *&c.*) they will all retain the same Position among themselves, as if they were affected by no Kind of Power at all; and thus they are said to be *relatively at rest*, or in a State of *Quietus* among themselves.

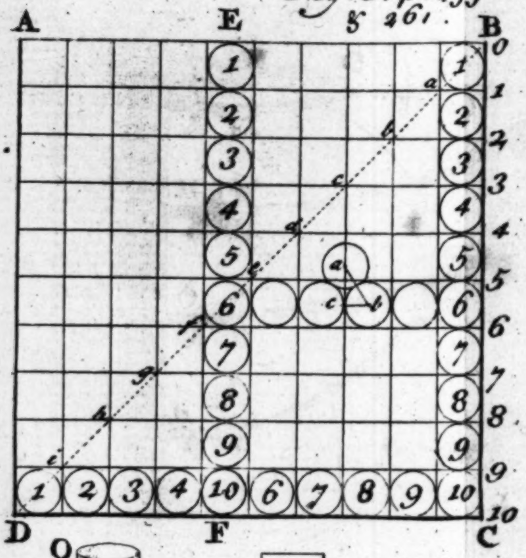
3. Since no one Particle of the same Fluid has a greater Share of the attracting Power than another, no one will tend to descend before another; and therefore among the Particles of the same Fluid, there is no such Thing as we call *Relative* or *Residual Gravity*, which is nothing but *the Excess of Gravity, by which one Body tends downwards more than another*, as will be more fully explained hereafter (See *Annotat.* LVIII).

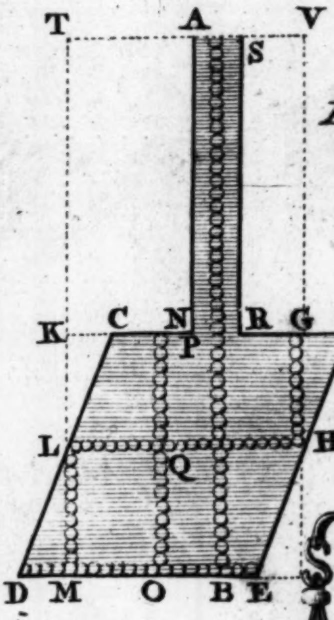
4. Now because Philosophers found that a Bucket of Water in Water weighed nothing (that is, that there was *no Relative Gravity in Water*) they very strangely inferred there was no *Absolute Gravity* of any Part or Particle of Water, whilst it remained in Water, but only became heavy when taken out, or separated from the rest. But their Mistake is easily evinced by the following Experiment; let a Bottle or Vial with Shot in it to make it sink in Water (when close corked) be hung at the End of a nice Balance, and then immersed into a Jar of Water; while thus hanging in Water, let it be counterpoised very exactly by Weights put into the Scale at the other End. Then pulling out the Cork

2 the

Plate XV.

Fig. 1. p. 255 & 261.





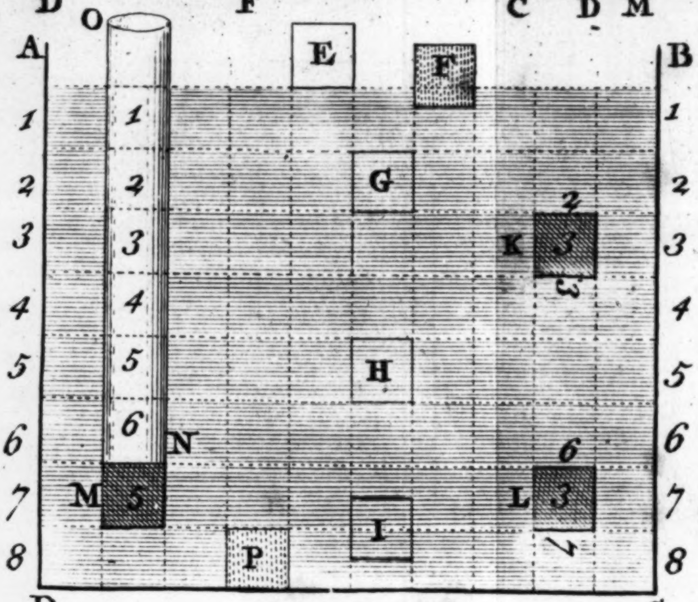


Fig. 3. p. 280.

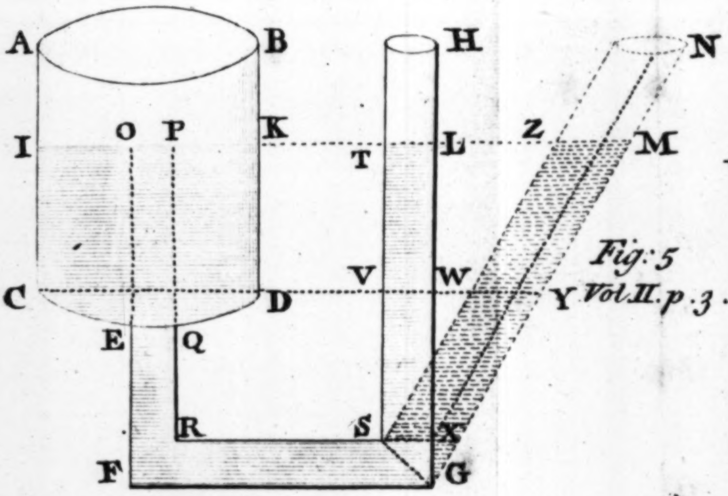


Fig: 5 Vol. II. p. 3.

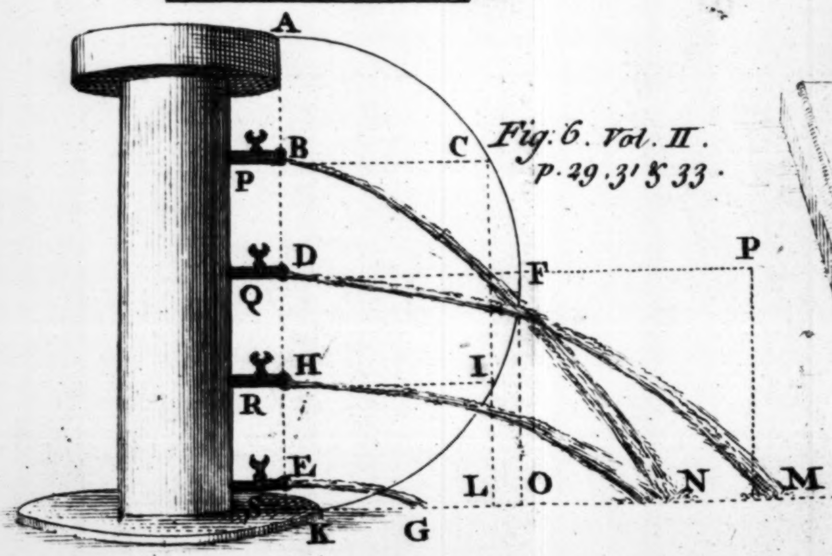


Fig: 6. Vol. II. p. 29. 31 & 33.

THE NEW HYDROSTATIC BALLANCE

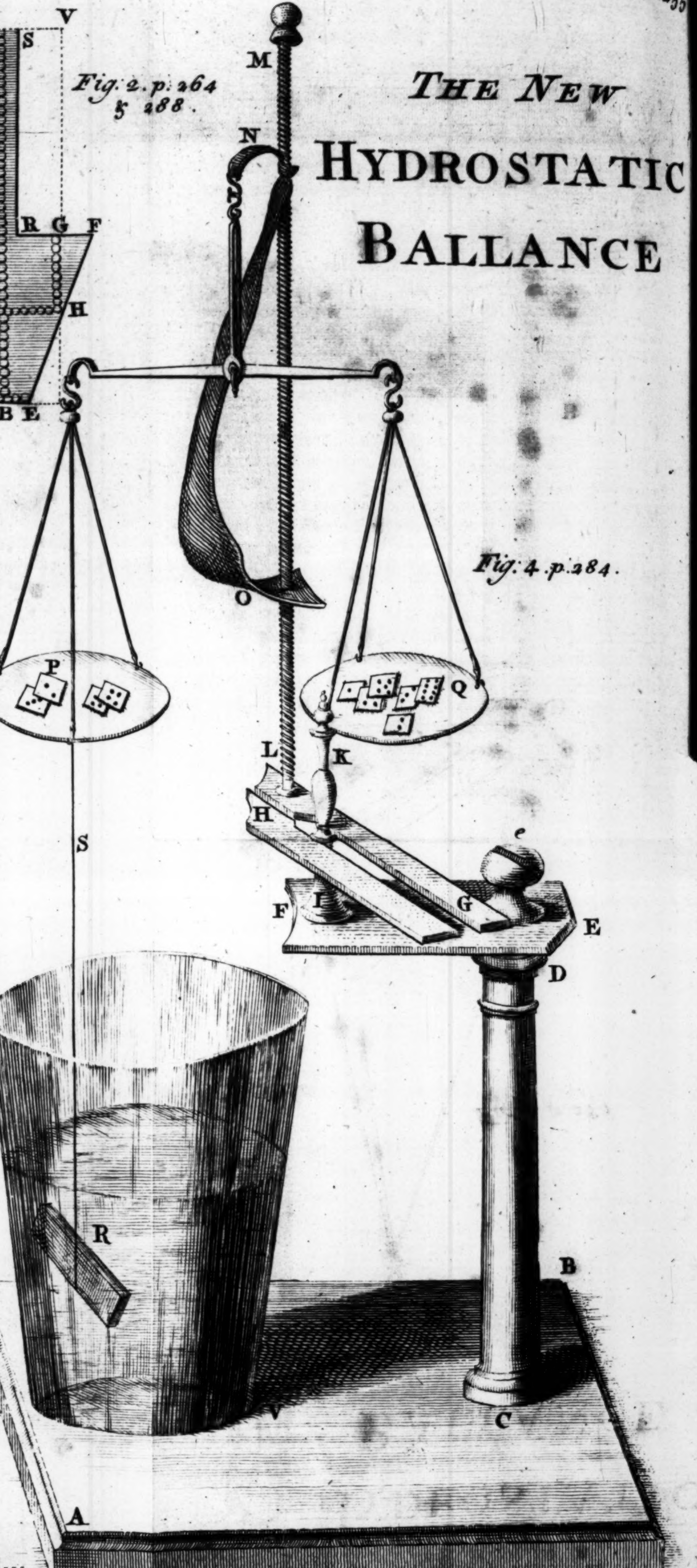


Fig. 2. p. 264 & 288.

Fig. 4. p. 284.

E: Bowen sc.

Whence the Weight and Preſſure of Fluids on the Bottoms of Veſſels, *&c.* muſt be equal.

IV. *The Preſſure of Fluids upwards is equal to the Preſſure downwards*, at any given Depth. To illuſtrate this, and the foregoing Propoſition, let ABCD be a Veſſel of Water, whoſe Altitude EF ſuppoſe to conſiſt of a Column of 10 aqueous Particles: Then, 'tis evident, the firſt or uppermoſt Particle 1 can affect the next Particle 2 only by its Weight or Preſſure, which therefore is as 1; and ſince that Particle 2 is immoveable, and Action and Re-action equal and contrary, the ſaid Particle 2 will re-act upwards upon the Particle 1 with a Force which is as 1. In the ſame manner the Particle 2 acts on the Particle 3 by Preſſure downwards, with 2 Degrees Pl. XV. Fig. 1.

the Water will ruſh into the Bottle and deſtroy the *Equilibrium*, by cauſing the End of the Balance to deſcend; which will be a plain Proof *that Water has Weight in Water*.

5. But to carry this Experiment farther; let now the *Equilibrium* be again reſtored, by adding more Weights to thoſe in the Scale: then taking the Bottle out of the Water let the Water be poured out, and weighed, and the Weight will be found exactly equal to that which was added to the Scale to reſtore the *Equilibrium* above-mentioned. This ſhews *that Water (and all Fluids) weigh the ſame in their own Element as out of it.*

Degrees of Force, arising from its own Weight and that of the Particle above it; and accordingly it is press'd upwards with an equal Force by the Re-action of the Particle 3: And so of all the rest. Wherefore the Propositions are manifest.

V. *The Pressure is upon all Particles of the Fluid at the same Depth equal in every Part*; or, *the Particles of a Fluid, at the same Depth, press each other every way, and in all Directions equally.* For if any Particle were press'd more on one Part than another, it must give way, or yield, till the Pressure became every way equal; otherwise an incessant intestine Motion of the Particles would ensue, which is absurd, and contrary to Experience (LIII).

VI. FROM

(LIII) 1. There are several Ways to shew that the Pressure of Fluids is every Way equal; but the most simple and evident, I think, is by taking a very long Glass Tube hermetically sealed at one End, and immersed in a perpendicular Position with the open End in another tall Tube or Jar of Water; the Air in the Tube being compressible will yield to the Pressure of the Water below, and admit the Water to rise in the Tube, to Heights which will be always proportional to the Altitudes of the Fluid above it; and this will be the Case when the lower Part of the immersed Tube is bent into a right Angle, that the Water may come upon it laterally: and thence it will appear *that the Pressure of Fluids is every Way equal, and proportional to the Altitudes.*

VI. FROM the mutual Pressure and equal Action of the Particles it follows, *that the Surface of a Fluid must be perfectly smooth and*

2. This likewise may be shewn from mechanical Principles, for let the Sphere A be supported by two others in an horizontal Position B and C; join the Centres a, b, g; and draw the Perpendicular af; let ec be a Tangent to the Sphere B in the Point c, where the Sphere A touches and presses it, intersecting the Perpendicular af in e; and let ae express the Force of A downwards; then since ae is resolvable into two Forces ac and ce, of which the latter being in the Tangent to B, does not at all affect it, and the other ac being perpendicular to the Surface; this other Force ac is that alone by which the Body A presses B. Pl. XIV. Fig. 3.

3. But since ac is an oblique Force, that is, neither perpendicular nor lateral, let it be resolved into the two Forces ad and dc; of which the former is perpendicular, and so does not affect the Body B in pressing it sideways; but the other dc being in an horizontal Direction is all the Force with which A presses B in a lateral Direction; but since dc is parallel to bf, we have $ac : dc :: ab : fb$; and since the Particles of a Fluid are equal, we have $fb = bc = ac$; therefore $fb = \frac{1}{2} ab$, and consequently $dc = \frac{1}{2} ac$; that is, *the Force with which* A *presses* B *laterally is just half the Force with which it presses it directly.*

4. And since by the third Law of Motion the Pressure of the Sphere D upwards is equal to that of A downwards, and so the Force upon the Bodies B and C in a lateral Direction is the same, that is, *half the direct Force*; therefore the whole Force with which the Body B or C is urged laterally by the joint Action of A and D *is equal to the whole Force with which either of them act upon them singly.* From what has been said, it appears, that the Force with which the Body A or D presses the Body B or C, is less than the Force of Gravity, or that by which they act perpendicularly, that is, ac is less than ae.

and even; for ſhould any Part ſtand higher than the reſt by any Force, as Attraction, &c. it would immediately ſubſide to a Level with the other Part by the Force of its own Gravity, when that Force is removed.

VII. THE Figure of the Surface of all Fluids is ſpherical or convex; for all the Particles equally gravitating towards the Centre of the Earth, will take their Places from it at equal Diſtances at the Surface, and ſo form a Part of the Superficies of a Sphere, equal to the Bulk of the Earth (LIV).

VIII.

(LIV) 1. Beſides the Reaſon of the Thing, we know by Experiment, that the Surface of large Waters, as thoſe of the Sea or Ocean, is convex; for a Perſon ſtanding on the Shore, and viewing a Ship under Sail, directly before him, will loſe Sight thereof by Degrees, the Hull or Body of the Ship firſt diſappearing, then
Pl. XIV. Fig. 4. the lower Parts of the Maſts, then the Tops of the lower Maſts, and laſtly, the top of the talleſt Maſt; as repreſented at A, B, C, where the Ship gradually deſcends below the horizontal Line H O, or Line which bounds the Sight, as it proceeds on the convex Surface of the Earth from A to C.

2. The Reaſon why we ſee not this Convexity in the Surface in Fluids of ſmall Extent, as in Veſſels, Ponds, &c. is becauſe the Superficies of a Sphere ſo large as that of the Earth will ſo nearly co-incide with a Plane, for a certain Space, as not to be diſcernable from it; and therefore the Surface of Fluids within that Space or Extent, will appear plain, or nearly ſo; and what that Extent of Surface is, as the Reader may be curious to know, I will ſhew by the following Calculation.

VIII. SINCE Fluids press equally every way, the Pressure of each Particle against the Side of a Vessel will be proportional to its Altitude; and consequently the Pressures of the Particles 1, 2, 3, 4, &c. of a perpendicular Column against the Side B C will

3. Let A C be the Semidiameter of the Earth = 4000 Miles, or 21120000 Feet; (for 5280 make one Mile) A E a Part of the Earth's Surface, A F a Tangent thereto at the Point A. Now it is found by Experience, that nothing is distinctly visible to the Eye that does not subtend an Angle of one Minute at least. Let B D then be the least Distance between the Convex Surface A E, and the Tangent Line A F, that is discernable to the Eye at A; then will D A B, contained between the Tangent and the Chord A B, be one Minute. If now on A B we let fall the Perpendicular C G, then in the right-angled Triangle A G C, we have all the Angles given, and the Side A C, to find the Side A G. For since D A C is a Right Angle, and D A B = 1 Minute, the Angle G A C = 89° 59′; and therefore the Angle A C G = 1 Minute also. Therefore we say, Pl. XIV. Fig. 5.

As Radius 90°	—— —— ——	10,000000
Is to the Angle A C G 1 Minute	—	6,463726
So is the Side A C = 21120000	—	7,324694
To the Side A G =	6144 —	3,788420

4. But 2 A G = A B = 12288 Feet, which is a little more than two Miles and a Quarter; and therefore, unless the Surface of Water be more than two Miles Extent, it will not appear different from a Plane; the Distance D B being in any less Extent insensible. Hence a Ship on the Sea will depart about two Miles and a Quarter before we sensibly lose Sight of any Part, because till then she will seem to be sailing on a Plane.

will be as a Series of Numbers in Arithmetical Progression, whose first Term is 0; therefore *the Sum of all the Pressures is equal to the Number of Pressures multiplied by half the greatest Pressure:* But the Number of Pressures is as the Number of Particles, or Altitude of the Fluid BC; also the greatest Pressure is as the same Altitude: *Wherefore the total Pressure against the Side of a Vessel is as the Square of the Altitude of the Fluid* (LV).

IX. HENCE

(LV) 1. That the perpendicular Pressure of Fluids on the Bottoms of Vessels is estimated by *the Area of the Bottom multiplied by the Altitude of the Fluid*, is evident, I suppose, to every Reader. For suppose a Vessel were 2 Feet wide, 3 Feet long, and 4 Feet deep, and fill'd with Water to the Brim; then the Area of the Bottom is $2 \times 3 = 6$ square Feet, and every square Foot being press'd by the Column of Water, containing 4 cubic Feet, 'tis plain the whole Bottom will be press'd by $6 \times 4 = 24$ cubic Feet, that is, by the whole Number of cubic Feet which the Vessel contains.

2. But the Quantity of the lateral Pressure, or that against the Side of the Vessel, is not so obvious; yet among the several Ways of computing it, I think none so easy, natural, and universal, as that I have mention'd, *viz.* by considering it in a single Column of Particles, as a Series of Quantities in *Arithmetical Progression*, because this depends entirely on the single Principle already established, *viz. that the Action or Pressure of Fluid Particles is every Way equal.* By which I shall endeavour farther to illustrate this Matter, and facilitate the Method of Computation.

3. Upon the uppermost Surface of the first Particle 1, which coincides with the Surface of the Fluid, there is

no

IX. HENCE if the Vessel A C be of a Cubical Form, the Pressure against a Side BC Pl. XV. Fig. 1.

no Gravity, and consequently no Pressure; therefore the Beginning of the Series, or first Term is 0, or nothing. The Body of the first Particle presses the second Particle 2 with its own Weight which is as 1, and it presses the Side of the Vessel with the same Force, and therefore the *second Term* of the Series will be 1. The second Particle presses the third Particle 3 with the Force of its own Weight, and the Weight of that above it, that is, with a Force as 2; and since it presses the Side of the Vessel with the same Force, the *third Term* of the Series will be 2. After the same Manner it is shewn, that the *fourth Term* will be 3, the *fifth Term* 4, and so on. Whence it is evident the several Pressures will be as the Series, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *&c.*

4. Now that the Sum of such a Series is equal to the greatest Term multiplied by half the Number of Terms, is known to every Person versed in common Arithmetick, and may be easily shewn by an Example. For suppose the Series were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, then the Number of Terms is 12, and the half thereof 6; also the greatest Term is 11; but $6 \times 11 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66 =$ the total Pressure against the Side of the Vessel. Now this is manifestly but half the Pressure upon a Line of the same Length in the Bottom of the Vessel, *viz.* a Line of 12 Particles, for since each Point sustains the Pressure of 11 Particles, the whole must be $12 \times 11 = 132$.

5. It may here be objected that I have taken 12 Particles in a Line at the Bottom, whereas there is but 11 at the Side, and therefore the Length of the Side and Bottom is not the same, as in the Supposition we make it. But it is to be considered, that when the Particles are supposed indefinitely small and numerous, as is the real Case of Fluids, the Difference of Length occasioned by one single Particle will be infinitely small, and therefore will make no Error in the Computation.

BC is half that upon the Bottom CD; and consequently, *the total Pressure against the*

6. Let the Sum of all the Pressures, or the total Pressure against the Side of a Vessel, be represented by S, the Number of Terms in the Series by N, and the greatest Term by G; then since $S = \frac{1}{2} NG$, and since N and G are ever proportional to each other, we shall have S as $\frac{1}{2}$ N N, or 2 S as N^2; and since Halves are in the same Ratio with their Wholes, we have S as N^2; that is, *the Sum or total Pressure will be as the Square of the Number of Terms, or Altitude of the Fluid.*

7. For Example, in the above Series, if we take the Sum of 4 Terms, it will be $0+1+2+3=6$; and then of 8 Terms, the Sum will be $0+1+2+3+4+5+6+7=28$, which is 4 Times 6, and 4 over. Now these Sums ought to be as 1 to 4, because the Altitudes were as 1 to 2; and though the Sum of any Number of Terms will always be a little more than 4 Times the Sum of half that Number of Terms in this Way of computing, yet when the Number comes to be exceeding great, the Excess will become indefinitely small, and therefore may be neglected in the Case of Fluids.

8. Or yet more clearly and accurately in Symbols thus; since S is always as $\frac{1}{2}$ GN or as GN; and in the Series above adapted to Fluids, $G=N-1$, therefore $GN=NN-N$; and so S will always be as $NN-N$; but when N is indefinitely great, N^2 will be infinitely greater than N, which therefore will vanish in that Case in the Expression $N N-N$; and so S will ever be as N^2, that is, the Sum or total Pressure will be as the Square of the Altitude of the Fluid.

9. This Way of considering the Quantity of lateral Pressure by the *Arithmetical Series* is universal, whereas the common Method restrains it to the Property of an equicrural right-angled Triangle, and to a Vessel of a cubical Form; which I shall here give for the sake of such as would see the Demonstration of a Thing in several Ways. ABCD is a Vessel of a cubical Form, that is, whose Side BC is equal to the Length of the Bottom C D; if then the Diagonal B D be drawn, we shall have the Lines $1a=B1$, $2b=B2$, $3c=B3$, $d4=B4$,

the Sides and Bottom is equal to three times the Weight of the Fluid on the Bottom of such a Vessel.

X. THE

=B 4, &c. but B 1, B 2, B 3, B 4, &c. being as the Altitudes of the Fluid, will represent the lateral Pressures in the Points 1, 2, 3, 4, &c. therefore also the Lines *a* 1, *b* 2, *c* 3, *d* 4, &c. will represent the same lateral Pressures; hence when the Distances B 1, 12, 23, 34, &c. are indefinitely small, the Lines *a* 1, *b* 2, *c* 3, *d* 4, &c. will be infinitely near each other; and so all those Lines drawn in the Triangle BCD will make the Area of that Triangle. Therefore the Sum of all the lateral Pressure against the Side BC, will be as the Area of that Triangle. But the Area of the Triangle BCD is as the Square of the Side B C, as we know from the Elements of *Geometry.* Consequently, the Sum of all the lateral Pressures is as the Square of the Altitude of the Fluid BC. *Q. E. D.*

10. Now the Pressure on the Bottom of the Vessel CD being equal to C D × B C, and the Area of the Triangle B D C being equal to B C × $\frac{1}{2}$ C D; it is plain, the Sum of all the lateral Pressures against any one Side, is equal to but half the Pressures on the Bottom of such a Vessel; and so the Sum of all the Pressures against the four Sides is twice that upon the Bottom; and that the whole Force of the Fluid is equal to three Times the Force of its Gravity.

11. Hence we see how very differently Fluids act from Solids, which act upon each other by their Gravity only, whereas Fluids act by Gravity and by Pressure jointly. Solids act only downwards, but Fluids every way equally. Solids act with a Force proportional to their Quantity of Matter, but the Force of Fluids is not according to their Quantity, but their Altitude only.

12. The Quantity of Pressure on the Bottom of Vessels, or Planes oblique to the Horizon, is easily estimated from what has been said above; for let A K be such an inclined Plane, and AE the Surface of the Water; then will the Pressure upon the Points A, G, H, I, K, be as the Altitudes BG, CH, DI, EK. If now on the said Points we erect the Perpendiculars G *b*, H *c*, I *d*,

Pl. XV. Fig. 2. X. THE *Weight, Pressure, or Effect of a Fluid upon the Bottom* DE *of any Vessel* ACDEF,

I d, &c. equal to those Altitudes respectively, and suppose the same Thing done for every other Point in the Line AK; it is evident the Triangle A e K will be as the whole Pressure on the Line A K. But the said Triangle is equal to AK $\times \frac{1}{2}\, e$K; and supposing AG $=$GH$=$HI$=$IK, and cf drawn parallel to AK; we shall have $\frac{1}{2}\, e$K$=f$K$=c$H$=$CH; therefore AK$\times$CH, will be as the total Pressure on the Line AK.

13. If AK were the Section of a Plane, then the Surface of that Plane multiplied by CH, will be the Expression for all the Pressure on that Plane, provided CH be the Depth of the Centre of Gravity H, from the Surface of Water. But to give a more general Theorem of the above Rule, let a, b, be two Weights hanging from an horizontal Plane, at the Distances ac, bd; and join their Centres by the Line ab, and let x be their common Centre of Gravity, and xo its Distance from the Plane cd, perpendicular to which draw ay and bz; then since $a : b :: bx : ax$, by the Property of the common Centre of Gravity; and by similar Triangles we have $bx : ax :: xz : xy$. Therefore $a \times xy = b \times xz$; but $xy = xo - yo = xo - ac$; and $xz = \overline{zo - xo} = bd - xo$; therefore $a \times \overline{xo - ac} = b \times \overline{bd - xo}$; that is, $a \times ac + b \times bd = \overline{a + b} \times xo$. That is, in Words, *The Product of the Weights multiplied by their Distances from the Plane is equal to their Sum multiplied by the Distance of their common Centre of Gravity from the Plane.*

14. Now that this holds true in Lines and Planes is evident, because the indefinitely small Particles of Lines and Planes may be consider'd as very small Weights, and as what has been demonstrated of two, holds equally for all, therefore the above Rule is applicable to all Sorts of Surfaces, or the Pressure upon the Bottoms of Vessels, however posited or figured, may be exactly computed thereby.

15. Thus suppose A B C D E F represent a Vessel of a Prismatic Form, whose Bottom is an oblique triangular

ACDEF, *is proportional to the Altitude* AE *only, and not to the Quantity of the Fluid in*

gular Plane DEF, in which let FG be drawn perpendicular to the Base DE, then if we make $Fx : xG :: 2 : 1$, the Point x will be the Centre of Gravity of the Plane, and xo its perpendicular Depth from the Surface of the Fluid ABC, when the Vessel is fill'd with Water. Now suppose AF = 1 Foot; FG = 6 Feet; ED = 2 Feet; CE = HG = 3 Feet; and let the Plane Fbc be parallel to the Surface of the Fluid ACB. Then we have $FG : Fx :: Gb : xa$; that is $6 : 4 :: 2 : 1\frac{1}{3} = xa$; therefore $xa + ao = xo = 2\frac{1}{3}$ Feet, the Depth of the Centre of Gravity of the Triangle FED, whose Area is $FG \times EG = 6 \times 1 = 6$ square Feet. Therefore $6 \times 2\frac{1}{3} = 14$ Cubic Feet of Water, whose Weight is equal to the Pressure on the Bottom of the Vessel. Pl. XIV, Fig. 7.

16. Thus in case of a Cylindric Vessel whose Bottom is either an Ellipsis, a Cone, a Segment of a Sphere, *&c.* if the Quantity of the Surface, and Centre of Gravity be found, the Content of the Vessel is then known by the Rule above. Thus also if the Surface of the Side of any Vessel in any Form or Position be multiplied by the Depth of the Centre of Gravity, the Product will be the Number of Cubic Feet, whose Weight is equal to the Pressure on such a Side or Surface. And since a Cubic Foot of Water weighs 1000 Ounces, or $62\frac{1}{2}$ *lb.* Averdupoise Weight, the Quantity of Pressure may be easily expressed in any Denomination of Weight, as will be more particularly shewn hereafter.

17. One Thing more the Reader should be appriz'd of, and that is the *Centre of Pressure*, which is that Point in which we may conceive the whole Pressure to be concenter'd or united; or to which if a Force were applied equal to the whole Pressure, and acting in a contrary Direction, it would exactly balance or restrain the Effect of the Pressure; or it would keep the Plane (otherwise free to move) *in Equilibrio* with the total Pressure, unequally distributed over all its Parts. Thus if ABCD be a Vessel of Water, and the Pressure on the Side Fig. 8.

in the Vessel: For every Column of Particles GH, which presses downwards on the

Side AB be equal to twenty Pounds; if *a* be the Centre of Pressure, and a String *abc* were fix'd thereto going over the Pully *b*, and sustaining a Weight *c* of twenty Pounds, then would the Side AB be kept *in Equilibrio* by those two equal and opposite Powers.

18. Now the *Centre of Pressure* is the same as the *Centre of Percussion*, as is evident from hence, that in the Line AK the percussive Force of every Point G, H, I, K, is as the Velocity, that is, as the Distance AG, AH, AI, AK, from the Point A considered as the Point of Suspension. But those Distances are as the Altitudes GB, HC, ID, KE, which are as the Pressures on the foresaid Points: Therefore since the Pressures, and percussive Forces are always alike, the Centre of one must be the same with that of the other, but the Centre of Percussion of the Line AB is two Thirds of its Length distant from the Point of Suspension A, therefore also the Distance of the Centre of Pressure *a* is $\frac{2}{3}$ of the Side AB below the Surface of the Water. See *Annot.* XXIX.

Fig. 5.

SCHOLIUM.

19. The Reason why a greater Force of Pressure is allow'd to an oblique Plane or Bottom of a Vessel than what is equal to the Gravity of the Fluid, is this, that every Point in the Line AK is press'd with a *perpendicular* and *lateral Force* at the same Time, and by both *obliquely*; and of those two oblique Forces, or Pressures, one *direct Force* is compounded, which is greater than either, and acts in a Direction perpendicular to the Plane. But since both the oblique Pressures are always as the Depth of the Fluid, the direct compound Pressure will be in the same Ratio likewise. Hence as there is a greater Number of Points in the Line AK than in the Line AE, and each is press'd perpendicularly with a Force proportional to the Depth, 'tis necessary that we estimate the whole Force by the Area of the Triangle A*e*K, and not of AEK, as we must have done if Fluids press'd like Solids by their Gravity only.

the Side of the Veſſel EF, has its Force deſtroyed by the equal Re-action of the ſubjacent Particle H in the Side, and ſo cannot at all affect the Bottom of the Veſſel. Again; the Preſſure of any Column of Particles LM upwards, againſt the Side of any Veſſel CD, is equally re-acted by the Particle of the Veſſel over it, and ſo its Force or Preſſure on the Bottom muſt be the ſame as that of another Column of Particles AB of equal Altitude with the Fluid: Whence the Propoſition is evident (LVI).

XII. HENCE

(LVI) 1. There is nothing in the Explication of the Hydroſtatic Paradox difficult to be underſtood, but the Manner how the Point M comes to be preſſed with the ſame Force as the Point B, which has the whole Height of the Fluid above it. But this Difficulty will ſoon vaniſh when we conſider *that the Preſſure among the Particles of a Fluid at the ſame Depth is every Way equal*; therefore if LH be parallel to the Bottom DE, 'tis plain there will be the ſame Preſſure upwards in every Particle H, I, Q, L, as there is downwards, and every where as the Altitude AI. The Particle at L therefore preſſes upwards againſt a Particle in the Side of the Veſſel with a Force equal to the Weight of the Column of Particles AI, which Force is deſtroyed by the equal Re-action of that Particle in the Side of the Veſſel. The ſame Fluid Particle L alſo preſſes the Particle immediately beneath it in the Column MI, with the ſame Force, which added to the Weight of the Column LM makes the ſame Preſſure on the Point M, as there is on the Point B ariſing from the Preſſure on the two Parts AI+IB.

2. That the Preſſure upward at the Point L is equal to the Weight of the Column AI, is evident from hence, that if it be removed (that is, if a ſmall Hole be there made) the Fluid will from thence be thrown

up

Pl. XV. Fig. 2. XII. HENCE *a very small Quantity of a Fluid, as* APRS, *may be made to counter-balance*

up in Form of a *Jet d'Eau* to the Height of T nearly, which is in the same horizontal Line with the Point A, or Level of the Fluid. All which well consider'd is, I presume, sufficient to evince that the Pressure on the Point M and every other Point in the Line DE is the same with that on the Point B.

3. From what we have said, 'tis also plain, that the Top of the Vessel CNRF is press'd upwards with a Force every where proportional to the Altitude of the Fluid AP, which Force of Pressure upon the whole Surface is equal to the Weight of a Body of Fluid of the same Base, and whose Altitude is AP. The Sides also are press'd every where outwards with a Force proportional to the Altitude of the Water, and in the same Manner as they would be at the Depth were the Fluid every where of the same Bulk as in the Body of the Vessel, and of the same Altitude as at A.

Pl. XIV. Fig. 9. 4. The Consequence of this will be, that if an Instrument be contrived with a Bottom AB and Top CD connected with pliant prepared Leather EF, so as to be Water-tight, and if in the Middle of the upper Part at G be inserted a small Tube GH, and Water pour'd in so as to raise the upper Part DC that it may float freely on the Surface of the contain'd Water, then any Weight I, laid thereon will cause the Water to rise so high in the small Tube as will be equal to the Height of a Cylinder of Water of the same Base DE, and whose Weight will be equal to that laid on; and if another Weight K be added, it will raise the Water to twice the Height in the Tube; if a third Weight L be added, it will rise to three Times the first Height, and so on. Whence it appears that since the Bore of the Tube may be exceeding small, the greatest Weight may be sustain'd or counterbalanced by the least assignable Quantity of Water in the Tube.

5. But this Matter will be best explain'd by Calculation and Example. Suppose then DC = 6 Inches in Diameter, the Area of the circular Surface will be

28,27

balance or be equivalent to the Weight or Force of any given Quantity TKGV, *how great soever.*

XIII. WHEN

28,27 Square Inches; and since a Square Inch of Water weighs $\frac{78}{1000}$ of an Ounce Averdupois, therefore a Pound will contain 27,7 Cubic Inches; consequently as this Number is but half a Cubic Inch less than the other, 'tis evident that a Cylindric Column of Water whose Base is 6 Inches in Diameter, and Height one Inch, will weigh nearly one Pound; if therefore I be one Pound, the Water by its Pressure will rise one Inch in the Tube; if K be another Pound, then I+K will raise the Water two Inches, and so on. If the Bore of the Tube be $\frac{1}{10}$ of an Inch Diameter, the Quantity in the Tube one Inch high is to that in the Vessel one Inch high, as the Square of $\frac{1}{10}$ to the Square of 6, that is, as $\frac{1}{100}$ to 36, as 1 to 3600; and in that Proportion are their Weights also; whence 'tis plain the Water in such a Tube will sustain 3600 Times its own Weight.

6. In like Manner we may consider how great an Effect might be produced by only blowing with one's Breath thro' the Tube HG into the Vessel ABCD. By this Means, if the Dimensions of the Vessel were large, and the Tube long and small, a Man with the Breath of his Mouth might raise any given Weight, how great soever. Suppose, for Instance, the Area of the Top DC were 1000 square Inches, and the Distance between the Top and Bottom AD $= \frac{1}{4}$ of an Inch, then will there be contain'd in the Vessel 250 Cubic Inches of Air, of the same Density with the external Air. If now more Air be blown thro' the Tube into the Vessel, it will condense the internal Air, and by increasing its Spring cause it to raise the moveable Top DE, though charged with a very great Weight.

7. Thus since the Weight of Air presses upon every Square Inch with a Force of 15 *lb.* (as will be shewn hereafter) if we place on the said Top DC, $1000 \times 15 = 15000$ *lb.* and then blow into the Instrument 250 Cubic Inches of Air, it will double the Density of the internal Air, and consequently its Spring, which

 will

XIII. WHEN any Body is immersed in a Fluid, *it loses just so much of its Weight as is equal to the Weight of an equal Bulk of the Fluid*; but *the Weight lost by the Body is gain'd*

will then hold the great Weight in *Equilibrio*, and if ever so little more be blown in it will raise it; for in that Case the Air will press upwards with a greater Force than that of the Weight downwards.

8. If 500 Cubic Inches of Air were blown into the Instrument it would sustain 30000 *lb.* and so on. But in such Cases we must suppose the Parts EF sufficiently strong to bear such a Pressure from within; for since the Air is a Fluid it presses every Way equally. Now because the Bore of the Tube is small (suppose the 100th Part of a square Inch) a Person blowing through the same would meet with no more Resistance than the Pressure of the Air on the Area of the Bore, which when the Air is of a double Density, or when 250 Cubic Inches are crowded in, will be but $\frac{15}{100}$ of a Pound, which may be easily overcome by the Force of the Muscles in blowing, especially if we use a Stop-cock to shut off the Air in the Tube every Time we take in fresh Air to our Mouths.

9. If the Top of the Instrument DC were fix'd, and a Wire passing thro' the Tube GH were fix'd to the Bottom, and suspended at the End of a Balance, then if the Bottom be nicely balanced by Weights in the Scale of the other End, and after that Water pour'd in till the Top and Bottom were separated to their utmost Distance; to balance this Water will require one Pound Weight for every Inch in Height in the Vessel, and it will after that require one Pound Weight for every Inch that it rises in the Tube, which plainly shews that the Force of Fluids is always proportional to the Height, and not the Quantity of the Fluid. *Note*, I have supposed in this Article that the Diameter of the Vessel is six Inches, as before in *Art.* 5. Another remarkable Instance of this Hydrostatic Paradox, I shall give when I come to consider the *Swimming of an heavy Body in a lighter*

gain'd by the Fluid, which will be so much heavier than before (LVII).

XIV. If

lighter Fluid, and which will be a very clear Illustration of the Matter. See *Annot*. LXI.

(LVII) This is the fundamental Principle of every Hydrostatic Process, particularly of the whole Doctrine of *Specific Gravities*; which therefore cannot be made too plain and easy to be understood. To this End, let ABCD be a Vessel filled with Water to the Height EF; and let I be a cylindric Body (heavier than Water) to be immersed therein, as at L. By this Immersion of the Body I, a Quantity of the Fluid *a b c d*, equal in Bulk to the Body, will be displaced by the superior Force or greater Gravity of the Solid: And this Quantity of Fluid must ascend, (as being confined towards the Bottom and Sides) and so raise the Surface of the Liquor from E F to G H; and then will the Quantity E F G H be equal to the Bulk of the immersed Solid *a b c d*. Pl.XIV. Fig. 10.

2. But as the Solid comes to enter the Fluid, each Particle of the Fluid by its *Vis Inertiæ* will resist the Solid, or endeavour to oppose its Descent with all its Power; and so the whole Body of the Fluid that is removed or displaced by the Solid will resist it by the united Force of all the Particles: But this Force is equal to the Gravity of the Fluid removed, as is evident from hence, that the Fluid so removed is obliged to ascend or move in a Direction quite contrary to Gravity; *Wherefore the Solid in its Descent will be resisted by a Force equal to the Gravity of an equal Bulk of the Fluid.*

3. A Consequence of this will be, that, since Action and Re-action are equal, there will be just as much of the Gravity of the descending Solid destroyed, as is equal to the renitent Force, that is, to the Gravity or Weight of an equal Bulk of the Fluid. Whence it is evident that all Bodies immersed in Fluids will weigh lighter if suspended therein at the End of a Balance, than in the Air; and lighter in the Air than *in Vacuo*, *where only the true, real, or absolute Weight of Bodies can be shewn or known.*

XIV. If any Body E could be found without Weight, *it would, if placed on the Surface*

4. And since the Force which resists the Descent of Solids is proportioned to their Bulk only, it follows, that equal Bodies immersed in Fluids lose equal Parts of their Weight; and therefore a lighter Body loses more of its absolute Weight, than a heavier one of the same Bulk. And consequently if two Bodies of unequal Bulk are *in Equilibrio* in the Air, that Equilibrium will be destroyed on their being immersed in the Fluid; because that which has the largest Bulk will lose most Weight in the Fluid.

5. Again; it is plain the Weight of the Fluid is augmented in the same Proportion, as that of the immersed Solid is diminished; for the Force or Action of the Fluid on the Bottom of the Vessel CD is before Immersion to that afterwards as the Altitudes CF to CH, or to the Bulks of the Fluid EFCD and GHCD. And since those Bulks act only by their Gravity, it is plain the Action of the Fluid is increased only by the additional Gravity of the Quantity GHFE, which is equal to that which the Solid loses by Immersion, as was before shewn in *Art.* 2. All which Cases answer very exactly by Experiments.

6. From what has been premised we may easily apprehend what is meant by *Specific Gravity*, viz. *that which is peculiar to any Sort or Species of Matter or Body, when consider'd or compar'd in any given Bulk or Magnitude*, as a *Cubic Inch*, for Instance. Thus we say, the specific Gravity of common Water is to the Specific Gravity of Lead, as the Weight of a Cubic Inch of Water to the Weight of a Cubic Inch of Lead. And as the *Absolute Weight* of Bodies is their whole Power of Gravity *in Vacuo*, so their *Relative Gravity* is that which they have in any resisting Medium, as Air, Water, &c. and is equal to the Absolute Weight diminished by the Weight of an equal Bulk of the Medium.

Pl. XIV. Fig. 11. 7. What relates to the *Absolute and Specific Gravities*, the *Magnitude*, *Density*, &c. of Bodies, will best be understood by symbolical Computation; in order to which

Surface of a Fluid A B, *float thereon without any Part immersed*; for being devoid of Gra-

to which let A and B be two Bodies of equal Bulk, but different Quantities of Matter; and let B and C be two other Bodies with equal Quantities of Matter, but of different Bulks.

$$\text{And let}\left\{\begin{array}{l} \mathrm{D} = \text{Density} \\ \mathrm{B} = \text{Bulk} \\ \mathrm{M} = \text{Quantity of Matter} \end{array}\right\}\text{in the Body A.}$$

$$\text{Also}\left\{\begin{array}{l} D = \text{Density} \\ B = \text{Bulk} \\ M = \text{Matter} \end{array}\right\}\text{in the Body B.}$$

$$\text{And}\left\{\begin{array}{l} d = \text{Density} \\ b = \text{Bulk} \\ m = \text{Matter} \end{array}\right\}\text{in the Body C.}$$

8. Then, because the Density of any Body is proportional to the Quantity of Matter under equal Bulks, we shall have $D : \mathrm{D} :: M : \mathrm{M}$: and, because when the Quantities of Matter are equal, the Bulks must be reciprocally as the Densities, therefore we have $D : d :: b : B$. Whence $D = \frac{\mathrm{D}M}{\mathrm{M}} = \frac{db}{B}$; consequently $\mathrm{D}MB = db\mathrm{M}$. But $B = \mathrm{B}$, and $\mathrm{M} = m$; therefore $\mathrm{D\,B}m = db\mathrm{M}$. Whence we have $\mathrm{D} : d :: b\mathrm{M} : m\mathrm{B}$; and $\mathrm{B} : b :: d\mathrm{M} : \mathrm{D}m$; and $\mathrm{M} : m :: \mathrm{DB} : db$.

9. The *Specific Gravity* of Bodies being as the Weights, that is, as the Quantities of Matter, in equal Bulks, will be as the Density: Therefore $\mathrm{D} : d :: \mathrm{S} : s$; and by Substitution of Ratios we have the general Theorem above become $\mathrm{S\,B}m = s\,b\,M$. And since the Absolute Weights (A, a,) of any two Bodies are as the Quantities of Matter, we have $\mathrm{S\,B}\,a = \mathrm{A}\,s\,b$. Wherefore $\mathrm{S} : s :: \mathrm{A}\,b : a\,\mathrm{B}$; that is, the Specific Gravities will be as the Absolute Weights directly, and the Bulks inversely, or as the Absolute Weights divided by the Bulks.

10. Also $\mathrm{A} : a :: \mathrm{S\,B} : s\,b$; that is, the Absolute Weights of Bodies are in the compound Ratio of their Specific Gravities and Bulks. Or the Absolute Weight of any Body is had by multiplying its Bulk and Specific Gravity together.

Gravity, it could have no Force to displace any Particles of the Fluid, and sink therein.

XV. IF *any heavy Body* F, *lighter than an equal Bulk of the Fluid, be placed on its Surface, it will sink or descend therein, till it has removed or displaced so much of the Fluid whose Weight is equal to that of the Body:* For then the Pressure upwards and downwards on the under Surface of the Body is equal; and consequently the Body will be there quiescent, or in *Equilibrio* with the Fluid. Hence *the whole Solid is to the immersed Part, as the Specific Gravity of the Fluid to that of the Solid* (LVIII).

XVI. IF

11. Again; because B : *b* :: A *s* : *a* S, it appears that the Bulk or Magnitude of Bodies will be as the Absolute Weights directly, and Specific Gravities inversely. Or the Magnitude of any Body is had by dividing its Absolute Weight by its Specific Gravity.

Pl. XIV. Fig. 12. (LVIII) 1. This Case is not strictly true but *in Vacuo*; for in the Air such a Body may be consider'd as sustain'd in two Mediums, *viz.* Air and Water; in one of which it will sink or descend, and in the other rise. And therefore to represent the true State of this Matter universally, we must raise a general Theorem in the Manner following: Let ABCD be a Vessel filled first with an heavy Fluid to the Level EF, and from thence with a lighter Fluid to AB. Suppose the Solid X sustain'd by those two Fluids; let the Part in the heavier be call'd A, and that in the lighter B; and let the Specific Gravities of the heavier and lighter Fluids be as *a* and *b*.

2. Then since the Part A displaces a Bulk of the Fluid equal to A, the Absolute Weight of that Bulk of Fluid will be A *a*, (by *Annot.* LVII. *Art.* 10.) and for the

XVI. If *a Solid, as* G, *equal in Weight to an equal Bulk of the Fluid, be immersed therein, it will take any Situation indifferently in any Part of the Fluid, as at* G, H, I, *without any Tendency to ascend or descend therein:* For being totally immersed, it must remove a Parcel of the Fluid of equal Bulk and Weight; and consequently the Pressure upwards

the same Reason the absolute Weight of a Bulk of the lighter Fluid equal to B will be Bb. Let c be the Specific Gravity of the Solid X; then the Sum of the Weights of the two Portions of the Fluids must be equal to the Weight of the Solid; otherwise it could not be sustain'd by them: Therefore $Aa + Bb = Xc = \overline{A+B} \times c$. Hence $Aa - Ac = Bc - Bb$. Consequently, $A : B :: c - b : a - c$; and compounding, $A : A+B\ (=X) :: c - b : a - b$.

3. These two Theorems are thus express'd in Words:
   1. As the Part of the Solid within the heavier Fluid is to the Part contain'd within the lighter: So is the Difference between the Specific Gravity of the Solid and lighter Fluid, to the Difference between the Specific Gravity of the Solid and the heavier.
   2. The Part of the Solid in the heavier Fluid is to the whole Solid, as the Difference between the Specific Gravity of the Solid and lighter Fluid, to the Difference between the Specific Gravity of the two Fluids.

4. Hence, if $b=o$, we have $A : X :: c : a$; that is, the Part immersed is to the whole Solid, as the Specific Gravity of the Solid to the Specific Gravity of the Fluid. And if the two Fluids were Water and Air, Water and Oil, or any other, and their Specific Gravities given, with that of the Solid, it will be easy to find the Parts of the Solid contained in either Fluid by the Theorems above-mentioned.

upwards is equal to the Tendency downwards on the lower Surface every where; and therefore it can have no Power to ſink: Alſo the Preſſure downwards muſt be equal to the Preſſure upwards on the upper Surface, whence it can have no Tendency to riſe or ſwim; it will therefore remain at Reſt in any Poſition, G, H, I, whereſoever in the Fluid (LIX).

XVII. LASTLY, IF *a Body* K *or* L, *heavier than an equal Bulk of the Fluid, be immerſed therein, it will deſcend by the Exceſs of its Gravity above that of the Fluid:* For, when immerſed, it will be reſiſted by the Force

(LIX) I have ſhewn, that while a Body is ſuſpended in a Medium, its abſolute Gravity is diminiſhed by the Reſiſtance of the Medium; it is therefore only the *relative* or *reſidual Gravity* of the Body that we find in ſuch a Caſe: And this being equal to the Difference between the Specific Gravities of the Body and the Medium, 'tis plain, where that Difference vaniſhes, that is, where the ſpecific Gravities of the Body and Medium are equal, there the relative Gravity will become Nothing; whence ſuch a Body ſuſpended by us in ſuch a Fluid has no ſenſible Weight. And this is the Reaſon why a Bucket of Water, while in the Water, ſeems to have no Weight; becauſe the Specific Gravities of the Water in the Bucket, and of the Wood of the Bucket, being the ſame with that of the Water in which it is ſuſpended, there can be no relative Gravity of the Bucket of Water experienced, and therefore no Gravity or Weight at all. The Want of conſidering this has been the Occaſion of many abſurd Poſitions and puerile Concluſions in Philoſophy; as *abſolute Levity*, a *Diminution of abſolute Gravity*, &c.

Force of an equal Bulk of the Fluid, which therefore will deſtroy juſt ſo much of the Gravity of the Solid; and conſequently, the Reſidue or Exceſs of Gravity in the Solid is that alone by which it muſt deſcend (LX).

FROM

(LX) 1. As thoſe Bodies ſwim which are ſpecifically lighter than Water, as above explained; ſo others ſink or deſcend in a Fluid by being ſpecifically heavier, that is, by their relative Gravity. Thus, if the ſpecific Gravity of the Solid be to that of the Fluid as 3 to 1, then $3 - 1 = 2$ is the relative Gravity by which it deſcends. If the Specific Gravities are as 7 to 1, then $7-1=6$ is the relative Gravity. Whence we obſerve,

2. That the Deſcent of Solids in a Fluid Medium is the very ſame with that of Bodies deſcending on an inclined Plane, becauſe in both Caſes the abſolute Gravity is only diminiſhed, by the Reſiſtance of the Medium in one Caſe, and by that of the Plane in the other; and therefore all the Properties of the Motion of a Body falling freely, belong to this Motion thro' a Medium likewiſe.

3. This relative Gravity of Solids, by which they ſink or ſwim, is uſually illuſtrated by the Deſcent and Aſcent of Glaſs Images and Bubbles included in a Jar of Water cover'd over with a Bladder, ſo as to include a ſmall Quantity of Air between the Bladder and Water. The Images, &c. have ſmall Holes in the Bottoms of their Feet, thro' which ſome Water is put into their Bodies, and that in ſuch Quantities as will render them but very little ſpecifically lighter than Water, but ſome more ſo than others, that they may not begin to move all together. Pl. XIV. Fig. 13.

4. The Images being thus put to float in Water, and the Bladder tied down, if the Hand be laid on the Bladder, and gently compreſſes the Air beneath, the Air by its Spring will act upon the Water, and cauſe it to compreſs the Air in the Bodies of the Images, by which Means more Water will be driven into their

FROM what has been premiſed of the Nature of Fluids, it will be eaſy to underſtand, *that*

Bodies; and when ſo much is got in as will make them ſpecifically heavier than the Water, then they will begin to deſcend one after another; and by varying the Degree of Preſſure you may keep them ſuſpended in any Part of the Fluid as you pleaſe.

5. It is to be obſerved, that the Matter of which the Bodies of thoſe Images and Bubbles are made ought to be ſpecifically heavier than Water, that they may ſink when fill'd with Water, which otherwiſe they could not do: They are therefore made with Glaſs, which is about three Times heavier than Water. Alſo the Holes in the Feet ought to be very ſmall, leſt the Water ſhould run out in the Air.

6. On the contrary, if the Images were but juſt heavy enough to ſink to the Bottom of the Jar, if then the Bladder, inſtead of being preſs'd down, were lifted up from the Surface of the Water, the Air would expand itſelf, and have a leſs Spring; and therefore the Air alſo in their Bodies would exert its Spring, (which is now greater than that under the Bladder) and drive ſome Water out of their Bodies, by which Means they will become ſpecifically lighter than the Water, and riſe in it to the Top.

7. The Image or Bubble will ſink or ſwim without the Artifice of a Bladder, if nicely managed; for if Care be taken to have juſt ſo much Water in the Image as will render it but very little higher than the Water, it will then ſwim at the Surface; if then you put it into a proper Depth, the natural Preſſure of the Fluid upwards will force ſo much Water more into it, as to make it heavier than Water, and from that Place it will, of its own Accord, ſink down to the Bottom. Whence it appears the ſame Body will ſink or ſwim in the ſame Medium, according to the different Circumſtances it is under from the Medium, and not from any Thing in itſelf.

Pl. XIV. Fig. 14. 8. If any Veſſel AB be fill'd half full with Salt Water to C, and a Bubble at B be made juſt heavy enough to ſink therein to the Bottom; and after that an Image

mage

that the lighteſt Body P *may be depreſs'd in the heavieſt Fluid,* by any Contrivance to keep the ſaid Fluid from preſſing on the under Surface of the light Body, by which Means only light Bodies are made to ſwim. Thus *Cork* or *Wood* will abide at the Bottom of a Veſſel fill'd with Quick-ſilver.

AGAIN: *On the other Hand, the heavieſt Body* M *may be made to ſwim in the lighteſt Fluid, by keeping the ſaid Fluid from preſſing on its upper Surface,* by means of the Tube NO: For when by this means it is immerſed ſo deep as to keep off an equal Weight of the Fluid, the Preſſure then of the Fluid acting upon its under Surface upwards will be equal to the Weight of the Solid tending downwards; and therefore if the Solid be ſunk ever ſo little deeper, it muſt ſwim

mage at C, having ſo much Water put into its Body that when the Hole in its Foot is ſealed up, it ſhall be juſt light enough to ſwim in the ſaid ſalt Water: Things being thus prepared; if ſo much hot freſh Water be put into the Jar AB as will fill it to the Top, the Conſequence will be, that the Fluid being now lighter, will not ſuſtain the Image C which will ſink to the Bottom; and the Heat of the Water will rarify the Air in the Bubble B, by which Means the Water in it will be in Part extruded, and the Bubble thereby becoming lighter, will riſe to the Top. Thus by this Artifice, the Image and Bubble will ſpontaneouſly (as it were) change Places; which pleaſant Experiment, or Hydroſtatic Problem, was firſt propoſed by Mr. *John Caſwell,* Aſtronomy Profeſſor at *Oxford,* near 40 Years ago.

ſwim by the ſuperior Force or Preſſure of the Fluid upwards (LXI).

HENCE alſo the Reaſon of *trying the different Gravity, Denſity, or Strength* (as it is commonly call'd) *of divers Fluids or ſpirituous Liquors by the* HYDROMETER *or Water-Poiſe:* For, ſince the ſtronger any Fluid is, the greater will be its Reſiſtance to any Solid immerſed, 'tis

Pl. XV. Fig. 3. (LXI) Thus for Inſtance, if the Body M be 5 Times heavier than Water of an equal Bulk, and if by Means of the Tube N O, placed on its upper Surface, the Water be kept from preſſing thereon, that it be immerſed to 7 Times its Thickneſs below the Surface of the Water, 'tis plain the Preſſure on the under Surface upwards will be as 7, but downwards only as 5; and therefore ſince there is the Exceſs of two Degrees of Preſſure upward, 'tis plain the Body cannot deſcend; but may very properly be ſaid to ſwim on the Water, and this will be the Caſe of all Bodies, however large or heavy, if placed at a proper Depth in the Water, with the above-mentioned Circumſtances.

2. But what is moſt remarkable, and at the ſame Time the cleareſt Proof of the Hydroſtatic Paradox, is, that if the Body M be immerſed to the Depth of 10 Times its Thickneſs below the Surface, with a Tube to keep off the Water from its upper Surface, then whatever be the Form of the Tube upwards, Water pour'd into the ſame will in all Forms thereof, at the Altitude of 5 Times the Thickneſs, make the Preſſure on the under Surface upwards and downwards equal, and conſequently, if the Altitude be ever ſo little increaſed, the Body will deſcend. If the Body be immerſed to a Depth, equal to 12 Times its Thickneſs, then the Altitude of the Water pour'd in muſt be equal to 7 Times its Thickneſs, becauſe $7+5=12$, and ſo on in all other Depths, let the Veſſel NO be ever ſo great or ſmall, provided its Baſe be equal to the Surface of the Body; the Preſſure of the Water pour'd in being proportional to the Altitude only, and not at all to the Quantity thereof.

'tis evident the Hydrometer cannot ſink ſo far into the heavy or ſtrong Fluids, as into thoſe which are lighter or weaker. The ſeveral Degrees of Strength, therefore, are eaſily ſhewn by the graduated Neck of this Inſtrument (LXII).

THE

(LXII) 1. The HYDROMETER is one of the moſt uſeful Inſtruments of the Philoſophical Kind; for tho' the *Hydroſtatic Balance* be the moſt general Inſtrument for finding the ſpecific Gravities of all Sorts of Bodies, yet the Hydrometer is beſt ſuited to find thoſe of Fluids in particular, and with the greateſt Eaſe, Conveniency and Expedition, as will appear from the following Account, and Deſcription thereof.

2. This Inſtrument conſiſts of four principal Parts, *viz.* (1.) A large round hollow Ball or Sphere A B C D, which is made of Ivory, Glaſs, or Metal, as Copper, Braſs, &c. (2.) An hollow Glaſs-Ball E C, of a ſmaller Size, partly fill'd with Quick-ſilver ſcrew'd on to the lower Part of the former, in order to render it but little ſpecifically lighter than Water, that it may nearly ſink in it. (3.) A long ſmall Stem or Shank AF fix'd into it on the upper Part, which by its Weight cauſes the Body of the Inſtrument to deſcend in the Fluid with Part of the ſaid Stem. (4.) A ſmall Weight F, ſcrew'd on upon the Top of the Stem or Wire AF, to cauſe the Inſtrument to ſink ſo far, that the Surface of the Fluid may always cut the Stem in its middle Point G.

3. Tho' the above be the uſual Structure and Compoſition of this Inſtrument, it is certainly not the beſt; for if the Body or Globe ABCD be of Ivory, it will imbibe Part of the Liquor, and ſo its ſpecific Gravity will be alter'd, and conſequently the Experiment cannot be juſt. If it be of Glaſs, it will be ſubject to break, and will ſtand in need of Screws, Solder, Leather, &c. to make the Parts tight, which will be found alſo both inaccurate and inconvenient. The beſt Way therefore is, to have it made of Metal, *viz.* of Copper; with a Braſs Stem above, ſolder'd on; and a Braſs

THE HYDROSTATIC-BALANCE is alſo an Inſtrument invented on the ſame Principle.

a Braſs Ball below to ſcrew on with a very nice Shoulder; then every Part will be durable, and very ſecure from any Alteration of its Weight.

4. When this Inſtrument is ſwimming in the Liquor, the Part of the Fluid diſplaced by it will be equal in Bulk to the Part of the Inſtrument under Water, and equal in Weight to the Weight of the whole Inſtrument. Suppoſe the Weight of the Whole were 4000 Grains, then 'tis evident we can by this Means compare together the different Bulks of 4000 Grains of various Sorts of Fluids. For if the Weight F be ſuch as ſhall cauſe the Hydrometer to ſink in *Rain-Water*, till its Surface comes to the middle Point of the Stem G; and if after this, it be immerſed in common *Spring Water*, and the Surface obſerved to ſtand $\frac{1}{10}$ of an Inch below the middle Point G, 'tis evident that the ſame Weight of each Water differs in Bulk only by the Magnitude of one Tenth of an Inch in the Stem.

5. Now ſuppoſe the Stem were 10 Inches long, and weigh'd 100 Grains; then every 10th of an Inch would be 1 Grain Weight, and ſince the Stem is of Braſs, and Braſs is about 8 Times heavier than Water, the ſame Bulk of Water will be equal to $\frac{1}{8}$ of a Grain; and conſequently to the $\frac{1}{8}$ of $\frac{1}{4000}$ Part, that is a 32000dth Part of the whole Bulk, which is a Degree of Exactneſs as great as can be deſired. Yet the Inſtrument is capable of ſtill greater, by making the Stem or Neck to conſiſt of a flat thin Slip of Braſs, inſtead of one that is round or cylindrical: By this Means we increaſe the Surface, which is the moſt requiſite Thing; and diminiſh the Solidity, by which the Inſtrument is render'd more exact.

6. In order to adapt this Inſtrument to all Sorts of Uſes, there ought to be two different Stems to ſcrew on and off in a ſmall Hole at A. One Stem ſhould be ſuch a nice thin Slip of Braſs, or rather of Steel, like a Watch-Spring ſet ſtrait, as I have juſt mentioned, on one Side of which ought to be the ſeveral Marks, or Diviſions, to which it will ſink in various Sorts of Water, as *Rain-Water*, *River-Water*, *Spring-Water*, *Sea-*

ciple. By it *we have a moſt uſeful and ready Method of finding the various comparative or ſpecific Gravities of Fluids and ſolid Bodies, to*

Sea-Water, *Salt-Spring-Water*, &c. And on the other Side you mark the Diviſion to which it ſinks in various lighter Fluids, as *Hot-Bath-Water*, *Briſtol-Water*, *Lincomb-Water*, *Cheltenham-Water*, *Port-Wine*, *Mountain*, *Madeira*, and various other Sorts of Wine. But in this Caſe, the Weight F on the Top muſt be a little leſs than before, when it was uſed for the heavier Waters.

7. But in Caſe of trying the Strength of ſpirituous Liquors, a common cylindric Stem will do beſt, becauſe of its Strength and Steadineſs; and this ought to be ſo contrived, that when immerſed in what is called *Proof-Spirit*, the Surface of the Spirit may be upon the middle Point G, which is eaſily done by duly adjuſting the ſmall Weight F on the Top, and making the Stem of ſuch a Length, that when immerſed in Water, it may juſt cover the Ball, or riſe to A; but when immerſed in pure Spirit, it may riſe to the Top at F; then by dividing the upper and lower Parts G A, G F, into ten equal Parts each, when the Inſtrument is immerſed into any Sort of ſpirituous Liquor, it will immediately ſhew how much it is above or below Proof.

8. This Proof Spirit conſiſts of half Water, and half Alcohol, or pure Spirit, that is, ſuch as when poured upon Gun-powder, and ſet on Fire, will burn all away, and permit the Powder to take Fire, which it will, and flaſh as in the open Air. But if the Spirit be not ſo highly rectified, there will remain ſome Phlegm or Water, which will make the Powder wet, and unfit to take Fire. This Proof Spirit of any Kind weighs ſeven Pounds twelve Ounces *per* Gallon.

9. The common Method of ſhaking the Spirits in a Vial, and by raiſing a Crown of Bubbles, to judge by the Manner of their riſing and breaking away, whether the Spirit be Proof or near it, is very precarious, and capable of great Fallacy. There is no way ſo eaſy, quick, certain, and philoſophical, as this by the *Hydrometer*, which will demonſtrate infallibly the Difference of Bulks,

to the last Degree of Accuracy; especially in the *New Structure and Method of using it*, as represented in *Plate* XV. *Fig.* 4. The Parts of which are as follow; AB, the Foot on which it stands; CD, a Pillar supporting a moveable Brass Plate EF, fastened thereto by the Screw in the Knob *e*. In the End of this Plate is fix'd an upright Piece IK, supporting another Plate GH, which slides backwards and forwards thereon, and is moveable every way about it. In the End of this Plate, at H, is fix'd (by a Nut beneath) a Wire LM, tap'd with a fine Thread from one End to the other; upon this moves the *Swan-Neck Slip* of Brass NO, to which a very exact Balance is hung at the Point N; to one of whose Scales P is appended the heavy Body R, by a fine Horse-hair or Piece of Silk S: The Weight of the said Body R in the Air is express'd by the Weights put into the Scale Q to make an *Equilibrium* therewith, which being destroy'd by immerging the Solid in the Fluid TV, contain'd in the Glass WV, is again restor'd by Weights put into the Scale P. *So that the Weights in the Scale Q compared with those in the Scale* P, *shew*

Bulks, and consequently specific Gravities, in equal Weights of Spirits, to the 30, 40, or 50 thousandth Part of the Whole, which is a Degree of Accuracy, beyond which nothing can be desir'd.

P, *shew at once the specific Gravity of the Solid* R *to that of the Fluid* TV (LXIII).

THE specific Gravity of Fluids is readily determined by weighing one and the same

(LXIII) 1. The Reason why I have given no Account or Figure of the *Common Hydrostatic Balance*, is because it is every where to be found in Books of this Sort; but principally, because I would advise Gentlemen to the Use of that exhibited in the Lectures, which they will find far more expeditious and exact, and therefore much better fitted to answer the End of such an Instrument. For this Method of suspending the Solid to be weighed by a Horse-Hair, or fine Silk, requires not the large heavy Glass-Bucket (as in the *Common Hydrostatic Balance*) to find the specific Gravities of Solids; or the heavy Glass-Bubble to find those of Fluids; and therefore the Weight being reduced, the Balance may be smaller and nicer, and consequently such as will turn with a lesser Difference of Weight.

2. The different Gravities of Solids being first compared with that of Water, are then easily compared with each other; for let S be the Weight of one Solid, *s* the Weight of any other, and W the Weight of Water, all of equal Bulks; and let S : W :: 5 : 1, and *s* : W :: 7 : 1; therefore 5 W = S, and 7 W = *s*; consequently S : *s* :: 5 W : 7 W :: 5 : 7; that is, the specific Gravities of the Solids S and *s* are as 5 to 7. And in the same Manner the specific Gravities of Fluids are compared, being first compared with that of any one Solid.

3. As the very Notion of *specific Gravity* implies Comparison, so there must be some Sort of Body fixed upon, whose Gravity must be made a Standard for the Gravities of other Bodies of equal Bulk to be compared with. This Standard Body must have two Properties; first, it must be easy to be had or come at upon all Occasions; and secondly, it must be of a fixed unalterable Nature, or at all Times the same, that there may be no Variation of its Gravity in equal Bulks, in different Times or Places. Now it is certain such a Body must be of the Fluid Kind, because the best Way of finding specific Gravities is by Immersion.

ſame ſolid Body in them ſeverally; for ſince we ſuppoſe the Balance *in Equilibrio* with

4. Among Fluids there are none which promiſe the Requiſites for a Standard ſo fairly as Water. Yet here we find various Kinds, and of different Gravities, and none which are quite unexceptionable. *Rain Water* is the moſt ſo of any, (but this is not always at Hand) its ſpecific Gravity being ſo nearly always the ſame, that could it be always had, it would anſwer all our Purpoſes very well. However, *common Water*, by means of the Hydrometer, might be always made a Standard in the following Manner.

5. Let there be a Quantity of Salt diſſolved in Spring-Water to give it a Body or Denſity, a little greater than any Water of that Kind can be ſuppoſed to have naturally. Then let a very exact Hydrometer be ſet to float therein, and obſerve at what Diviſion of the Neck the Surface of Water ſtands; and that will be the Point, to which the Hydrometer ought to ſink in the Water deſigned for theſe Experiments, and which it may eaſily be made to do by the Solution of Salt therein, or a Mixture of Salt-Water therewith. And a better Method than this, for procuring a *Standard Fluid*, I am not able to think of.

6. By this Method I only propoſe to fix a Standard for very nice Enquiries; but for common Uſes, *common Water* will do, whoſe Gravity muſt be repreſented by Unity or 1, or (in caſe of conſtructing Tables with great Accuracy) by 1,000, where three Cyphers are added to give room to expreſs the Ratios of other Gravities in larger Numbers in the Table. In doing this, we have a twofold Advantage; the firſt is, that by this Means we can expreſs the ſpecific Gravities of Bodies to a much greater Degree of Accuracy and Exactneſs. The ſecond is, that the Numbers of the Table do alſo expreſs the Ounces *Averdupois* contained in a cubic Foot of every Sort of Matter therein ſpecified, becauſe a cubic Foot of common Water is found, by Experiment, to weigh very nicely 1000 Ounces.

7. Now an Ounce *Averdupois* weighs $437\frac{1}{2}$ Grains, and

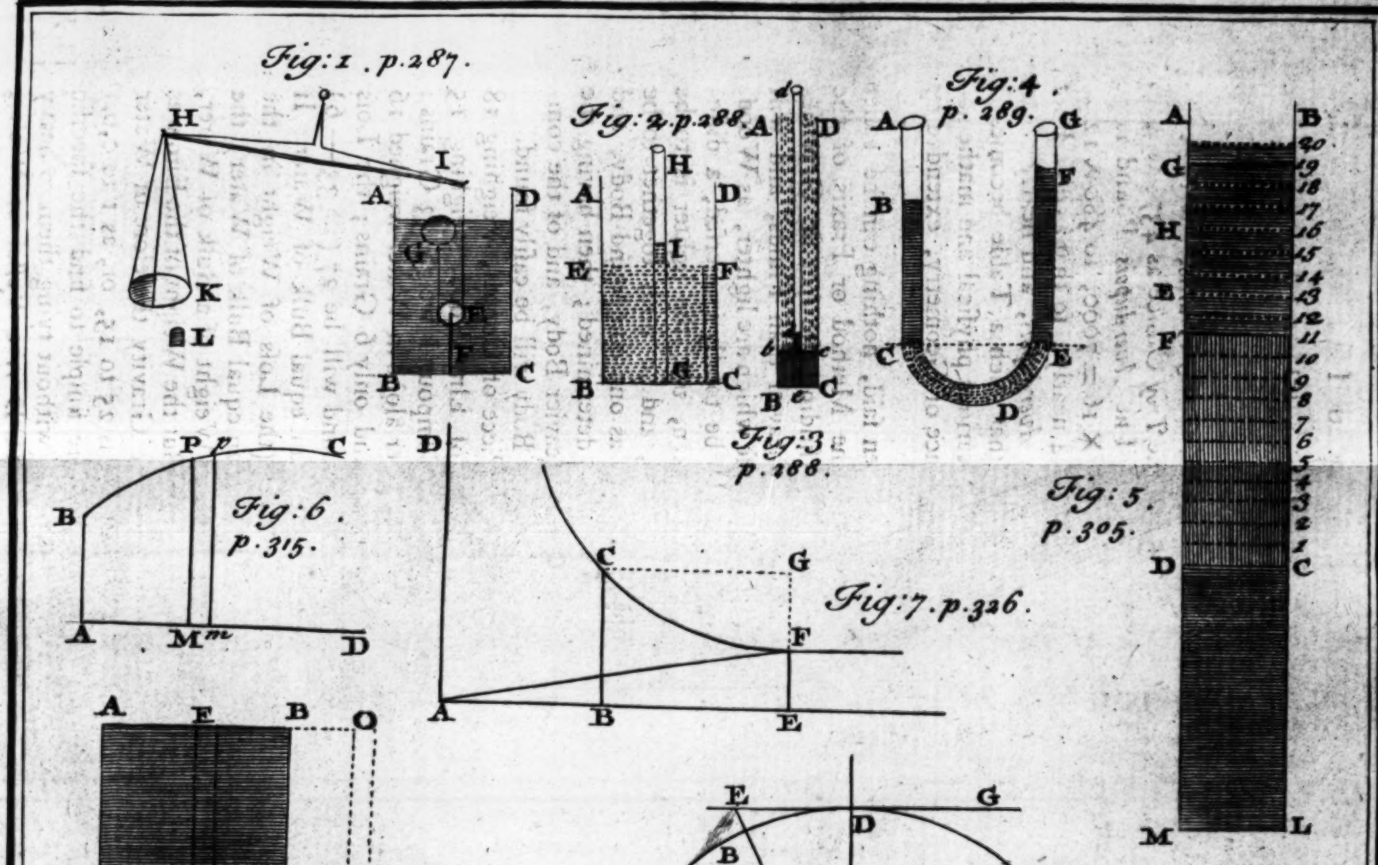


Fig:1 . p.287.
Fig:2.p.288.
Fig:3
p.288.
Fig:4 .
p. 289.
Fig: 5.
p.305.
Fig:6 .
p.315.
Fig:7.p.326.
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

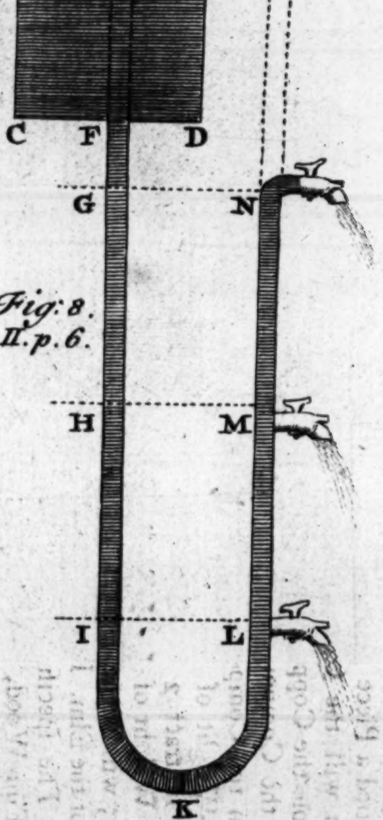


Fig: 8.
Vol. II. p. 6.

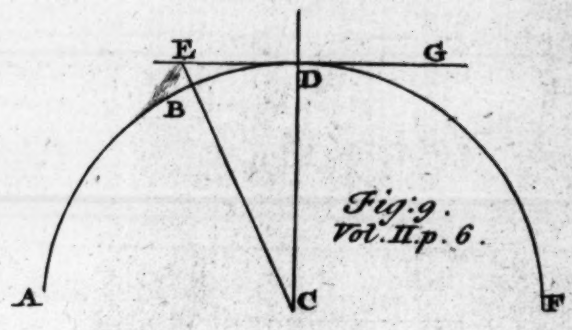


Fig: 9.
Vol. II. p. 6.

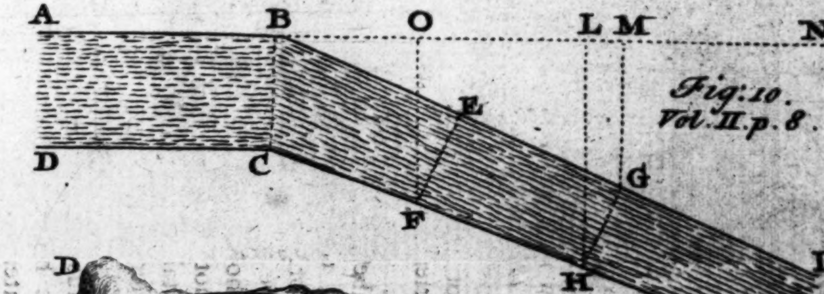


Fig: 10.
Vol. II. p. 8.

Fig: 11.
p. 10.

with the Body suspended in the Air, the Equilibrium will be destroyed when the Solid

and an Ounce *Troy* contains 480 Grains, therefore the *Averdupois* Ounce is to the *Troy* Ounce as $437\frac{1}{2}$ to 480, or, as 51 to 56 nearly. The *Averdupois* Pound is to the *Troy* Pound as $437\frac{1}{2} \times 16 = 7000$, to $480 \times 12 = 5760$, that is, as 17 to 14 nearly; so that a cubic Foot of Water weighs $62\frac{1}{2}$ *lb. Averdupois*, and nearly 76 *lb. Troy*. And hence it is that such a Table becomes so useful in the Solution of various physical and mathematical Problems, and Science of Geometry, extended by Hydrostatic Principles.

8. From what has been said, nothing can be easier to be understood, than the Method or Praxis of the Hydrostatic Balance, for finding the specific Gravities of all such Solids as are heavier than Fluids, and will sink in them; but for those which are lighter, as Wood, Cork, &c. and will not be totally immersed, a different Method must be taken, *viz.* the lighter is to be connected to an heavier, and so both together to be immersed into the Fluid, as one compound Body, and its specific Gravity thus determined; then having the specific Gravity of the heavier Body, and of the compound, that of the lighter Body will be easily found.

9. For suppose to a Piece of Copper, weighing 18 Grains, be tied a Piece of Elm Wood, weighing 15 Grains, then will the Compound weigh 33 Grains: Again, suppose the Copper alone in Water weighed 16 Grains, and the Compound only 6 Grains; the Loss of Weight in the Compound will be 27 ($= 33 - 6$) which is the Weight of an equal Bulk of Water: If from this we subtract 2 (the Loss of Weight in the Copper, and Weight of an equal Bulk of Water) the Remainder 25 will be the Weight of a Bulk of Water, equal to that of the Elm. But the Weight of the Elm was 15 Grains: The specific Gravity therefore of Water is to that of Elm-Wood, as 25 to 15, or, as 1 to 0,6.

10. Another Way more simple to find the specific Gravities of light Bodies (without tying them to heavy ones) is as follows, ABCD is a Vessel of Water, in Pl. XVI. Fig. 1.

Solid is immersed in the Fluid, and must be then restored by Weights put into that Scale

in which is placed a small Pulley E, on a Foot F; G is a light Body floating on the Surface of Water; HI is a Balance, I E G a Horse-Hair going round the Pulley, and connecting the light Body G to the End of the Balance. Now let the Vessel be placed in such a Manner, that the Body G refusing to go under Water, may draw the Balance H I out of an horizontal Position; then such a Weight L, placed on the Scale K, as will draw the Body G under Water, and restore the horizontal Position, or Equilibrium of the Balance, will discover the specific Gravity of the Body G.

11. For since Bodies ascend, as well as descend, by the Differences of specific Gravities, or relative Weight; it is plain the Weight L, that detains the Body under Water, must be equal to the Excess of the Weight of the Fluid above that of the Solid under equal Bulks, and therefore the Weight L, added to that of the Body, will give the Weight of an equal Bulk of Water, and of Course the specific Gravity. For Example, let the Body G be a Piece of Elm-Wood weighing 36 Grains, and the Weight L sufficient to detain or keep it under Water, will be found equal to 24 Grains; then $36 + 24 = 60 =$ the Weight of Water equal in Bulk to the Elm. Consequently the specific Gravity of Water is to that of Elm, as 60 to 36, that is, as 1 to 0,6; the same as was found before. *Note*, the Weight of the Scale K must be added to the Weight L, because it is all the Force upon the Point H that keeps the Body under Water.

12. There is yet another Way, and that a very good one, for finding the specific Gravities of such Liquors as will not mix with each other, as Water and all Kinds of Oil; it is thus: Let A B C D be a Vessel of Water to the Height E F, and G H a Tube dipt in Oil
Pl. XVI. Fig. 2. first, for the Oil to rise a sufficient Height therein, when the upper Orifice H is to be stopped close with the Finger, and thus immersed into the Water of the Vessel; then the Finger being taken away, the Oil will

Scale to which the Body is appended. Theſe Weights will ſeverally expreſs the Gravities

will ſubſide and ſtand at the Altitude G I in the Tube. Now ſince this Column of Oil is balanced or ſuſtained by a Column of Water of an equal Baſe and Altitude C F, and the Denſities of Fluids are reciprocally as the Bulks, the ſpecific Gravities (which are as the Denſities) will be as the Bulks, that is, the Altitudes reciprocally, (becauſe the Quantity of Matter is the ſame in both) therefore if G I=100, and C F=87, the ſpecific Gravity of the Oil will be to that of the Water, as 87 to 100, or that of the Water to that of the Oil, as 1,000 to 0,87.

13. In like Manner, if A B C D be a long Tube with a little Mercury poured in to a ſmall Height *b c*, and then a ſmall Tube *a e*, open at both Ends, be put into it; and laſtly, if Water be poured into the large Tube upon the Mercury to the Height of 14 Inches, it will by its Preſſure raiſe the Mercury to the Height *b* of one Inch in the ſmall Tube above the Surface of that in the large one; which ſhews that the ſpecific Gravity of Mercury is to that of Water nearly as 14 to 1. As will appear by the Experiment if accurately made.

14. Once more; another Way, which in ſome Caſes may do very well, to find the ſpecific Gravities of any different Liquids, is by Means of a recurved Tube A D G, in which if Quickſilver be poured firſt to fill the Bottom Part, and then one Liquid into one Leg, and another Liquid into the other Leg, in ſuch Manner that they preſs on the Quickſilver on each Side equally, which will appear by the Surface on each Side being in the ſame horizontal Line C E; then will the ſpecific Gravity of the Liquid in the Leg A C be to that in the Leg E G, as the Altitude of the latter E F to the Altitude of the former C B.

15. By the Table of ſpecific Gravities it appears that Gold is the heavieſt Body in Nature, and that Mercury is next to it in Gravity; and conſequently Gold

Gravities of an equal Bulk of the reſpective Fluids; and conſequently, they may thus

only will ſink in Mercury; therefore by weighing Gold in Mercury and Water, the ſpecific Gravities of theſe two Fluids may be determined by the Hydroſtatic Balance: But ſince Mercury will readily adhere to Gold, it will create us ſome Trouble in getting it off again, which muſt be done by Fire, or *Aqua Fortis*; as the latter is not always at hand, it is common to put Gold into the Fire, but Care muſt be taken not to put it into a Coal Fire, becauſe the acid Spirit of the Sulphur will leſſen the Coheſion of the Particles of the Gold, and ſo render it very brittle, and apt to break, eſpecially when laid on a cold Stone to cool; the beſt way therefore to find the ſpecific Gravities of theſe Fluids is that above directed in *Art.* 13.

16. *A TABLE of SPECIFIC GRAVITIES.*

Of METALS.

Fine or pure Gold,	19,640
Gold of a Guinea of *George* II.	17,150
Gold of a Moidore,	17,140
Silver fine or pure,	11,091
Silver of a Shilling of *George* II.	10,000
Lead,	10,130
Copper,	9,000
Braſs caſt,	7,850
—— wrought,	8,000
Steel tempered,	7,704
Iron,	7,645
Tin,	7,550

17. *MINERALS, ORES, &c.*

Copper Ore,	3,775
Lead Ore,	6,800
Biſmuth,	9,700
Turbith Mineral,	8,235
Antimony from *Germany*,	4,000

thus be compared with each other, or all of them with the Gravity of *common Water*,

Antimony from *Hungary*,	4,700
Speltar,	7,065

18. *STONES, FOSSILS, &c.*

Adamant or Diamond,	3,400
A Pſeudo-Topaz,	4,270
A Pſeudo-Hyacinth,	2,631
A Pſeudo-Jaſper,	2,666
A *Bohemian* Granate	4,360
Swediſh Granate,	3,978
Onyx-Stone,	2,510
A Cornelian,	2,568
An *Engliſh* Agate,	2,512
Turcois Stone,	2,508
Sardrachates,	3,598
A golden Marcaſite,	4,589
Rock Cryſtal,	2,659
Iſland Cryſtal,	2,720
Lapis Nephriticus,	2,894
Lapis Lazuli,	3,054
Lapis Hæmatites,	4,360
Lapis Calaminaris,	5,000
Lapis Judaicus,	2,500
Lapis Manati,	2,270
Lapis Amianthus, or Aſbeſtos from *Wales*,	2,913
—— ditto, from *Italy*,	2,360
Glaſs of the common Sort,	2,666
Flint,	2,542
Black *Italian* Marble,	2,704
A White *Italian* ditto,	2,707
A fine Marble,	2,700
Another *ditto* of *Italy*,	2,718
A pellucid Pebble,	2,641
A Selenitis,	2,322
Mundick, or Gold Spar,	4,430
Kidney Stone,	3,600
Blue Stone,	2,740

ter, as usual, and disposed in a proper Table; making that of Water 1,000.

Star Stone,	3,450
Hard Paving Stone,	2,460
Burford Stone,	2,049
Alabaster,	1,875
Rag-Stone,	2,470
Rotten-Stone,	1,980
Copperas-Stone,	4,300
Chalk,	2,370
Slate,	2,740
Oil-Stone,	2,380
An Hone,	2,388
China,	2,270
Piece of Brown Stone Bottle,	1,777
Piece of White Stone Mug,	2,250
Talc,	2,657
—— of *Venice*,	2,780
—— of *Jamaica*,	3,000
Armenian Bole, or Earth,	2,727
Common Sea Coal,	1,272
Magnet, or Loadstone,	1,840
Piece of *Stonehenge*, very hard,	2,618
—— *ditto*, of a softer Sort,	2,500
Bristol Stone,	2,510

19. *ANIMAL SUBSTANCES.*

Bone of an Ox,	1,656
Ivory,	1,826
The Tip of a Rhinoceros's Horn,	1,242
——— of an Ox Horn,	1,840
——— of a Stag's Horn,	1,875
Calculus humanus,	1,700
Ditto	1,240
Ditto	1,433
Ditto	1,660
Oyster-Shell,	2,092
Murex-Shell,	2,590

IN the ſame Manner, if divers Solids are firſt weighed in Air, and then afterwards

A Cockle-Shell,	2,520
Mother of Pearl,	2,480
A Piece of hard Fiſh Skin,	1,621
A Piece of dried Fleſh of Fiſh,	1,129
The Quill Part of a Feather,	1,330

20. *VEGETABLE SUBSTANCES.*

Dry Box-Wood,	1,030
Dry Oak,	0,925
Dry Elm,	0,600
Dry Aſh ſappy,	0,734
Ditto more dry, about the Heart,	0,845
Dry Maple,	0,755
Dry Fir,	0,546
Dry Cedar,	0,600
Dry Walnut-tree,	0,631
Dry Yew,	0,760
Beach meanly dried,	0,854
Crab-tree meanly dried,	0,765
Lignum Vitæ,	1,327
Lignum Nephriticum,	1,200
Lignum Aloes,	1,777
Lignum Brazilicum,	1,030
Lignum Rhodium,	1,125
Lignum Aſphaltum,	1,179
Lignum Guaiacum,	1,337
Saſſafras Wood,	0,482
Red Wood,	1,031
Red Santalum Wood,	1,128
White *ditto*,	1,041
Citrine *ditto*,	0,809
Speckled Wood of *Virginia*,	1,313
Maſtic Wood,	0,849
Ebony,	1,177
Cork,	0,240

wards immersed in the same Fluid, as Water, for Instance, the *Equilibrium* will be destroy'd;

Good Wheat of the last Year,	0,757
White Oats,	0,472
Blue Pease,	0,795
White Pease very dry,	0,807
Barley of the last Year,	0,658
Malt made of the same,	0,485
Field Beans very dry,	0,807
Wheaten Meal unsifted,	0,495
Rye Meal unsifted,	0,454
Wood-Ashes,	0,930

21. *MISCELLANEOUS SUBSTANCES.*

Amber,	1,040
Jet,	1,238
Bezoar Oriental,	1,530
——— Occidental,	1,500
Sulphur Common,	1,800
——— Vivum,	2,000
Borax,	1,720
Wood petrified,	2,341
Coral Red,	2,689
—— White,	2,500
Corallachates,	2,605
Cinnabar Natural,	7,300
————Artificial,	8,200
————of Antimony,	6,044
The reputed Silver Ore of Wales,	7,464
The Metal thence extracted,	11,087
Ceruse,	3,156
Tartar Common,	1,849
—— Emetic	2,246
—— Vitrioli,	2,298
Cream of Tartar,	1,900
Camphire,	0,995
Mercury Crude,	13,593

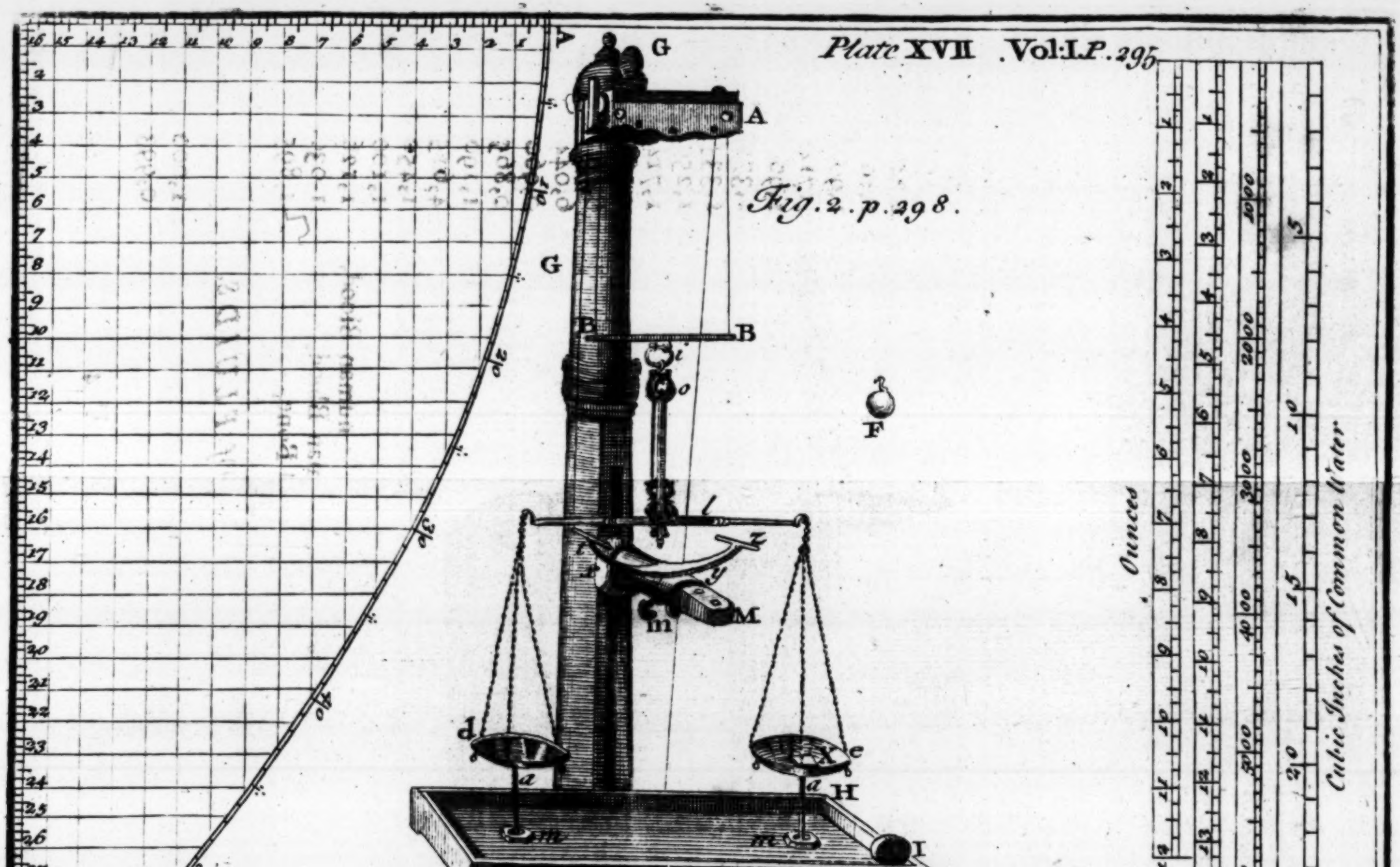

A
G
B
i
o
l
z
y
x
t
M
m
F
d
e
a
H
I
20
30
40
Ounces
1000
2000
3000
4000
5000
5
10
15
20
Cubic Inches of Common Water

Fig. 2. p. 298.

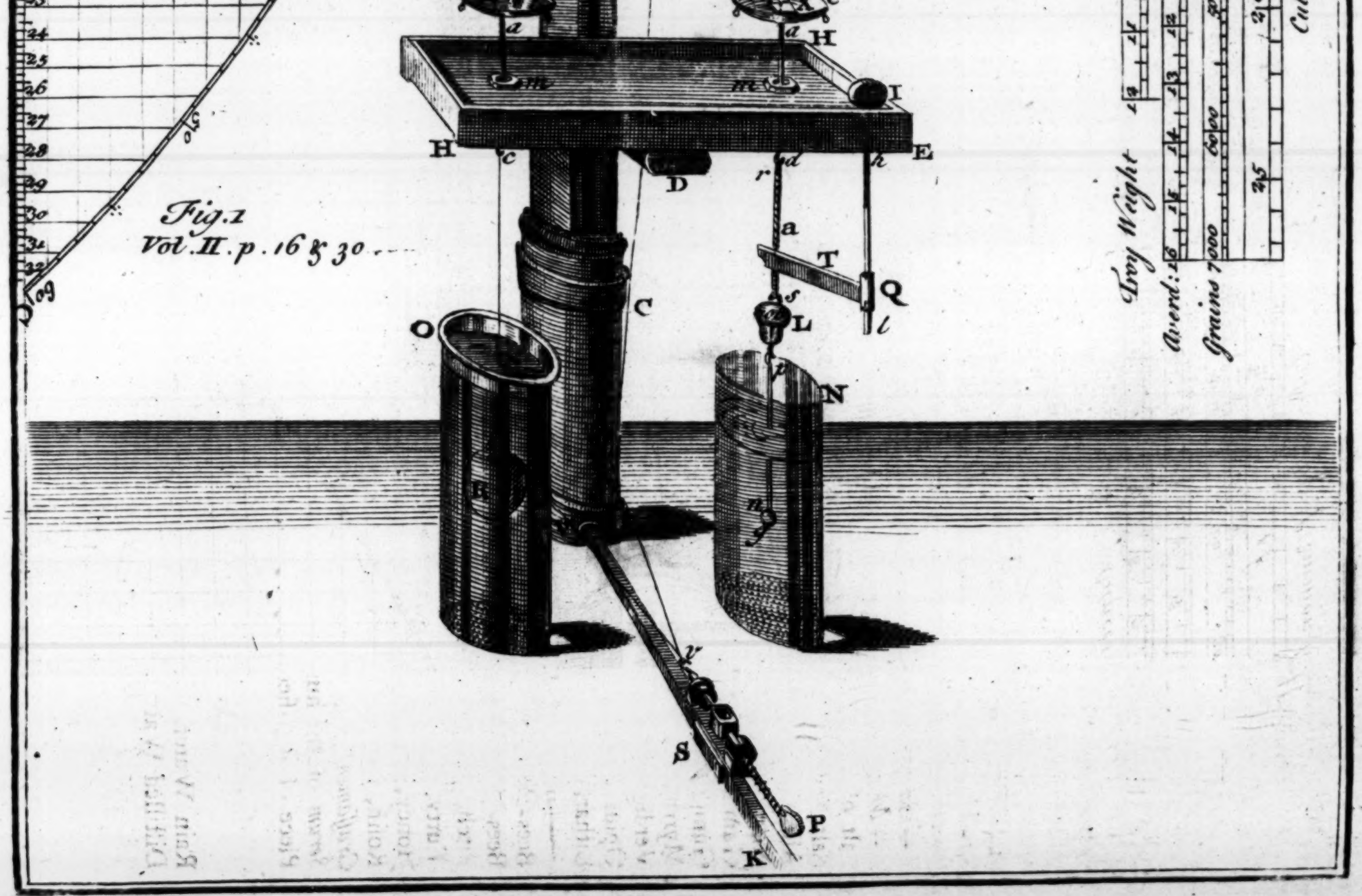


Fig.1
Vol.II.p.16 & 30
Troy Weight
Averd:
Grains 7000
6000
H
E
D
C
O
N
T
Q
L
I
S
P
K

destroy'd; which will be restored, as before, by putting in so much Weight as is equal

Mercury distilled once,	13,570
—— Sublimed 511 Times,	14,110
Glass of Antimony,	5,280
Vitriol of *Dantzick*,	1,715
—— *English*,	1,880
—— White,	1,900
Sal Gemmæ,	2,143
— *Prunellæ*	2,148
— *Polychrestum*,	2,148
— *Ammoniacum*,	1,453
— *Mirabile Glauberi*,	2,246
Salt of Hartshorn	1,496
Salt of Vitriol,	1,900
Alum,	1,714
Nitre,	1,900
Gum *Arabic*	1,375
Gum Tragacanth,	1,333
Myrrh,	1,250
Verdigrease,	1,714
Opium,	1,363
Litharge of Gold,	6,000
—— of Silver,	6,044
Bees-Wax Yellow,	0,960
Bees-Wax White,	0,865
Pitch,	1,190
Tutty,	4,615
Honey,	1,450
Rosin,	1,100
Crassamentum of the human Blood,	1,126
Serum of the human Blood,	1,030
Piece of petrified Bone,	1,895

22. *Of FLUIDS.*

Rain Water,	1,000
Distilled Water,	0,993

equal to the Weight of the same Bulk of Water: The Gravity, therefore, of every Solid

Well or Spring Water,	0,999
River Water,	1,009
Sea Water,	1,030
Aqua Fortis,	1,300
Aqua Regia,	1,234
Oil of Vitriol,	1,700
Oil of Clove Gilliflowers,	1,034
Oil of Amber,	0,978
Oil of Aniseed,	0,994
Oil of Caraway-seed,	0,940
Oil of Linseed,	0,932
Oil of Mint,	0,975
Oil of Olives,	0,913
Oil of Orange,	0,888
Oil of Origany,	0,940
Oil of Rosemary,	0,934
Oil of Sassafras,	1,094
Oil of Spikenard,	0,936
Oil of Turpentine,	0,792
Spirit of Turpentine,	0,874
Spirit of Wine rectified,	0,840
Ethereal Spirit of Wine,	0,732
Spirit of Vitriol,	1,203
Spirit of Amber,	1,030
Spirit of Hartshorn,	1,073
Spirit of Urine,	1,120
Spirit of Honey,	0,895
Spirit of Nitre,	1,315
——— *Ditto* rectified,	1,610
Spirit of Sea-Salt,	1,130
Spirit of Tartar,	1,073
Tincture of Antimony,	0,866
Butter of Antimony,	2,470
Balsam of *Tolu*,	0,896
Lixivium of Salt of Tartar	1,550
Burgundy Wine,	0,953

Solid is thus compared with Water, and consequently with each other; as in the Table below.

THE

Canary,	1,033
Red Wine from *Pontac*,	0,993
White Wine Vinegar,	1,011
Distilled Vinegar,	1,030
Milk of Goats,	1,009
Cow's Milk,	1,030
Urine,	1,030

23. Since all Bodies are subject to expand with Heat, and be condensed with Cold, it must follow, that the specific Gravities of Bodies cannot be precisely the same both in Summer and Winter. This was first observed in Experiments by M. *Homberg*, and after him by M. *Eisenschmid*, who found the absolute Weight of a cubic Inch of several Sorts of Bodies in Summer and Winter were sensibly different in each other, but upon examining the Numbers in his Table I find they are very erroneous, and have left them out in this Edition, and shall only observe, that this Affair may be rendered more apparent by taking the *specific Gravities* of these Liquors, (1.) when they are very cold; (2.) when they are pretty warm; and (3.) when they are very hot; for in these several Cases the Subject will discover a much less Density or Weight in equal Bulks, either by the Common or Hydrostatic Balance. But there is no Method in this Case so accurate as examining the different Bulks and Gravities of Fluids by the Divisions of *Farenheit*'s Thermometer from the Degree of Cold in a *freezing Mixture*, to Heat of *boiling Water*, and tabulating the Observations.

24. And as in this, and many other Cases, it is required to be very exact in weighing Bodies, even beyond what is attainable by the nicest Mechanism of the Instrument itself, I shall here give the Reader an Account of an Improvement of the Common Balance in this Respect, from the late learned *S'Gravesande*; and it

THE *Application of Hydrostatics* to the several Uses of Life will be evident from the follow-

it will be the more pertinent in this Place, as it depends on an hydrostatic Principle. This Instrument serves equally for Exactness in common as in hydrostatical Matters.

25. The Figure of the Machine represents the Balance in its hydrostatic Use. I shall first describe the Machine; then shew the new-contrived Artifice for Exactness; and, lastly, give an Instance of its universal Use. V C G is the Stand or Pillar fixed in the
Pl. XVII. Fig. 2. Table. From the Top at A hangs, by two silken Strings, the horizontal Piece or Bar B B; from which is suspended, by a Ring at *i*, the fine Beam of a Balance *l*, which is kept from descending too low on either Side by the gentle springing Piece *t x y z*, fixed on the Supporter M. The Harness is annulated at *o*, to shew distinctly the perpendicular Position of the *Examen* by the small pointed Index fixed above it.

26. The Strings by which the Balance is suspended passing over two Pulleys, one on each Side the Piece at A, go down to the Bottom on the other Side, and are hung over the Hook at *v*; which Hook, by means of a Screw P, is moveable about $1\frac{1}{2}$ Inch backwards and forwards, and therefore the Balance may be raised or depressed as much. But if a greater Elevation or Depression be required, the Sliding-Piece S, which carries the Screw P, is readily moved to any Part of the square Brass Rod V K, and fixed by means of a Screw.

27. The Motion of the Balance being thus provided for, the rest of the *Apparatus* is as follows: H H is a small Table fixed upon a Piece D, under the Scales *d* and *e*, and is moveable up and down in a long Slit in the Pillar above C, and fastened at any Part with a Screw behind. At the Point in the Middle of the Bottom of each Scale is hung by a fine Hook a Brass Wire *a d*, *a e*. These pass though two Holes *m*, *m*, in the Table; and to the Wire *a d* is suspended a curious cylindric Wire *r s*, perforated at each End for that Purpose.

following Instances; having first premised, (what is found by Experiment) *that a Cubic Foot of common Water weighs very*

pose. This Wire *r s* is covered with Paper graduated by equal Divisions, and is about five Inches long.

28. In the Corner of the Table at E is fixed a Brass Tube, in which a round Wire *h l* is so adapted as to move neither too hard nor too freely by its flat Head I. Upon the lower Part of this moves another Tube Q which has Friction enough to cause it to remain in any Position required; to this is fixed an Index T, moving horizontally when the Wire *h l* is turned about, and therefore may be easily set to the graduated Wire *r s*.

29. To the lower End of the Wire *r s* hangs a Weight L, and to that a Wire *p n* with a small Brass Ball *g*, about $\frac{1}{4}$ of an Inch in Diameter. On the other Side, to the Wire *a c* hangs a large Glass Bubble R by a Horse-hair. Let us at present suppose the Weight L taken away, and the Wire *p n* suspended from S; and on the other Side let the Bubble R be taken away, and the Weight F suspended in its Room at *c*. This Weight F we suppose to be such as will keep *in Equilibrio* with the several Parts appended to the other Scale, at the same Time that the middle Point of the Wire *p n* is in the Surface of the Water in the Vessel N.

30. The Wire *p n* is to be of such a Size, that the Length of one Inch shall weigh four Grains. Hence it is evident, since Brass is eight Times heavier than Water, (see *Art.* 16,) that for every Inch the Wire sinks in the Water, it will become half a Grain lighter, and half a Grain heavier for every Inch it rises out of the Water: Consequently, by sinking two Inches below the middle Point, or rising two Inches above it, the Wire will become one Grain lighter or heavier.

31. And therefore, if when the middle Point is at the Surface of the Water *in Equilibrio*, the Index T be set to the middle Point *a* of the graduated Wire *r s*, and the Distance on each Side *a r* and *a s* contain 100 equal Parts; then, when in weighing Bodies the Weight

very exactly 1000 *Ounces Averdupois, or* 62 *Pounds and a half*; which may be reduced to

Weight is desired to the *hundredth Part* of a Grain, it may easily be had by proceeding in the following Manner. Let the Body to be weighed be placed in the Scale *d*, and put the Weights in the Scale *e*; and let these be so determined, that one Grain more shall be too much, and one Grain less too little. Then the Balance being gently moved up or down by the Screw P, till the Equilibrium be nicely shewn at *o*; and then if the Index T be at the middle Point (a) of the Wire *rs*, it shews the Weights put into the Scale *e* are just equal to the Weight of the Body.

32. But if the Index T stand at any Part between a and *r*, it shews the Number of Grains in the Scale *e* were more than equal to the Weight of the Body in the Scale *d*, because the Wire *pn* is now made lighter by sinking below the middle Point. Thus, suppose the Weights put into the Scale *e* were 1095 Grains, and the Index T cuts the 36th Division above a, it shews that 36th hundredth Parts of a Grain are to be added, or that the Weight of the Body is 1095,36 Grains.

33. On the other Hand, had the Index stood at 36, the Division below a, it would have shewn the Weights in the Scale *e* were more than equal to the Weight of the Body by 36 Hundredths of a Grain; and that there the Weight of the Body was 1094,64 Grains. By this Method we find the absolute Weight of the Body; the relative Weight is found by weighing it hydrostatically in Water as follows.

34. Instead of putting the Body in the Scale *d*, as before, let it be appended with the Weight F at the Hook *c*, by a Horse-hair as at R, supposing the Vessel of Water O were away; then the Equilibrium being made, the Index T standing between a and *r*, at the 36th Division, shews the Weight of the Body 1095,36 Grains. As it thus hangs, let it be immersed in the Water of the Vessel O, and it will become lighter by much; the Scale *e* will descend till the Beam of the Balance

to *Troy* Weight, by considering, that the *Averdupois* Pound is to the *Troy* Pound as 17 to 14, and the *Averdupois* Ounce to the *Troy* Ounce as 51 to 56.

HENCE, *to find the Quantity of Pressure against the Sluice or Bank that pens the Water*, we have this Rule: *Multiply the Area of the Sluice under Water by the Depth of the Centre of Gravity in Feet, and that Product again by* $62\frac{1}{2}$; *the Product will be the Number of Pounds required. Example:* Admit

Balance rests on the Supporter *z*. Then suppose 100 Grains put into the Scale *d* restored the Equilibrium precisely, so that the Index T did again point to the 36th Division above a[*]: It is plain the Weight of an equal Bulk of Water would in this Case be exactly 100 Grains.

35. But if the 100 Grains in the Scale *d* cause it to preponderate a little, then by turning the Screw P the Balance may be raised, till the Wire *p n* becoming heavier restores the Equilibre. Let now the Index T cut the 6th Division above a; then $36 - 6 = 30$, which shews that the Wire *p n* is now $\frac{3}{10}$ of a Grain heavier than before; therefore the Weight of the Water is only 99,7 Grains: Whence its Gravity to that of the Body is as 99,7 to 1095,36; as required.

36. After a like Manner may this Balance be applied to find the specific Gravities of Fluids; which will not be difficult to those who apprehend what has been already said. In Practice, it will be necessary to use great Precaution in every Particular; the Wire *p n* should be oiled, and then wiped as clean as possible; enough will remain to prevent the Water adhering thereto: Also the Balance ought to be raised very gently, and when come to an Equilibrium should be gently agitated, to see if it will come so again.

mit the Length of the Sluice be 20 Feet, the Depth of Water 5; then will the Area under Water be 100 square Feet; which multiplied by $2\frac{1}{2}$, the Depth of the Centre of Gravity, gives 250 cubic Feet; which again multiplied by $62\frac{1}{2}$, gives 15625 *lb.* equal to 7 Tons nearly (LXIV).

AGAIN: *Since the Weight of Bodies is always as the specific Gravity in equal Bulks*, it follows, that the Numbers in the Table do also express the Number of *Averdupois* Ounces

(LXIV) 1. That the Area under Water multiplied by the Depth of the Centre of Gravity gives the Number of solid Feet of Water pressing against the Sluice, is evident from *Annot.* LV. *Art.* 13, 14, 15, &c. And this Product is to be multiplied by $62\frac{1}{2}$, because so many Pounds is the Weight of one solid or cubic Foot of Water. See *Annot.* LXII. *Art.* 7.

2. This Example gives the Numbers which I took from a Pen that was made across a River; and though it may seem wonderful that all the Water in the River should affect the Pen with no Force of Pressure, yet this will be found consistent with the Laws of the Action of Fluids, when that is understood which we have delivered in *Annot.* LVI. For the Water of the River does no otherwise influence the Pen, than as it sustains a Plane of fluid Particles to a certain Height in immediate Contact with the Pen. This single Plane of Particles it is that gives all the Pressure on the Pen; and they would do the same, could they be otherwise supported against it, if all the rest of the Water in the River were taken away. Thus the Waters of the Sea or Ocean lying against any Dam or Bank will press it with no greater Force than the least Quantity of Water standing to the same Height, over the same Extent of the Dam.

Ounces contained in a cubic Foot of each refpective Sort of Matter therein mention'd. Therefore, *if the Magnitude of any Body be multiplied by the fpecific Gravity, the Product will be its abfolute Weight.* Thus, fuppofe I would know what Weight of Lead will cover a Church whofe Area is 30000 Feet, and the Thicknefs of the Lead $\frac{1}{100}$ of a Foot: Then *per* Rule, ($30000 \times \frac{1}{100} =$) $300 \times 11325 = 3397500$ Ounces, or $97\frac{4}{5}$ Tons; the Weight required.

ANOTHER ufeful Problem is, to *find the Magnitude of any Thing, when the Weight is known*; which is done *by dividing the Weight by the fpecific Gravity in the Table, the Quotient is the Magnitude fought.* For Inftance, What is the Magnitude of feveral Fragments of *Coral* whofe Weight is 7 Ounces? Divide 7 by the fpecific Gravity 2690, the Quotient is $\frac{26}{10000}$ of a cubic Foot: then $\frac{26}{10000} \times 1728 = 4\frac{1}{2}$ cubic Inches, very nearly the Magnitude required.

ALSO, *by knowing the Magnitude and Weight, we can find the fpecific Gravity, by dividing the Weight by the Magnitude in cubic Feet.* Thus fuppofe a Piece of Marble contain 4 cubic Feet, and weighs 603 *lb.* or 10800 Ounces; then $\frac{10800}{4} = 2700$, the fpecific Gravity required, as *per* Table.

HAVING

HAVING given the ſpecific Gravity of Gold to Silver as 19 to 11, and ſuppoſe any Compound thereof, as King *Hiero*'s Crown, whoſe ſpecific Gravity is 16; to determine the Proportion and Weight of the Gold and Silver employed in making it, ſay, *As the Difference of the ſpecific Gravities of the Compound and the lighter Ingredient*, viz. 5, *is to the Difference of the ſpecific Gravities of the heavier Ingredient and the Compound*, viz. 3, *ſo is the Bulk of Gold to that of Silver made uſe of.* That is, if the whole Crown were divided in 8 Parts, the Gold would conſiſt of 5, and the Silver of 3: Then the Magnitudes 5 and 3, multiplied by the ſpecific Gravities 19 and 11 ſeverally, will give the Numbers 95 and 33, which expreſs the *Proportion of the Weight of the Gold to that of the Silver* (LXV).

SINCE

(LXV) 1. The Rules here given for ſolving the three firſt Problems are only thoſe Theorems expreſſed in Words which you find in *Annot.* LVII. *Art.* 9, 10, 11. But that which concerns the fourth, relating to King *Hiero*'s Crown, I ſhall here demonſtrate as follows:

2. Let A and B be any two Sorts of Matter, and a and b denote their ſpecific Gravities; the Compound made of thoſe two Bodies will be $A + B$; and let c be the ſpecific Gravity thereof. Then ſince the abſolute Weight of a Body is equal to its Bulk multiplied by its ſpecific Gravity (See *Annot.* LVII. *Art.* 10.) we ſhall have $A\,a =$ Weight of the Body A, and $B\,b =$ Weight

SINCE Bodies of different specific Gravities, equiponderating each other in Air, … upon

Weight of the Body B, and $\overline{A+B}\times c =$ Weight of the Compound. But $A\,a + B\,b = \overline{A+B}\times c = A\,c + B\,c$; whence $A\,a - A\,c = B\,c - b\,B$; consequently $A : B :: c - b : a - c$, which is the Rule above laid down where A is the Quantity of Gold, and B that of Silver, and A + B the Composition of *Hiero*'s Crown.

3. When King *Hiero* (suspecting the Workmen had allayed the Gold with more Silver than was necessary) sent to *Archimedes* to examine into, and detect the Fraud, if there were any; this great Mathematician was long at a loss to think of any Method of doing it; till one Day getting into a Tub full of cold Water to bathe himself, he observed, that as he entered the Tub the Water ran out, and he immediately saw it must follow, that if the Tub were full, the Water which ran out upon his Immersion, must be equal in Bulk to his Body.

4. Hence the Philosopher began thus to reason: If I immerse the Crown in a Vessel full of Water, it will protrude as much as is equal to its Bulk. If after this I immerse the same Weight of pure Gold, and pure Silver, I shall get Bulks of Water equal to each; consequently having the Bulks of Gold, Silver, and Crown of equal Weight, I have the specific Gravities, which must be as the Bulks inversely. Then I proceed to find the Ratio, or Proportion of Gold to the Silver in the Crown, as follows.

5. Suppose A M L B be a Vessel filled with Water to the Height D C, and that the Mass of Gold equal in Weight to the Crown, being immersed into the Water, raises the Surface thereof to E; and after that, the Mass of Silver of the same Weight immersed raises the Surface to G; then if the Height of the Vessel above C be divided into equal Parts, and D F = 11, and D G = 19; it is plain the Bulks of the Gold and Silver will be as D F to D G, and the specific Gravities as D G to D F. Lastly, if the Crown be immersed, Pl. XVI. Fig. 5.

upon being immersed into Water, will have the *Equilibrium* immediately destroyed by

it will raise the Surface to E, so that D E = 10. Whence the Proportion of the Bulks of the Gold and Silver in the Crown may be determined.

6. For since the Difference of specific Gravities of the Gold and Silver is D G — D F = F G = 8; if the Bulk of the Crown be divided into 8 equal Parts, it is evident, that since the specific Gravities of the debased and pure Gold Crowns will be as the Bulk inversely; that is, as D F to D E, we can easily find the Point H, which will express the specific Gravity of the former; for D E : D F :: D G : D H. Now the Point H always divides the Difference F G into two Parts G H, H F, which have the same Proportion as the Parts of Silver in the Crown have to the Parts of Gold; for as the Point E ascends, the Point H descends, and when E coincides with G, H falls upon E, and the Crown becomes all Silver; as on the contrary, when E descends to F, and H ascends to G, the Crown becomes all Gold; therefore every where F H will be to H G as the Parts of Gold to the Parts of Silver in the Crown.

7. Therefore, in the present Case, because the Crown, when immersed, raises the Water to the Height D E, and H is 3 Divisions below G, it shews that 3 of the 8 Parts of the Crown are Silver, and consequently the other 5 Parts are Gold, as H is 5 of the Divisions above F. Hence the Bulk of Gold in the Crown is to that of the Silver as 5 to 3, as before determined. That *Archimedes* took some Method like this is certain; and it is said, he offered to *Jupiter* a Hecatomb of Oxen for inspiring him with the Thought.

8. To the Problems in the Lecture, I shall add the following. Let there be two Bodies A and B, one heavier, the other lighter than Water; let their specific Gravities be *a* and *b*, and let the Weight of A be given, and denoted by W; and let it be required to find what Weight *(w)* of the lighter Body B must be connected with the heavier, that so the Compound shall have the same specific Gravity with Water. Now because

by the greater Resistance of the Fluid, and consequently the greater Loss of Weight in the

cause $A = \frac{W}{a}$ and $B = \frac{w}{b}$, therefore $A + B = \frac{W}{a} + \frac{w}{b} = \frac{Wb + wa}{ab}$ = Bulk of Water, which multiplied by its specific Gravity (c) gives the Weight of Water $\frac{Wbc + wac}{ab} = W + w =$ the Weight of both the Bodies. Whence we have $Wbc + wac = Wab + wab$; and so $Wbc - Wab = wab - wac$; consequently $\frac{Wbc - Wab}{ab - ac} = w$; or if $c = 1$, then we have $\frac{Wb - Wab}{ab - a} = \frac{b - ab}{ab - a} \times W = w =$ Weight of the Body B required.

9. For Example; suppose the specific Gravity of a Man's Body, Water, and Cork be as 10, 9, $2\frac{1}{4}$; then $a = 10$, $c = 9$, and $b = 2\frac{1}{4}$; let $W = 150$ *lb.* = Weight of the Man's Body; then will $\frac{Wab - Wbc}{ab - ac} = w = 5$; therefore if such a Man were to take 5 *lb.* of Cork with him into the Water, he would be of the same specific Gravity of the Water, and ever so little more would make it impossible for him to sink; whence the Art of learning to swim might by this Problem be greatly facilitated.

10. *To find the Weight of a Globe of Water one Inch in Diameter.* We shall here chuse *Troy* Weight, where one Cubic Foot weighs 76 *lb.*; and therefore one cubic Inch weighs $\frac{76}{12 \times 12 \times 12}$ of a Pound, or $\frac{76 \times 12 \times 20 \times 24}{12 \times 12 \times 12}$ Grains, that is, $\frac{76 \times 40}{12}$ or $\frac{760}{3}$ Grains. But in Geometry we prove the Cube is to its inscribed Sphere as 1 to 0,524; therefore as $1 : 0,524 :: \frac{760}{3} : 132$ Grains = the Weight of a Globe of Water one Inch Diameter.

the lighteſt and moſt bulky Body; therefore it follows, that in *buying Gold*, which is

11. *To find the Diameter of a Globe by weighing it hydroſtatically.* Let it firſt be weighed in Air, and then in Water, and the Difference of thoſe Weights will be the Weight of an equal Bulk or Globe of Water, which call W; now the Weights of homogeneous Bodies are as their Bulks, which (in Spheres) are as the Cubes of their Diameters. But the Weight of a Globe of Water 1 Inch in Diameter is 132 Grains (by *Art.* 10.) Therefore we ſay, As $132 : W :: 1^3 : D^3$; whence $\frac{W}{132} = D^3$, conſequently $\sqrt[3]{\frac{W}{132}} = D =$ Diameter of the Globe required.

12. *To find the ſolid Content in Cubic Inches of any irregular Body.* Let W = Difference of its Weight in Water and Air = Weight of an equal Bulk of Water. Then as the Weight of a cubic Inch of Water $\frac{760}{3} : W :: 1 : \frac{3}{760} \times W =$ ſolid Content of the Body required.

13. *To find the Proportion of Magnitude between any two Bodies propoſed.* Let each be weighed in Air and Water, and the Loſſes of Weight they each ſuſtain will be the Weights of two Bulks of Water, and equal to thoſe Bodies reſpectively; and conſequently will expreſs the Ratio of the Magnitudes of thoſe Bodies.

SCHOLIUM.

14. The Excellency of this Hydroſtatic Method, above all others, appears from hence, that it is univerſal, and equally adapted to all Sorts and Shapes of Bodies, which common Geometry is not. How would it puzzle a Geometrician to exhibit the juſt Dimenſions or Bulk of a Fiſh, for Inſtance? How does the aukward Figure of his Body, the Appendage of Fins and Tail, elude the Principles of his Art? Whilſt the Philoſopher immediately, and without any Trouble, gives the Anſwer by his *Hydroſtatic Balance.* Again, were a Geometer

is *heavier than Braſs*, we ſhould chuſe the *lighteſt Air*, *i. e.* when the Mercury in the Barometer ſtands loweſt; but in *buying precious Stones*, *Pearls*, &c. which are *lighter than Braſs*, the beſt Time to do it in is *when the Air is heavieſt and moſt buoyant*, *viz.* when the Quickſilver ſtands higheſt in the Barometer: But in *ſelling Gold or Jewels*, the contrary Rules are to be obſerved in regard to the Gravity of the Air (LXVI).

ONCE

meter aſk'd, Which is biggeſt, *a Guinea or a Shilling*, and what the Difference? He would find it vain to conſult his Art, and muſt borrow his Aid and Anſwer from this moſt uſeful Science, which excels equally in Accuracy as Univerſality.

(LXVI) 1. Since a cubic Inch of Gold weighs 10,$\frac{36}{100}$ Ounces *Troy*, and a cubic Inch of Air $\frac{2}{7}$ of a Grain, when the Air is of a mean Gravity; if we ſay, As 10,36 : $\frac{2}{7}$ :: 12 : $\frac{24}{72,52}$ = the Parts of a Grain which 12 Ounces or a Pound of Gold will loſe when the Air is in a mean State. Now ſince the Air, when it ſupports a Column of Mercury 28 Inches high, is lighter by $\frac{1}{20}$ Part than when it ſuſtains a Column of 30 Inches Height; therefore $\frac{24}{72,52} + \frac{1}{20} = \frac{552,52}{1450,4}$ = the Parts of a Grain which ſo much Air weighs when heavieſt, as is equal in Bulk to a Pound of Gold; and twice the Quantity, *viz.* $\frac{1105,04}{1450,4}$ = the Weight of Air equal in Bulk to two Pounds of Gold. But this is a little leſs than one Grain; let us ſuppoſe it one Grain.

2. Then two Pounds of Gold will loſe of its Weight one Grain; and ſince Gold is twice as heavy as Braſs that

 it

ONCE more: Since *the Goodness of Mineral Waters, Drugs, Metals, precious Stones, &c. is best shewn by their specific Gravity*, it will at once appear of what vast Importance the Hydrostatic Balance is, and how absolutely necessary in the Hands of every judicious Dealer in any such Kind of Commodities (LXVII), (LXVIII).

it is weighed against, the Brass Weights will have twice the Bulk, and will there lose two Grains of its absolute Weight. If an Equilibrium be now made, when the Air is lightest, or less by one Tenth of its former Weight, the Gold will lose one Tenth of a Grain, and the Brass will lose two Tenths of a Grain; consequently the Buyer will get one Tenth of a Grain of Gold in this last State of the Air, more than he would have had in the former, or heaviest State; because now one Tenth of a Grain more must be added to the Brass to make an Equilibrium. And this in Value is the fifth Part of a Penny, reckoning Gold at Two-pence *per* Grain; or one Part in 115200.

3. Since the specific Gravity of Diamond is to that of the Brass Weights as 1 to 3, if we make an Equilibrium between the two Pounds of Brass and Diamond, when the Brass loses two Tenths of a Grain, the Diamond will lose six Tenths of a Grain; consequently four Tenths of a Grain must be subducted from the Brass for an Equilibrium in the lightest State of the Air, and so much the Buyer will lose of Diamond more than when the Air is heaviest of all; which is about one Part in 28800 of the Whole. Whence it appears, that a Regard to the State of the Air is a Matter of more Curiosity than Importance; the Advantage being so very inconsiderable in either Case.

(LXVII) 1. How great the Usefulness and Importance of Hydrostatic Knowledge is to *Physicians*, *Chemists*, *Apothecaries*, *Jewellers*, *Goldsmiths*, &c. will appear by reading Mr. *Boyle*'s excellent *Medicina Hydrostatica*; in which

which Book the skilful Author proposes the following Uses to be made of Hydrostatic Knowledge, *viz.*

2. *First*, To explore the Nature and Difference of Fossils by finding their specific Gravities. For since the most pure and homogeneous Kind of Stones are in Gravity to Water as about $2\frac{1}{2}$ to 1; and Tin, the lightest of Metals, is to Water in Gravity as about 7 to 1; if a stony Substance be found to have a greater Proportion of Gravity than that of $2\frac{1}{2}$ to 1, it must be probable that it has in it some adventitious Matter of a metalline Nature, or is at least commixed with some mineral Body more heavy than pure Stone, and may therefore very probably be usefully applied to some medicinal Purposes. For Instance of this Kind, he mentions *Lapis Hæmatites* or Blood-stone, *Lapis Lazuli*, the Load-stone, and *Lapis Calaminaris*; all which have their Uses in Physic.

3. *Secondly*, He proposes this Method as very certain to determine whether a Body, supposed to be a Stone of the mineral Kind, be so indeed. Thus Coral, which, says he, some take to be a *Plant*, and others a *Lithodendron*, but most reckon it among precious Stones, is in Gravity to Water as 2,68 to 1, which favours the last Opinion. Thus a *Calculus humanus* and a *Bezoar* were found as 1,7 and 1,5 to 1, and therefore too light to be of the same Species with common Stone.

4. A *Third* Use which he proposes is to discover the Resemblance or Difference between Bodies of the same Denomination, and thereby to collect and ascertain the several Degrees of Goodness respectively. Whence he argues the Necessity of this Sort of Knowledge to Physicians, Chemists, Apothecaries, Druggists; to the Goldsmith, the Merchant, the Miner, *&c.*

5. A *Fourth* Use is to discern genuine Stones from counterfeit ones, which may be of great Help to Jewellers. Here he gives Instances of factitious Coral and factitious Gems, and a Bezoar, which he found out that Way not to be genuine, though a great Price was set on the latter.

6. Hence Mercury is found to have a different Gravity, being sometimes but $13\frac{1}{2}$, and sometimes above 14 Times heavier than Water; and hence a notable Difference may arise in two Weather-glasses at the

same Time, and in the same Place, even to a whole Inch, from the different Gravity of the Mercury in one and the other. Therefore those who publish Registers of the Weather ought to find out and declare to the World the specific Gravity of the Quick-silver they use in their Barometers.

7. These Uses he enumerates over and above what we have taken Notice of, of a Mechanical and Geometrical Nature: And to let us know the high Value he had for this Science, he thus expresses himself: "As little Skill as I have in Hydrostatics, I would not be debarred from the Use of them for a considerable Sum of Money, it having already done me acceptable Service, and on far more Occasions than I myself first expected, especially in the *Examen* of Metals and mineral Bodies, and of several mineral Productions." With much more to the same Purport.

(LXVIII) 1. To render this Lecture on *Hydrostatics* more compleat, I shall here subjoin what relates to the Resistance of Mediums, or to the Motion of Bodies moving in a resisting Medium: and to determine rightly in this Case, I shall consider every particular Circumstance by itself, *cæteris paribus*, and then represent the Whole connected together. We shall suppose the Case of *a Globe moving in a Fluid of an uniform Density for a given Time.*

2. First, Let the same Globe move with the same Velocity, first in a denser Fluid, and afterwards in a rarer; then it is plain, the denser the Fluid is, the more Particles the Globe will meet with, and strike in a given Time; and therefore the greater will be the Re-action or Resistance of the Fluid: Therefore in this Case *the Resistance of the two Fluids will be as their Densities.*

3. In the next Place, suppose two different Globes, of the same Quantity of Matter, to move in the same Fluid with the same Velocity. As these Globes move, it is evident the Number of Particles they meet and strike, and therefore by which they are resisted, will be as their Surfaces, or rather as their Half-Surfaces; and they are as the Squares of their Diameters: Consequently, *the*

the Resistance which these Globes meet with will be as the Squares of their Diameters.

4. Lastly, Let the same Globe move in the same Fluid with different Velocities. Now it is easy to conceive, that if one Globe moves with a Velocity double to that of the other, it will strike twice the Number of Particles in the same Time, and its Resistance will be twice as great on that account. Again: The Globe which moves with a double Velocity will strike each Particle with a Force twice as great as the other does with half the Velocity; the Re-action therefore of the Particles on the former will be twice as great as upon the latter. Therefore the Resistance to the Globe moving with the double Velocity will be four Times as great: And since this will be the Case universally, *the Resistance to a moving Body is as the Square of its Velocity.*

5. Now putting all these Cases together, the Resistance to a Globe moving uniformly in an uniform Fluid will be in the compound Ratios of the Density of the Medium, the Square of the Diameter, and the Square of the Velocity. So that if A and B are two Globes, whose Diameters are C and d, and Velocities V and v, moving in Mediums whose Densities are N and n; then will the Resistance of the Globe A be to that of B, as $N \times D^2 \times V^2$ to $n \times d^2 \times v^2$.

6. In this Computation we have considered only that Resistance which arises from the Re-action of Matter by its *Vis Inertiæ*: But there is some small Resistance arising to Bodies moving in a Fluid from two other Causes, *viz.* one from the Friction of the Body against the Parts of the Fluid as it moves, the other from the Tenacity of the Particles of the Fluid, which arises from their Attraction of Cohesion. But both these are very small when compared with the other, and quite inconsiderable to Bodies moving with any considerable Swiftness. The Resistance peculiar to Air, arising from its Elasticity, will be considered in another Place.

7. In the above Cases we have supposed the Bodies moving in the Fluid to be homogeneal, or of the same Matter and Density; but if they are heterogeneal, or of different Densities, then will the Resistance be further variable; for it will be inversely as the Densities

or

or Quantities of Matter which the Bodies contain; because the greater the Quantity of Matter, the greater will be the *Momentum* of the moving Body, and the less the Resistance of the Medium compared therewith, and so will be the more easily overcome by it. Hence, if the two Globes mentioned in *Art.* 3. have not equal Quantities of Matter, (as is there supposed) the Resistance will be as their Surfaces directly, and inversely as the Quantities of Matter; that is, as $D^2 \times \frac{1}{D^3}$ to $d^2 \times \frac{1}{d^3}$, or as $\frac{1}{D}$ to $\frac{1}{d}$, or as d to D. That is, *the Resistance to Globes of unequal Diameters and Quantities of Matter will be* (cæteris paribus) *inversely as their Diameters.*

8. Since a Body moving in a resisting Medium must every Moment have its *Momentum* or Quantity of Motion ($Q = MV$) abated or diminished by the Re-action of the Fluid; and since in a given Time this Loss or Decrement of Motion is always as the Re-action, that is, as the Resistance (R) of the Fluid: But the momentary Increase or Decrease of Motion is as its Fluxion $\dot{Q}$ or $\dot{V}M$. Wherefore the Resistance of the Fluid for any equal Parts of Time will be $R = \dot{Q} = \dot{V}M$; or when the Mass of Matter M is given, we have $R = \dot{Q} = \dot{V}$.

9. But when the Moments of Time are not given, that is, not equal, the Resistance R will as the Decrement of Motion $\dot{Q}$ directly, and the Moment or Fluxion of the Time ($\dot{T}$) inversely; for twice the Resistance will destroy the same Quantity of the Motion in half the Time, and three Times the Resistance will destroy it in one Third of the Time. Therefore $R = \left(\frac{\dot{V}M}{\dot{T}} = \right)\frac{\dot{Q}}{\dot{T}}$; consequently, when the Mass M is given, we have $R = \frac{\dot{V}}{\dot{T}}$; that is, the Resistance is directly as the Fluxion or momentaneous Decrement of the Velocity, and inversely as the Fluxion of the Time.

10. We have before shewn, (*Annot.* XXI. *Art.* 5.) that in uniform Motions *the Velocity is as the Space di-*

rectly, and the Time inversely, or $V = \frac{S}{T}$; and because in every Kind of Motion, whether *in Vacuo* or in any Sort of resisting Medium, the momentaneous Velocity may be consider'd as equable or uniform, or proportional to the Space described each Moment of Time; and therefore $V = \frac{\dot{S}}{\dot{T}}$, and so $V\dot{T} = \dot{S}$, and $\dot{T} = \frac{\dot{S}}{V}$; and taking the Fluents, we shall have the *Sum of all the Rectangles* $V\dot{T} = S =$ the Space described, and the Sum of all the Quotients $\frac{\dot{S}}{V} = T =$ the Time.

11. Therefore if the Curve BPC be so described that its Ordinates MP, *mp*, (perpendicular to the Axis AD) expound the Velocity V, and the *Abscissæ* (from the fix'd Point A) AM, A*m*, the Time T; the Perpendicular AB being erected in the Point A, and meeting the Curve in B; the Area ABPM will express the Space S described in the Time T. Let *mp* be infinitely near to MP, then will $Mm = \dot{T}$; therefore the Fluxion of the Area ABPM will be the Areola MP*pm* $= V\dot{T} = \dot{S}$. Pl. XVI. Fig. 6.

12. In like Manner, if the Abscis AM expounds the Space described $= S$, and the Ordinate MP be as the Velocity inversely, that is, if MP be as $\frac{1}{V}$, the Area ABPM will expound the Time T in which AM is described; for the Fluxion of the Time being $\dot{T} = \frac{\dot{S}}{V} = \dot{S} \times \frac{1}{V} = Mm \times MP$, the Sum of all these Fluxionary Areolas MP*pm* will be the Area ABPM expressing the Time.

13. Because when the Mass of Matter in the moving Body is given, we have $R = \frac{\dot{V}}{\dot{T}}$ (*Art.* 9.); therefore $R\dot{T} = \dot{V}$, or *the Fluxionary Decrement of Velocity is as the Resistance and the Fluxion of the Time conjointly.* But *the Fluxionary Increment of the Space will be as the Velocity and its Fluxionary Decrement directly, and the Resistance*

sistance inversely, that is, $\dot{S} = \frac{V \times \dot{V}}{R}$; for as above $\dot{T} = \frac{\dot{V}}{R}$, and $\dot{S} = V\dot{T}$ (*Art.* 10.); therefore $\dot{S} = V \times \frac{\dot{V}}{R}$ or $\dot{S} = \frac{\dot{V} \times V}{R}$.

14. Hence $R = \frac{V \times \dot{V}}{\dot{S}}$, or *the Resistance is as the Velocity and its Fluxionary Decrement directly, and the Fluxion of the Space inversely.* Also $R\dot{S} = V \times \dot{V}$; that is, *the Velocity multiplied by its Fluxionary Decrement is as the Resistance and Fluxionary Increment of the Space conjointly.* Because the Velocity has been hitherto consider'd as decreasing, it ought to have the Fluxion $\dot{V}$ express'd negatively in the above Equations; that is, $R\dot{T} = -\dot{V}$, and $R\dot{S} = -\dot{V} \times V$.

15. We have also hitherto consider'd the Resistance R as comprehending all that retards the Motion of the Body in the Medium. But this retarding Power may arise from different Principles, as the Density of the Medium, or its particular Resistance; and also from a centripetal Force, as that of Gravity: Thus a Stone thrown upwards is retarded by the Resistance of the Air, and Power of Gravity. Let the Centripetal Force be C, the Resistance of the Medium *R*; then will $R = C + R$ when these Forces conspire, and $R = C - R$ when they act contrarily.

16. Thus in the Motion of an ascending Body, where $R = C + R$, we have $R\dot{T} = C\dot{T} + R\dot{T} = -\dot{V}$; and $C\dot{S} + R\dot{S} = -\dot{V} \times V$. But in the Descent of the Body, where the Centripetal Force C is greater than that of the Resistance, we have $C - R$; and then the above Theorems will be $C\dot{T} - R\dot{T} = \dot{V}$, and $C\dot{S} - R\dot{S} = \dot{V} \times V$; for now the Velocity is increasing, and its Fluxion positive. But if the Central Force be less than the Resistance, we have $R - C$; then it will be $R\dot{T} - C\dot{T} = -\dot{V}$, and $R\dot{S} - C\dot{S} = -\dot{V} \times V$.

17. If in these Theorems we suppose $R = o$, they will be changed into others, by which the Motion of Bodies in non-resisting Mediums may be determined. Thus for

for the Ascent they will be $C\dot{T} = -\dot{V}$, and $C\dot{S} = \dot{V} \times V$; and for the Descent $C\dot{T} = \dot{V}$, and $C\dot{S} = \dot{V} \times V$. And thus the Motion of a Body in a resisting Medium may be compared with the Motions of the same Body in a non-resisting Medium, or *in Vacuo*.

18. If the Resistance be equal to the central Force, that is, if $R = C$, then in descending Bodies we have $C\dot{T} - R\dot{T} = \dot{V} = 0$; consequently, as the Fluxion of the Velocity is equal to Nothing, the Velocity can neither increase nor decrease, but is in that Case uniform or equable.

19. Since, till this happens, the Velocity is continually increasing, (because its Fluxion $\dot{V}$ is always as $C - R$ for any given Time) therefore when $C = R$, or $\dot{V} = 0$, the Velocity of the descending Body is a *Maximum*, or the greatest possible: But this Velocity Bodies descending in a Fluid, though they constantly approach to it, will never attain; as will be seen in the Sequel hereof.

20. In order to estimate the Velocity, the Time, and the Space described by Bodies falling in an uniform Fluid, such as Waters, whose Parts have no considerable Tenacity, and where the Retardation of the Body is the Effect only of the Resistance it meets with from the *Vis Inertiæ* or Re-action of the Fluid; it will be necessary first to observe, that when any Body moves in a Fluid with a given Velocity, the Re-action of the Fluid upon the Body is the very same as its Action upon it would be, were the Body to be at Rest in the Fluid moving against it with the same Velocity: Because the Magnitude of the Stroke is always as the relative Velocity, which is here the same in both Cases. (See *Annot.* XXIII. 8.)

21. Also that a circular Plane, a Cylinder, a Globe, a Spheroid, a Cone, &c. whose Breadth is the same, and against which the Fluid moves in a perpendicular Direction, are equally acted upon by the Fluid; or, if they move in the Fluid, are equally resisted by it. For let P Q be the Diameter of any of those Bodies placed in a Canal E F T S, thro' which the Water flows from the large Vessel A B C D, into another below fill'd with stagnant Water, touching the Plate XVIII. Fig. 6.

the Bottom of the former CD. Then ſince the Capacity of the Canal is alike ſtraiten'd or diminiſh'd by each of thoſe Bodies, it is plain the Fluid flowing through it muſt ſuffer the ſame Retardation from all of them, and therefore muſt act alike upon each; conſequently, were the Fluid at Reſt, and thoſe Bodies moving through it, they would all be equally retarded.

22. It muſt be farther obſerved, that all Reſiſtance ariſing from the Aſperity of the Surfaces of Bodies, is not here conſider'd; alſo it is ſuppoſed, that the Fluid is contiguous, infinitely compreſs'd, and whoſe Parts have no Elaſticity, Tenacity, nor Friction; and then the Caſe will be very nearly the ſame with *very deep Water*.

23. Now Sir *Iſaac Newton* has ſhewn, that if the Veſſel ABCD be indefinitely large or wide, the Velocity of the Fluid in the annular Space about the circular Plane P O will be equal to that which a Body acquires in falling through the Height GH; becauſe it would be ſo, were the Plane P Q placed in the Orifice EF, the Motion of the Fluid through the Canal being uniform, and not accelerated by Reaſon of the ſtagnant Water beneath.

24. He farther ſhews, that if this Plane were placed in the Orifice E F at the Bottom of the Veſſel, as at p, q, it would ſuſtain a Portion of the deſcending Fluid, or Cataract AEFB, like to that repreſented by pHq, which is leſs than two Thirds of a Cylinder of the ſame Baſe pq, and Altitude GH, but greater than one third Part thereof: And that therefore it is nearly an Arithmetical Mean between them, or equal to Half of the ſaid Cylinder.

25. This is upon Suppoſition that the Plane pq is exceeding ſmall; but if it be augmented till it becomes equal to the Hole EF, it will then ſuſtain the Weight of the perpendicular Column of Water over it, that is, it will ſuſtain the Weight of a Cylinder of Water of the ſame Baſe and Altitude G H: And that therefore in all other Caſes the Weight which the Plane pq ſuſtains is to the Weight of a Cylinder of Water, whoſe Baſe is equal to the Plane and Altitude $\frac{1}{2}$ G H, as EF^2 to $EF^2 - \frac{1}{2}pq^2$.

26. The Reſiſtance (R) therefore to the Plane pq moving in the Orifice E F, and conſequently of the Plane

Plane PQ moving in the Canal, will be in all Dimensions of the Canal and Plane compared with the Weight (W) of a Column of Water whose Base is equal to the Plane, and Altitude $\frac{1}{2}$ GH, as the Area of the transverse Section of the Canal to the Excess of this Area above $\frac{1}{2}$ the Area of the Plane; that is, $R : W :: EF^2 : EF^2 - \frac{1}{2}PQ^2$.

27. Let now the Orifices of the Canal EF and ST be closed up, and let the Plane PQ ascend in the Fluid compress'd on every Side; and in its Ascent let it oblige the superior Fluid to descend by the annular Space between it and the Canal; then will the Velocity (V) of the ascending Plane be to the Velocity (v) of the descending Water, as $EF^2 - PQ^2$ to PQ^2; that is, $V : v :: EF^2 - PQ^2 : PQ^2$; whence $V : V + v :: EF^2 - PQ^2 : EF^2$.

28. But since the Motion of the Water is contrary to that of the Plane, the relative Velocity, with which they act on each other, is the Sum of both, *viz.* as $V + v$. (See *Annot.* XXIII.) Now let this relative Velocity be equal to that with which the Water before descended in the annular Space by the Plane PQ at Rest, (as in *Art.* 23.) *viz.* that which is acquired by falling through GH. Then will the Action of the Water upon the Plane be the same as before at Rest, and consequently we have here also $R : W :: EF^2 : EF^2 - \frac{1}{2}PQ^2$; and $V : (V + v =) v :: EF^2 - PQ^2 : EF^2$.

29. If now we suppose the Canal to be augmented in Width *ad infinitum*, PQ will then be infinitely small in respect of EF; whence PQ^2 and $\frac{1}{2}PQ^2$ will vanish in the Terms of the above Ratios, which will then become equal; and so $R = W$, and $V = v$. That is, *the Resistance of the Plane will now become equal to the Weight of a Cylinder of Water, whose Base is equal to that of the Plane, and Altitude* $\frac{1}{2}$ GH: Also, *the Velocity of the Plane is equal to that which such a Cylinder would acquire by falling through the Height* GH.

30. Let A = Area of the Plane PQ, then will $\frac{1}{2}GH \times A$ = the Cylinder of Water, whose Weight W is equal to the Resistance R of the Plane. But the Cylinder $\frac{1}{2}GH \times A$, moving with the same Velocity V of the Plane, will have the same Resistance. The Space through

through which it must descend to acquire that Velocity is twice its Length, *viz.* GH; and with that Velocity, by an uniform Motion, it would pass over a Space equal to 2 GH = 4 Times its Length $\frac{1}{2}$ GH. *Wherefore the Resistance of the Cylinder is equal to the Force (or Weight* W) *by which (in falling freely) a Motion equal to that of the Cylinder may be generated, (in acquiring the Velocity* V) *in the Time in which the said Cylinder will describe a Space equal to four Times its Length.*

31. Then because the Motion M of the Cylinder is as its Length L, its Base A, its Density D, and its Velocity V, that is, M : L×A×D×V; (see *Annot.* LVII. 8.) therefore when (as in this Case) the Quantities A, D, V, are given, we have M : L; and since the Time T (in which the Motion M is generated, or 4 L is described) is also as L, therefore both M and T will vary with L equally, and consequently will be proportional to each other. But the Force which generates Quantities of Motion proportional to the Time is in itself constant and immutable, and is therefore equal to the Resistance of the Cylinder, which also remains unchanged under all Degrees of Motion.

32. If the Density D of the Cylinder be variable, so will the Quantity of Motion M be likewise, and also the Force W, by which an equal Quantity of Motion in the same Time may be generated or destroyed. Therefore, let d = Density of a Cylinder of Water, D = Density of a Cylinder of Lead of the same Base, w=Force which would generate the Motion of the Cylinder of Water in the Time it would describe four Times its Length, and W the Force that would generate the Motion of the Cylinder of Lead in the same Time: Then d : D :: w : W. But we have shewn w=R, the Resistance of each Cylinder: Consequently, R : W : : d : D, that is, *the Resistance of a Cylinder of Lead is to the Force which will generate its whole Motion while it describes four Times its Length, as the Density of Water to the Density of Lead.*

33. Hence, since the Resistance of a Globe and Cylinder moving in a Fluid is the same, and since a Globe is $\frac{2}{3}$ of its circumscribing Cylinder, the Force which will generate or destroy the whole Motion of the Cylinder while it moves over 4 Times its Length, will generate or destroy the whole Motion of the Globe while

while it moves uniformly through $\frac{2}{3}$ of 4 Diameters, or of its Diameter.

34. From what has been said, if the Velocity of a Globe or Cylinder moving in Water be known, and also its Diameter; then the Resistance it meets with from the Fluid, while it moves uniformly through, is known also. Thus let its Velocity V = 16 Feet *per* Second, and its Diameter three Inches. Then a Body falling through the Space of 4 Feet acquires that Velocity, (see *Annot.* LXXV. 6.) whence GH = 4; and $\frac{1}{2}$ GH = 2 Feet, the Length of a Column of Water, whose Base being 3 Inches in Diameter will weigh about 6 *lb.* and is equal to the Resistance of the Globe.

35. Hence also it is easy to find the Height from which if the Globe falls *in Vacuo* it shall acquire the Velocity which shall be the greatest it can possibly acquire by descending in the Fluid by the Force of its *relative Weight. Which Height will be to $\frac{4}{3}$ Parts of its Diameter, as the Density of the Globe to the Density of the Fluid.*

36. For, let R = Resistance; D = Density of the Globe, d = that of the Fluid, and F = Force which will generate the Motion of the Globe in the Time it moves over $\frac{8}{3}D$, where D = the Diameter of the Globe; then we have R : F :: d : D (in *Art.* 33.) whence F = $\frac{DR}{d}$. Now let 2 S = Space which the Globe describes uniformly in the Time of the Fall, and with the Velocity acquired thereby. Then will 2 S : $\frac{8}{3}$ D :: D : d (*per Art.* 32.) : : S : $\frac{4}{3}$ D. But T and t, the Times in which uniform Spaces are described, are as those Spaces, *viz.* T : t : : 2S : $\frac{8}{3}$ D; whence T : t : : D : d. Again, since in the Times T and t equal Quantities of Motion are produced, *viz.* the whole Motion of the Globe; for F produces that Motion in the Time t, and in the Time T it is produced by the Force or Relative Weight of the Globe, which call P. Then, since the greater the Time is, the less will be the Force acting uniformly to produce a given Effect, we have F : P : T : t; hence F : P : : D : d; and so F = $\frac{PD}{d} = \frac{RD}{d}$; whence P = R, that is, the Resistance being equal to the Weight of the Body, it can be no longer accelerated.

37. As Sir *Isaac Newton* was the Author of this Theory, so he instituted a Series of Experiments to confirm the same, for which Reason, as well as to exemplify the foregoing Method of Computation, I shall here repeat some of the Principal, made with Globes of Wax and Lead included, let fall in Water through the Depth of 112 Inches. His first Experiment was with a Globe $\frac{84}{100}$ Parts of an Inch Diameter $= D$, whose Weight *in Vacuo* was $156\frac{13}{38}$ Grains, and in Water 77 Grains, therefore $156\frac{13}{38} - 77 = 79\frac{13}{38}$ Grains = the Weight of an equal Bulk of Water. Wherefore $d : D :: 79\frac{13}{38} : 156\frac{13}{38} :: \frac{8}{3}D$ ($=2,24597$ Inches) : $2S = 4,4256$ Inches; and so $S = 2,2128$ Inches.

38. The Globe falling *in Vacuo*, with its whole Weight of $156\frac{13}{38}$ Grains, will in one Second of Time describe $193\frac{1}{3}$ Inches. And by its relative Weight of $79\frac{13}{38}$ Grains in Water, it would in the same Time by falling *in Vacuo* describe 95,219 Inches. For the Spaces described in the same Time will be as the accelerating Forces or Weights of the Bodies. (See *Annot.* XXVII. 2, 3.)

39. But the Time T, in which the Globe by descending *in Vacuo* with its relative Weight of $79\frac{13}{38}$ Grains, will describe the Space $S = 2,2128$, is to the Time $t = 1$ Second, in which it described the Space 95,219, in the subduplicate Ratio of those Spaces; that is, $T : t :: \sqrt{2,2128} : \sqrt{95,219}$, wherefore $T = \sqrt{\frac{2,2128}{95,219}} = 0'', 15244 =$ the Time in which the Globe by descending *in Vacuo* acquires the greatest Velocity V with which it can descend in the Water.

40. In the next Place, to find the Velocity and the Space described in the Fluid in the whole Time of the Fall (F) Sir *Isaac* has this Method; let the Number N be found for the Logarithm $0,43429448\text{1}9\,\frac{2F}{T}$; then is the Velocity of the Globe at the End of the Fall $= \frac{N-1}{N+1} \times V$. Again, let L be the Logarithm of the Number $\frac{N+1}{N}$; then will the Space described be $\frac{2SF}{T} -$

138629S+4,60517 LS. These Velocities and Spaces described are calculated and disposed into a Table by our Author for any Number of Seconds to 10 T. The Demonstration of the above Rules is too long and intricate for this Place, but may be seen in the Commentary on the *Principia* by *Le Seur* and *Jacquier*. This Table will be inserted at the End of this Note.

41. By this Table you see how soon Bodies acquire their greatest Velocity in descending in Water; for Instance, in $5T = 0'',76 = 46'''$, the Globe has acquired a Velocity which is to the greatest as 99990920 to 100000000. And in $10T = 1'' : 30'''$ the Velocity of the Globe is to the greatest Velocity as 99999999⅗ to 100000000. Which Difference is insensible to the most critical Eye; and from thence (having fallen through 37 Inches) it descends with an uniform Velocity to Appearance, though it really for ever increases.

42. If the Time of Descent in the Water be less than 10 T, the Distance descended is shewn by the third Column; but if it be greater, as in the present Experiment it is $4''$, then say, as $T : 2S :: 4'' : \frac{8S}{T} = 116,1245$ Inches, because Spaces described with an uniform Velocity are as the Times. This uniform Space must be diminished by substracting the constant Number 1,38629 S. Therefore $116,1245 - 1,38629\ S = 113,0569$ Inches, which the Globe should describe in the Water in $4''$ by the Theory.

43. But in the Experiment it described only 112 Inches, which Difference of 1 Inch in 112 is inconsiderable, and would have given no Disturbance to any but Sir *Isaac Newton*. But he being as nice in Practice as skilful in Theory, attempted another Set of Experiments in a deeper Vessel, *viz.* of 15,3 Feet or 183,6 Inches. I shall here give the Reader a View of the Event of his several Experiments in both Vessels in the following Table.

Experiment.	Weight in Air.	Weight in Water.	Number of Globes.	Times of their Descent in Experiments.					Time by the Theory.
In the Vessel 112 Inches deep.									
	Grains.	Grains.		′	′	″			″ ‴
1	$76\frac{1}{3}$	$5\frac{3}{16}$	3	15	15	15			14:48
2	121	1	3	46	47	50			40
In the Vessel 183 Inches deep.									
3	$139\frac{1}{4}$	$7\frac{1}{8}$	4	$19\frac{1}{2}$	50	51	53		50
4	$54\frac{3}{8}$	$21\frac{1}{2}$	4	$28\frac{1}{2}$	29	$20\frac{1}{2}$	30		29
5	$212\frac{3}{8}$	$79\frac{1}{8}$	5	15	$15\frac{1}{2}$	16	17	18	15
6	$293\frac{1}{8}$	$35\frac{7}{8}$	4	$29\frac{1}{2}$	30	$30\frac{1}{2}$	31		28
7	139	$6\frac{1}{2}$	4	50	$50\frac{1}{2}$	51	52		52
8	$273\frac{1}{4}$	$140\frac{3}{4}$	4	12	$12\frac{1}{2}$	12	13		$11\frac{1}{3}$
9	384	$192\frac{1}{2}$	4	$17\frac{1}{4}$	18	$18\frac{1}{2}$	19		$15\frac{5}{9}$
10	48	4	3	$44\frac{1}{2}$	45	46			$46\frac{5}{9}$
11	141	$4\frac{3}{8}$	3	63	64	65			$64\frac{1}{2}$

44. In all the Experiments of the deep Veſſel, the Times are expreſſed in *Half Seconds*, that is, in all but the two firſt. And the ſmall Differences from the Theory, Sir *Iſaac* very eaſily accounts for from the oſcillatory Motion with which thoſe which deſcended moſt ſlowly were found to move, as in the 5th, 8th, and 9th Experiments. But in thoſe which deſcended more ſwiftly, the Times very nearly correſpond with the Theory. Theſe Experiments were made in Water; others were made in Air, to ſhew that the Phænomena agree with the Theory in an elaſtic and rare Medium, as well as in a non-elaſtic and denſe one.

45. In order to this the Denſity of Air to that of Rain Water is aſſumed as 1 to 860; and 5 Globes formed of Hog's Bladders, blown in a wooden Mould, while green, and taken out when dry, were let fall from the Dome of St. *Paul*'s to the Pavement, from the Height of 272 Feet, and the Time meaſured by Half Second Pendulums; the Event of all the five Globes is ſhewn in the following Table. I ſhall here ſubjoin a Calculation for one of them, *viz.* the 3d, as follows.

46. The Diameter of this Globe was $5{,}3 = D$, and it weigh'd 137,5 in Air. A Globe of Air of the ſame Diameter weighs 23 Grains, very nearly; therefore $137{,}5 + 23 = 160{,}5$ Grains, the Weight of the Globe *in Vacuo*. Wherefore, as $23 : 160{,}5 :: \frac{8}{3} D : 2S = 98{,}626$. Again, as $160{,}5 : 137{,}5 :: 193\frac{1}{3} : 165{,}628$ = Inches the Bladder would fall in the Air in one Second. But with the ſame relative Weight $137\frac{1}{2}$, it will *in Vacuo* deſcend thro' the Space $S = 49{,}313$ in the Time $0'',5456 = T$, and acquire the greateſt Velocity with which it can deſcend in Air. The Time of the Fall was $18\frac{1}{2}''$, in which with that Velocity the Globe would deſcribe 278 Feet 8 Inches by an uniform Motion; for $0'',5456 : 98{,}626$ Inches $:: 18\frac{1}{2}'' : 278\frac{2}{3}$ Feet. Laſtly, ſubduct $1{,}3863\ S = 5\frac{2}{3}$ Feet, and there remains 273 Feet, though in the Table, by a more accurate Calculation, it is but 272 Feet, 7 Inches, and by the Experiment it was 272 Feet.

47. The following Table ſhews, in the firſt Column, the Weight of each Globe or Bladder; the ſecond its Diameter; the third the Time in which they ſeverally deſcended through the Height of 272 Feet; the fourth ſhews the Spaces they ſhould have deſcended through by the Theory; and the laſt Column ſhews the Difference between the Theory and Experiments.

The Weight in Grains.	*Diameter in Inches.*	*Times of the Fall.*	*Spaces per Theory.*		*Difference.*	
			Feet.	Inch.	Feet.	Inch.
128	5,28	19″	271 —	11	0 —	1
156	5,19	17	272 —	$0\frac{1}{2}$	0 —	$0\frac{1}{2}$
$137\frac{1}{2}$	5,3	$18\frac{1}{2}$	272 —	7	0 —	7
$97\frac{1}{2}$	5,26	22	277 —	4	5 —	4
$99\frac{1}{8}$	5	21	282 —	0	10 —	

48. According to this Theory, having found the Reſiſtance which a Globe meets with in deſcending in any reſiſting Medium, agreeing to its greateſt Velocity, it is eaſy then to find its Reſiſtance for any leſſer Velocity propoſed; becauſe the Reſiſtance for any given Velocities, will be as the Squares of thoſe Velocities.

49. The Globe has been hitherto ſuppoſed to begin its Motion in the reſiſting Medium from a State of Reſt; but we may ſuppoſe it alſo projected into ſuch a Medium, or to enter it with a certain Degree of Motion or Velocity; and in this Caſe we may determine the Velocity, the Reſiſtance, and the Space deſcribed in the Medium, from any given Time by knowing the Denſity of the Globe and Medium, and the Velocity with which it enters it; as alſo what Part of its Motion the Globe ſhall have loſt in that Time.

Pl. XVI. Fig. 7. 50. But in order to this, we muſt take to our Aſſiſtance the *Hyperbola*. Therefore let AB = Time in which the Globe would loſe its whole Motion by the Reſiſtance it meets with (upon entering the Fluid,) if uniformly continued. In the Points A and B erect the Perpendiculars AD and BC; and let BC repreſent the Motion or Velocity, and alſo the Reſiſtance of the Globe at its Entrance into the Fluid; thro' the Point C, let the Hyperbola be deſcribed to the Aſymptotes AD and AB continued to any Point E; in E erect the Perpendicular EF meeting the Hyperbola in F; compleat the Parallelogram CBEG; and draw AF cutting BC in H.

51. Then if the Globe in any Time BE deſcribes *in Vacuo* the Space repreſented by the Parallelogram BCGE by its firſt Velocity BC uniformly continued, it will in a reſiſting Medium deſcribe the hyperbolic Space or Area BCFE; and its Motion or Velocity at the End of the Time BE will be repreſented by the Ordinate EF, and the Part loſt by FG. Laſtly, its Reſiſtance at the End of that Time will be BH, and the Part loſt HC.

52. For $BC^2 : EF^2 :: BC : \frac{EF^2}{BC}$ = the Reſiſtance at the End of the Time BE, as BC is that at the Beginning. And from the Nature of the Hyperbola, BC : EF :: AE : AB; and by ſimilar Triangles AE : AB :: EF : BH; wherefore $HB = \frac{EF^2}{BC}$, the Reſiſtance at the End of the Time BE.

53. Hence if the Globe in any Time T loſes its whole Motion M by the Reſiſtance R uniformly continued;

tinued; the same Globe shall in a resisting Medium, by the Resistance R decreasing in the duplicate Ratio of the Velocity, in the Time t lose the Part $\frac{tM}{T+t}$ of its whole Motion M; and then $M - \frac{tM}{T+t} = \frac{TM}{T+t}$ will be the Part of the Motion remaining. For let m be the residual Motion, then $T : t :: AB : BE$ (by *Art.* 50, 51.) and hence $T+t : T :: AE : AB$; and moreover $M : m :: CB : FE :: AE : AB$; whence $T+t : T :: M : m$; consequently $m = \frac{TM}{T+t}$.

54. Lastly, the Space (S) described in the resisting Medium in the Time t will be to the Space (2S) described by the uniform Motion M, as the Logarithm of the Number $\frac{T+t}{T} \times 2{,}302585$ to $\frac{t}{T}$. For let $AB = a$, $BC = b$, $BE = x$, and $AE = a+x$, and L=Logarithm. Then (*per* Conics) the Area $BCEF = ab \times L\frac{a+1}{a}$, and the Rectangle $BCGE = bx$, whence $s : 2S :: ab \times L\frac{a+x}{a} : bx$; and dividing by ab, we have $s : 2S :: L\frac{a+x}{a} : \frac{x}{a} :: L\frac{T+t}{T} : \frac{t}{T}$; but the Expression $L\frac{T+t}{T}$, as it here stands, is an hyperbolical Logarithm, which is to the common tabular Logarithm as 2,302585 to 1. Consequently the tabular $L\frac{T+t}{T} \times 2{,}302585$=Logarithm sought.

55. The Spaces, Times, and Velocities of Bodies moving either by their *Vis insita* alone, or conjointly with an accelerating Force, as Gravity; in resisting Mediums of various Density and Ratio of Resistance, may be analytically investigated as follows. Let the initial Velocity, or that with which it begins its Motion in the Fluid be V, the Time t, the Space described s, and the Velocity at the End of the Time v, the Density of the

the Medium d, and the Resistance r as $\frac{dv^n}{a^n}$. Then (by *Art.* 14.) $r\dot{s} = -\dot{v}v$, wherefore $\frac{dv^n\dot{s}}{a^n} = -\dot{v}v$; and so $d\dot{s} = a^n\dot{v}v^{1-n}$.

56. Whence if the Density be uniform, as of Water, then $d=1$. And if $n=2$, then $\dot{s} = a^2\dot{v}v^{-1}$, $= -\frac{a^2\dot{v}}{v}$. Now s is the Fluent of the Fluxion $\dot{s}$, and the Fluent of $-\frac{a^2\dot{v}}{v}$ is $Q-a^2Lv$, where Q is some constant Quantity, to find the Value of which, suppose $s = Q-a^2Lv=o$, then $Q=a^2Lv$, but when $s=o, v=V$; therefore $Q=a^2LV$. Wherefore $s=a^2LV-a^2Lv= a^2L\frac{V}{v}$.

57. The Time t is found by the *Formula* in *Art.* 10. where we had $\dot{t} = \frac{\dot{s}}{v}$, but because when d is given, $\dot{s} = a^n\dot{v}v^{1-n}$ (*Art.* 55.) therefore $\dot{t} = \frac{\dot{s}}{v} = \frac{a^n\dot{v}v^{1-n}}{v}$ $= a^n\dot{v}v^{-n}$. Whence taking the Fluents we have $t = \frac{Q-a^nv^{1-n}}{1-n}$; and putting $t=o$, we have $v=V$, wherefore $Q=a^nV^{1-n}$; consequently substituting for Q its Value, we have $t = \frac{a^nV^{1-n}-a^nv^{1-n}}{1-n}$. Hence, if $n=2$, $t = \frac{a^2V-a^2v}{Vv} = a^2 \times \frac{V-v}{Vv}$.

58. We had above $\dot{s} = a^n\dot{v}v^{1-n}$; and taking the Fluents on both Sides, we have $s = \frac{Q-a^nv^{2-n}}{2-n} = \frac{a^nV^{2-n}-a^nv^{2-n}}{2-n}$. Let $2-n=m$; then $ms = a^nV^m - a^nv^m$; hence $a^nV^m - ms = a^nv^m$; therefore $v = \frac{\overline{a^nV^m-ms}^{\frac{1}{m}}}{a^{\frac{n}{m}}}$. Which Theorem therefore gives the Velocity when the Space s is known.

59. In these Theorems we see the Agreement with the former Methods of determining these Quantities. Thus the Space by Theorem in *Art.* 56. is $s = a^2 \, L \frac{V}{v}$

$2{,}302585 \, L \frac{T+t}{T}$ (in *Art.* 54.) = BCFE, the Hyperbolic Space in the Figure. Whence, in this Case, $a^2 = 2{,}302585 \times \frac{8}{3} D$.

60. To determine the Space s which the Globe shall have passed by its *Vis insita* when it has lost half its Velocity or Motion, that is, when $V : v :: 2 : 1$; we have $L \frac{V}{v}$, or the Logarithm of $\frac{2}{1} = 0{,}301030$, which multiplied by 2,302585 gives 0,6931; and this again, multiplied by $\frac{8}{3} D = 2{,}6 D$, gives $1{,}84832 D = s$, which is not quite 2 Diameters.

61. The Time t being always as $a^2 \times \frac{V-v}{Vv}$, or (because a^2 is constant) as $\frac{V-v}{Vv}$; therefore when $v = \frac{1}{2} V$, we have $\frac{V-v}{Vv} = \frac{1}{2} = t$, which shews the Time in which the Globe loses half its Motion is half the whole Time; that is, BE=AB (in the Figure) the Time in which by an uniform Velocity B C it would describe $\frac{8}{3}$ Parts of its Diameter.

62. If the Medium be similar, or of an uniform Density, where $d = 1$; and if the Resistance be in the simple Ratio of the Velocity, then $n = 1$; and by the Theorems above, $s = aV - av$, and $t = a \, L \frac{V}{v}$, and $v = V - \frac{s}{a}$.

63. If in the foregoing Equations the Velocity $v = o$, then $s = \frac{a^n V^{2-n}}{2-n}$, when n is less than 2. But when $n = 2$, $s = a^2 \, L \frac{V}{o}$ = Infinite; if n be greater than 2, then also $s = \frac{a^n V^{n-2} - a^n v^{n-2}}{\overline{n-2} \times V^{n-2} \times v^{n-2}} = \frac{a^n}{\overline{n-2} \times o}$ = Infinite. That

is,

is, there is but one Case in which the Globe moving in the Fluid can lose its Velocity, or come to a State of Rest in describing a finite Space, and consequently in a finite Time, *viz.* when n is less than 2. In all other Cases the Motion will never be totally destroyed. And when $n = 1$, then, though the Space be finite, *viz.* $s = aV$, yet the Time of describing it will be infinite, *viz.* $t = a\, \mathrm{L}\, \frac{V}{o} =$ Infinite.

64. After a like Manner we may raise Theorems for the Spaces, Times, and Velocities of a Body descending by the Force of Gravity in any resisting Medium. For suppose the Medium of an uniform Density, and the Resistance in any multiplied Ratio of the Velocity, as $r = \frac{v^n}{a^{n-1}}$; then for a descending Body we have (by *Art.* 16.) $c\dot{s} - r\dot{s} = v\dot{v}$, and therefore $c\dot{s} - \frac{v^r \dot{s}}{a^{r-1}} = v\dot{v}$; whence $\dot{s} = \frac{v\dot{v}\, a^{n-1}}{c\, a^{n-1} - v^n}$.

65. Also because (by *Art.* 10.) $\dot{t} = \frac{\dot{s}}{v}$, therefore we have $\dot{t} = \frac{\dot{v}\, a^{n-1}}{c\, a^{n-1} - v^n}$. And in like Manner the Theorems for ascending Bodies are $\dot{s} = \frac{-v\dot{v}^{n-1}}{c\, a^{n-1} + v^n}$, and $\dot{t} = \frac{-\dot{v}\, a^{n-1}}{c\, a^{n-1} + v^n}$.

66. If the Resistance be as the Velocity, then $n=1$; and in the descending Body $\dot{s} = \frac{v\dot{v}}{c - v} = -\dot{v} + \frac{c\dot{v}}{c - v}$; the Fluents of which are $s = Q - v - c\,\mathrm{L}.\,\overline{c - v}$, but since when $s = o$ it is $v = V$, therefore then $Q = V + c\,\mathrm{L}.\,\overline{c - V}$; and so $s = V - v + c\,\mathrm{L}.\,\frac{c - V}{c - v}$. Also the Time is had from the Equation $\dot{t} = \frac{\dot{v}}{c - v}$; of which the Fluents are $t = Q - \mathrm{L}\,\overline{c - v}$

$= \mathrm{L}$

$= L = \frac{c-V}{c+v}$. And for ascending Bodies $s = V - v$ $+ c L \frac{c-V}{c-v}$; and $t = L \frac{c+V}{c+v}$.

67. If the Resistance be as the Square of the Velocity, then $n = 2$, and $r = \frac{v^2}{a}$. Let $V =$ the greatest Velocity the Body can acquire by descending in the Fluid, and because then the Resistance is equal to its Gravity or Force of its Weight (by *Art.* 36.) we have $c = \frac{V^2}{a}$, (for in that Case $v = V$,) whence $rc = V^2$. Let S = Space described *in Vacuo* to acquire the Velocity V; then because (by *Art.* 17.) in that Case $c\dot{S} = V\dot{V}$, therefore $cS = \frac{1}{2}VV$, and so $2cS = VV = ac$, consequently $a = 2S$.

68. Therefore since (*Art.* 64.) $\dot{s} = \frac{av\dot{v}}{ac - v^2} = \frac{2Sv\dot{v}}{V^2 - v^2}$ and putting $Vv - vv = xx$, and taking the Fluxions $v\dot{v} = -x\dot{x}$, and therefore $\dot{s} = -\frac{2Sx\dot{x}}{xx} = -\frac{2S\dot{x}}{x}$; and taking the Fluents, $s = Q - 2SL. x = Q - SL. x^2 = Q - SL. \overline{V^2 - v^2}$. Wherefore when $s = 0$, we have $v = V$, the initial Velocity, then $Q = SL. VV - V^2$. And consequently $s = SL. \frac{V^2 - V^2}{V^2 - v^2}$.

69. Let $L. h = 1$; then $sL. h = SL. \frac{V^2 - V^2}{V^2 - v^2}$; $\frac{s}{S}$ $Lh = Lh^{\frac{s}{S}} = L\frac{V^4 - V^2}{V^2 - v^2}$, therefore $h^{\frac{s}{S}} = \frac{V^2 - V^2}{V^2 - v^2}$ whence we get $v^2 = \frac{V^2 h^{\frac{s}{S}} + V^2 - V^2}{h^{\frac{s}{S}}}$. Wherefore the Velocity v is from thence easily found.

70. The Times t is obtain'd by the Equation $t = \frac{a\dot{v}}{t - vv}$ (in *Art.* 65.) $= \frac{2\,S\,\dot{v}}{V^2 - v^2} = \frac{\frac{S}{V}\dot{v}}{V+\dot{v}} + \frac{\frac{S}{V}\dot{v}}{V-\dot{v}}$; and taking the Fluents we have $t = Q + \frac{S}{V}L.\overline{V+v} - \frac{S}{V}L\,\overline{V-v} = Q + \frac{S}{V}L.\frac{V+v}{V-v}$; and putting $t = o$, and so $v = V$, we shall find $Q = -\frac{S}{V}L.\frac{V+V}{V-V}$; and therefore at length we have $t = \frac{S}{V}L.\frac{\overline{V+v} \times \overline{V-V}}{\overline{V-v} \times \overline{V+V}}$.

71. If the Body descends in the Fluid from a State of Rest, then $V = o$; and so the Space $s = S\,L.\frac{V^2}{V^2 - v^2}$; the Time $t = \frac{S}{V}L.\frac{V+v}{V-v}$; and the Velocity

$$v = \sqrt{\frac{V^2\, h\frac{s}{S} - V^2}{h\frac{s}{S}}}.$$

And in like Manner are found the Equations for the Spaces, Times, and Velocities of Bodies ascending in a resisting Medium, which are the same with these having only the Signs of V and v changed.

72. Thus I have given a Specimen of the several Ways that have been used to represent and compute the Motion of Bodies moving in resisting Mediums of any Density, and according to any Law of Resistance. The Theorems in the latter Articles express most of the Cases in *Prop.* I, II, III, VI, VIII, XI, of Sir *Isaac Newton*'s second Book of *Principia*; I could have inserted many more, but have already extended this Note to a great Length; and shall refer the Reader to the admirable Commentary on our Author by Messrs. *Le Seur* and *Jacquier*, from whence I have collected most of these Articles.

73. I shall conclude with inserting the Table refer'd to in the foregoing Computations, which is as follows.

Time of the Fall.	*Velocities acquired in the Fluid.*	*Spaces described by falling in the Fluid.*	*Spaces described with the greatest Velocity.*	*Spaces described by falling in Vacuo.*
0,001 T	99999$\frac{29}{30}$	0,000001 S	0,002 S	0,000001 S
0,01 T	999967	0,0001 S	0,02 S	0,0001 S
0,1 T	9966799	0,009983 S	0,2 S	0,01 S
0,2 T	19737532	0,039736 S	0,4 S	0,04 S
0,3 T	29131261	0,088681 S	0,6 S	0,09 S
0,4 T	37994896	0,155907 S	0,8 S	0,16 S
0,5 T	46211716	0,240229 S	1,0 S	0,25 S
0,6 T	53704957	0,340270 S	1,2 S	0,36 S
0,7 T	60436778	0,454540 S	1,4 S	0,49 S
0,8 T	66403677	0,581507 S	1,6 S	0,64 S
0,9 T	71629787	0,719660 S	1,8 S	0,81 S
1 T	76159416	0,867561 S	2 S	1 S
2 T	96402758	2,650005 S	4 S	4 S
3 T	99505475	4,618657 S	6 S	9 S
4 T	99932930	6,614376 S	8 S	16 S
5 T	99990920	8,613796 S	10 S	25 S
6 T	99998771	10,613718 S	12 S	36 S
7 T	99999834	12,613707 S	14 S	49 S
8 T	9999980	14,613706 S	16 S	63 S
9 T	9999997	16,613705 S	18 S	81 S
10 T	9999999[illegible]	18,613705 S	20 S	100 S

The END *of the* FIRST VOLUME.

BOOKS printed for T. CARNAN and F. NEWBERY, junior, at N°. 65, in St. Paul's Church-yard, London.

NEW *Principles of Geography and Navigation*; in two Parts. Part 1. containing the Theory of the true Figure and Dimensions of the Earth, deduced from actual Mensuration, and applied to a just Construction of Maps and Charts for Land and Sea Use. The whole exemplified in a new geographical Chart for Europe, and a large Sea Chart to seventy Degrees of Latitude; both which are adapted to the spheroidical Figure of the Earth, and the Degrees of each are divided into Minutes, with a Solution of all the Cases of Sailing on this new Elliptic Chart. Part 2. containing a Table of Meridional Parts, calculated for the Spheroid, to every Minute of Latitude, from the Measure of a Degree at the Equator. By Don George Juan, and Don Antoine de Ulloa, with a Solution of the several Cases of Sailing by it. Also new astronomical Principles of Navigation, and an improved mechanical Theory of Working a Ship, with a Table of the Sun's Declination, and Place in the Ecliptic. By Benjamin Martin. Price 10s. 6d. half bound.

Lingua Britannica Reformata; or a new Universal English Dictionary, under the following Titles, viz.

1. Universal; Containing a Definition, and Explication of all the Words now used in the English Tongue, in every Art, Science, Faculty or Trade.

2. Etymological; Exhibiting and explaining the true Etymon or Original of Words, from their respective Mother Tongues, the Latin, Greek, Hebrew and Saxon; and their Idioms, the French, Italian, Spanish, German, Dutch, &c.

3. Orthographical; Directing the true Pronunciation of Words, by single and double Accents; and by indicating the Number of Syllables in Words where they are doubtful, by a numerical Figure.

5. Diacritical; Enumerating the various Significations of Words in a proper Order, viz. Etymological, Common, Figurative, Poetical, Humorous, Technical, &c. in a Manner not before attempted.

6. Philological; Explaining all the Words and Terms, according to the modern Improvements in the

various

various Philological Sciences, viz. Grammar, Rhetoric, Logic, Metaphysics, Mythology, Theology, Ethics, &c.

7. Mathematical; not only explaining all the Words in Arithmetic, Algebra, Logarithms, Fluxions, Geometry, Conics, Dialling, Navigation, &c. according to the modern Newtonian Mathesis; but the Terms of Art are illustrated by proper Examples, and Copper Plate Figures.

8. Philosophical; explaining all Words and Terms in Astronomy, Geography, Optics, Hydrostatics, Acoustics, Mechanics, Perspective, &c. according to the latest Discoveries and Improvements in this Part of Literature. By Benjamin Martin. Price bound 6s.

The History of Astronomy, with its Application to Geography, History and Chronology; occasionally exemplified by the Globes. By George Costard, M. A. Vicar of Twickenham in Middlesex. In one Volume, Quarto. Price 10s. 6d. bound in the Vellum Manner.

Chronological Tables of Universal History, Sacred and Profane, Ecclesiastical and Civil, from the Creation of the World to the Year 1743. With a Preliminary Discourse on the short Method of Studying History; and a Catalogue of Books necessary for that Purpose; with some Remarks on them. By Abbé Lenglet Dufresnoy. In two Parts. Translated from the last French Edition, and continued down to the Death of King George the Second. Price bound 12s.

A New and accurate System of Natural History, containing, 1. The History of Quadrupedes, including amphibious Animals, Frogs and Lizards, with their Properties and Uses in Medicine. 2. The History of Birds, with the Method of bringing up those of the singing Kind. 3. The History of Fishes and Serpents, including Sea-Turtles, Crustaceous and Shell Fishes; with their medicinal Uses. 4. The History of Insects, with their Properties and Uses in Medicine. 5. The History of Waters, Earths, Stones, Fossils, and Minerals; with their Virtues, Properties, and medicinal Uses: To which is added, the Method in which Linnæus has treated these Subjects. 6. The History of Vegetables, as well foreign as indigenous, including an Account of the Roots, Barks, Woods, Leaves, Flowers, Fruits, Seeds, Resins, Gums, and concreted

Juices;

Juices; as also their Properties, Virtues, and Uses in Medicine; together with the Method of cultivating those planted in Gardens. By R. Brookes, M. D. Author of the General Practice of Physic. In Six Volumes. Price bound 1 l. 1 s.

The Elements of Heraldry; containing a clear Description and concise historical Account of that ancient, useful, and entertaining Science.

The Origin, Antiquity, and divers Kinds of Coats of Arms, with their essential and integral Parts considered separately. The several Sorts of Escutcheons, Tinctures, Charges and Ornaments used for Coats of Arms. The Marks whereby Bearers of the same Coats of Arms are distinguished from each other. Charges formed of Ordinaries, Celestial Figures, Animals, Birds, Fishes, Vegetables, Artificial and Chimerical Figures. The Laws of Heraldry; practical Directions for Marshalling Coats of Arms, and the Order of Precedency. Embellished with several fine Cuts, and 24 Copper Plates, containing above 500 different Examples of Escutcheons, Arms, &c. and interspersed with the Natural History, and allegorical Signification of the several Species of Birds, Beasts, Fishes, &c. comprehended in this Treatise. To which is annexed a Dictionary of the Technical Terms made Use of in Heraldry. By Mark Anthony Porny, French Master at Eton College. Price bound 6 s.

Interest improved; Shewing from a small Table, by a New, Easy, and Concise Method, 1. Rules for finding the Number of Days from one Month to another, &c. in all the different Cases. 2. The Interest for any Sum, Rate and Number of Days; likewise its Use in Discompt; also what any annual Sum is per Day, and daily Sum per Annum, &c. 3. A new Method of obtaining the Parts expressing Money; Interest for Years; with the Solution of several new Questions in Stocks, Brokerage, &c. 4. A Supplement, giving the Construction of the Table; where, by one easy Process only, the same is performed by the Pen. Making the whole useful, for all Academies, Schools, Merchants, Brokers, Clerks, and Accomptants. By Charles Brent. Price bound 2 s.

[illegible] together with [illegible]
[illegible]
[illegible] of the General [illegible]
[illegible] Price [illegible]

[illegible] *[illegible]*
[illegible] article [illegible]
[illegible] containing Some [illegible]
[illegible] [illegible]
[illegible] the [illegible]
[illegible] Tables [illegible]
[illegible] Charges and [illegible]
[illegible]
[illegible]

[illegible]
[illegible]
[illegible] lowest [illegible]
[illegible] containing [illegible]
[illegible] &c.
[illegible]
[illegible]
[illegible]

[illegible] Perry, [illegible]
[illegible]

[illegible]
[illegible]